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IMPLEMENTATION OF THE MODERN DIMENSIONAL ANALYSIS IN ENGINEERING PROBLEMS - BASIC THEORETICAL LAYOUTS

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Abstract: The authors analyze the ways how the results of the measurements obtained on a model can be transferred to the prototype. Here the advantages and the limits of the most known classical methods, as Geometrical Analogy, Theory of Similitude, and as well as the Classical Dimensional Analysis are evaluated. Taking into consideration that the above-mentioned methods have several difficulties, the authors choose that version of Dimensional Analysis, which was developed by the author's of contributions, named in the following as Modern Dimensional Analysis. This method offers a simple, easy and secure engineering approach, due to the fact, that it assures the establishing of: all dimensionless parameters involved in the analyzed phenomenon, the Model Law, as well as of the Dimensional Modelling, by a unitary and simple manner. Consequently, using this method, it became possible to foresee the prototype's behaviour with good accuracy, based on a reduced number of experimental investigations performed on the adequate model. In the succeeding parts of this contribution, the authors will describe how this efficient method in the modelling of static and dynamic calculi can be applied.
Keywords: geometric analogy, theory of similitude, dimensional analysis, modelling

1. INTRODUCTION

In the last period, due to its incontestable advantages, the extrapolation of experimental measurements' results obtained on model became widely applied instead of the directly measurements on prototype. In the case of a great size prototype, the direct measurements on it reveal several obvious difficulties in comparison with the reduced scale model's ones, e.g.:

- Greater costs;
- More difficult operating conditions;
- More numerous high-qualified operators, as well as
- Difficulties like environmental conditions, adequate enclosure for the measurements, etc. within test reproducibility.

On the contrary, for relative small prototypes, a larger scaled model assures more suitable conditions for the measurements, like easier access in the corresponding narrow areas of the prototype.

In order to obtain an adequate correspondence between the prototype's and the model's behaviours, during the last decades several theoretical approaches, starting from the Geometric Analogy (GA), Theory of Similitude (TS), ending with the so-called Classical Dimensional Analysis (CDA) [1; 2; 3; 4; 5; 6; 10; 11; 12; 13] were conceived.

In the case of the GA, the basic set consists of geometric similitude between the model and prototype, providing the proportionalities of their lengths and angular equalities.

Consequently, homologous points, surfaces and volumes of the prototype with the model [1; 2; 10; 11; 13] are defined.

TS pays attention to the basic features of the functional similarity, accepting development of similar processes at prototype and model, i.e. in homologous times, in homologous points the following phenomenon will be observed [1; 2; 5; 10; 11; 12; 13]. This guarantees individually each measured physical quantities ω constant ratios S_ω, named Scales Factor of physical quantities, on the model (ω_2), respectively on prototype (ω_1)

$$S_\omega = \frac{\omega_2}{\omega_1}[-] \qquad (1)$$

Here, instead of an effective analysis of the complex mathematical relationships, one will look for finding useful correlations between dimensionless quantities (i.e.: criteria relationships), through forming adequate groups of the physical quantities (named similitude criteria). These criteria relationships, based on the effective measurements on model, will provide the prediction of the prototype's response, the simplicity of the phenomenon analysis, as well as the diminishing of the necessary experimental investigations' number.

One has to mention, that the worked out criteria relationships for a particular phenomenon can't be applied for another one.

Unfortunately, in the case of complex phenomena, the TS became too difficult.

For enhancement of these comparative investigations prototype-model, the so-called Classical Dimensional Analysis (CDA) [1; 2; 3; 4; 5; 6; 10; 11; 12; 13] was developed.

Based on the Buckingham's theorem and involving both the π_j dimensionless groups and their $S\omega$ Scale Factors, one will obtain not only the maximal number of these groups, but also simplicity of the phenomenon's analysis even in comparison with TS by diminishing of the necessary number of the experimental investigations and their graphical representation.

By the means of the experimental investigations' results, performed on the model, one will obtain the numerical value for each π_j dimensionless group. Through preserving these numerical quantities constant and changing the involved physical quantities individually, it became possible to perform numerous virtual experiments.

One has to underline several disadvantages of TS and CDA related mainly to the obtaining of these π_j dimensionless groups, such as:

- It requires a high-level knowledge of the analysed phenomena;
- Their conceiving is a relatively arbitrary process and it is exclusively based on the ingenuity of the specialists;
- This process of obtaining the complete set of these π_j dimensionless groups represents a result of an assiduous work and it will rarely happen.

This is the reason why a unified and easy method is advised, which seems to be in the authors' opinion the named Modern Dimensional Analysis (MDA) developed by the author of the contributions [8; 9], briefly presented in the following.

2. BASIC THEORETICAL LAYOUTS OF THE MODERN DIMENSIONAL ANALYSIS (MDA) [8; 9]

In this case only a well-grounded knowledge of the involved physical variables and not of the analysed phenomenon is necessary, having a more or less influence on this process.

The main steps of MDA are the following:

- The enumeration (specification) of the whole potentially involved variables ($V_2, V_3, \ldots, V_h$), as well as of the so-called basic dependent variable V_1, which presents the main interest in this process-evaluation (i.e.: displacement, eigen-frequency, etc.)
- The setting of their dimensions $(d_1, d_2, \ldots)$;
- The formation of the so-called Dimensional Matrix (DM), has within its columns the whole variables $V_1, V_2, \ldots V_h$, as follows: compulsory, in the first column the basic dependent V_1 and their lines constituted from the exponents of the involved dimensions of the mentioned variables;
- Starting from the upper right corner of DM, we are looking for the maximum rank's invertible matrix , denoted by A;
- The variables included in matrix A will represent (not only for the prototype but also for the model, too) the so-called independent variables, with their dimensions: the main dimensions;
- The remaining lines, which aren't included in matrix A, will be omitted and the corresponding variables (named dependent variable) will build-up the matrix B;
- In the case, that by the operation of the neglecting of the supplementary lines some of the dependents variables cannot be described by the main dimensional set, then one has to reconsider the former main dimensional setup to obtain an uniform dimensional set for all variables;
- In this way one will obtain the so-called Reduced Dimensional Matrix (B+A);
- One will define the Dimensional Set, built from the Reduced Dimensional Matrix (B+A), completed with matrix $C = -\left(A^{-1} \cdot B\right)^T$, as well as one adequate, n-ranked unit matrix $D \equiv I_{nxn}$;
- In the following, one will define the Dimensional Lot, with its parts, described by relation (2)

As many lines remained as main dimensions $k = N_d$ after defining matrix A	1. 2. 3. 4. ... k.	B	A	
So many lines as dependent variables (columns) has the matrix B (i.e.: n); The number of these lines is identical with the $\pi_j\ \ j = 1, 2, \ldots, n$ dimensionless variables' number	1. 2. 3. 4. ... ... n.	$D \equiv I_{nxn}$	$C = -\left(A^{-1} \cdot B\right)^T$	(2)

One has to mention that the formula

$$C = -\left(A^{-1} \cdot B\right)^{T}, \tag{3}$$

is valid only for the case when all π_j are dimensionless variables and D is an unit matrix ($D \equiv I_{nxn}$);

- Using the lines $j = 1, 2, ..., n$ of the D and C matrixes one can determine the whole lot of the requested π_j $j = 1, 2, ..., n$ dimensionless variables in the following manner: each line contains the exponents of one dependent variable (from the corresponding column of matrix B, having upper index equal with 1) and of all the independent ones (from the corresponding columns of the matrix C); each π_j dimensionless variable will be equal with the product of these variables existent in the mentioned line;

Useful observations:

- In contributions [8; 9] the possibilities, how these π_j dimensionless variables' analysis can be simplified are indicated;
- The set of the whole $\pi_j = 1$ $j = 1, 2, ..., n$ dimensionless variables represents the Model's Law, where each initial ω variable will be substituted by their S_ω Scale Factors and from each of these new relations the corresponding dependent variables will be expressed;
- The obtained Model Law assures the precise manner of how to obtain the dependent variables magnitudes for the model (because for the prototype they are given a priori);
- The variables order within the matrices A and B do not influence the obtained Model Law;
- One can conclude, that the briefly described MDA method has the following main advantages, due to the fact that it assures:
  - » the whole set of the (possible) variables, which can influence the analysed phenomena in some manner, is to be included in the Dimensional Set; at a subsequent step the dispensable variables can be eliminated;
  - » the easiest-, and unitary way of prediction of the complete set of the $\pi_j = 1$ $j = 1, 2, ..., n$ dimensionless variables and of the Model Law, which cannot be obtained regularly nor by TS not by CDA approaches;
  - » the ruling-out of those practically random grouping of the physical variables in order to obtain the dimensionless variables, applied in TS and CDA; how it was mentioned before, the TS and CDA assume a very solid theoretical knowledge of the analyzed phenomena;
  - » the obtaining of the best results without solid theoretical background of the phenomena, because in this case of the MDA only the identification of the possible variables which can have some influence on the analyzed phenomena is requested;
  - » simulation of a broad variety of virtual experiments by keeping constants the S_ω Scale Factors of the involved independent variables in the Model Law's expressions;
  - » An easier, and much simplified graphic representation of the phenomena, because a single nomographic chart can be equivalent with a large number of curves from the TS or CDA.
- A variable is dimensionally irrelevant, if in matrix C, its corresponding column, contains only zero values; this variable can be eliminated from the Dimensional Set without changing the final expressions of the Model Law;
- A variable is physically irrelevant, if its contribution is much less than the other ones to the analysed phenomena; this variable also can be eliminated from the Dimensional Set without changing the final expressions of the Model Law;
- The dimensional similarity assumes that all $\pi_j = 1$ $j = 1, 2, ..., n$ dimensionless variables, involved in the analysed phenomena (both at the prototype and the model) are respectively identically not only in their mathematical expressions, but also in their numerical values; by adequately selecting the variables from matrix A, MDA offers several facilities, which can assure a good dimensional similarity between the prototype and model without geometric similarity;
- The dimensional similarity represents the base of the Dimensional Modelling, runned by the Model Law;
- The author of the contributions [7; 8] classifies the involved variables in the Dimensional Modelling in three categories:
  - » Category I - their values are either a priori given or can be freely chosen in advance of starting the modelling process;
  - » Category II – their values are obtained only by Model Law;
  - » Category III – their value are obtained by effective measurements on the model.
- The diminishing of the variable Category II represents the main path of the Dimensional Modelling optimisation, which will be illustrated in the next issue of the authors.

In order to optimize the modelling strategy, the author of the mentioned contributions recommends a synthesis of the obtained data in a tabular form (see below).

Table 1. Synthesis of the obtained data

Variable					Scale Factor	Variable Category	
Name	Symbol	Dimension	Value		Model/ /prototype	Prototype	Model
			Prototype	Model			
V_1							
V_2							
V_3							
...							
V_m							
Dimen-sionless variables	π_1						
	π_2						
	...						
	π_n						
Categories of Variables	I.	Given or freely chosen					
	II.	Obtained by means of Model Law					
	III.	Determined by effective measurements on the model					

3. CONCLUSIONS

— The authors presented the main advantages of the MDA in outlining the process of the prototype's behaviour through the behaviour of the model;

— In the authors opinion, the MDA represents the easiest, most advanced and unitary way to predict the complete set of the $\pi_j = 1 \quad j = 1, 2, ..., n$ dimensionless variables and of the Model Law, in order to find out, with good accuracy a useful correlation between the prototype's and the model's behaviour;

— Also, this method allows an exclusive model-focused smaller number of simplified experimental investigations;

— By a careful laying out of the Dimensional Set, MDA offers several facilities, which can assure a good dimensional similarity between the prototype and model without geometric similarity;

— The MDA can be applied with the same efficiency both for static and dynamic processes, which will be illustrated in the next issue of this work.

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