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THE APPLICATION OF THE MODERN DIMENSIONAL ANALYSIS ON STATIC CALCULI – THE DISPLACEMENT EVALUATION OF/IN A STRAIGHT BAR

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Abstract: In the authors' previous contributions the theoretical layouts of the Modern Dimensional Analysis (MDA) were described, based on the main references. How it was illustrated, the described MDA method can be applied in many engineering problems. In this contribution, the authors illustrate the main advantages of this method in a static case, namely in the displacement's evaluation of a straight bar, loaded by a concentrate force, perpendicular-disposed to its longitudinal axis. Several variants based on different suppositions of the Dimensional Set will be analysed, mainly how one can toggle the main variables from the whole potentially involved variables, which have some influence on the analysed phenomena. Finally, it will be proved, that one can find such Dimensional Sets, in which the corresponding sets of the independent variables can assure dimensional similarity between the prototype and model without geometric similarity. It can represent one of the main advantages of the MDA method.
Keywords: Modern Dimensional Analysis (MDA), static case, displacement's evaluation, straight bar

1. THE PROPOSED STUDY: THE FREE END'S VERTICAL DISPLACEMENT OF A CANTILEVER BEAM SUBJECTED TO A VERTICAL CONCENTRATED LOAD

A steel cantilever beam (a straight bar fixed on the end and free in the other) is given, having length L, a rectangular cross-section $(a \cdot b)$, loaded by a vertical force F (perpendicular disposed to its longitudinal axis) and related to a tri-rectangular system of axis $xGyz$ (Figure 1).

This beam, having relatively large dimensions, is considered to represent the prototype. Our goal consists in finding out the most suitable dimensional analysis model, on which one can perform measurements, in order to predict accurately the prototype's free-end's vertical displacement „ v_1 ".

How it was mentioned in the contributions [7; 8] as well as in [12], all parameters related to the prototype are indexed with "1" and those to the model with "2".

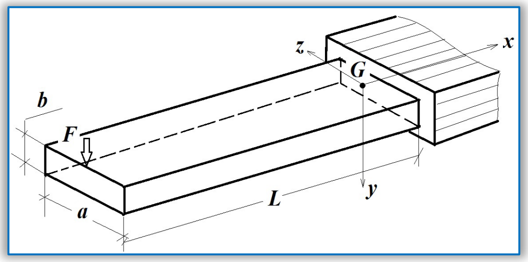

Figure 1. The analysed static-loaded cantilever beam

In order to forecast the requested vertical displacement of the prototype „ v_1 ", the advantages of the Modern Dimensional Analysis (MDA) will be applied. Here, by setting up the most suitable version of the Model Law, it became possible, by measurements on model of the corresponding displacement „ v_2 ", to obtain the magnitude of „ v_1 "by calculus.

2. DIMENSIONAL SETS PROPOSED BY AUTHORS

In the first approach such reasonable variables as (v, E, L, F, a, b, g) were accepted, as well as the lengths' separation along the main rectangular directions (x, y, z), i.e. m_x, m_y, m_z was applied [7; 8], in order to increase the number of the involved dimensions and to diminish of the dimensionless variables π_j in the Model Law,.

Based on the theoretical layouts, presented in [7; 8] as well as in [12], finally the first version (V1) of the Dimensional Set was obtained, where as independent variables (L, F, a, b, g) were selected, contained in matrix A, as well as the dependent ones (v, E), located in matrix B. Also one can observe the obtained matrixes C and D, as a result of the regular calculi from the mentioned contributions.

		B		A				
		v	E	L	F	a	b	g
V1	m_x	0	1	1	0	0	0	0
	m_y	1	-1	0	1	0	1	1
	m_z	0	-1	0	0	1	0	0
	kg	0	1	0	1	0	0	0
	s	0	-2	0	-2	0	0	-2
	π_1	1	0	0	0	0	-1	0
	π_2	0	1	-1	-1	1	2	0
		D		C				

One can observe that in matrix C, the column of the variable "g" contains only zero values. Consequently, this variable is physically irrelevant, so it can be eliminated definitively from matrix A, due to the fact that it will not influence the obtained Model Law.

Taking the above-mentioned fact into consideration, the obtained dimensionless variables (π_1, π_2) were expressed. From there on, one will equalize each one of them with "1", i.e. $\pi_j = 1,\ j = 1,2$ and express the corresponding dependent variable from them. As a next step, the involved variables ω will be substituted with their Scale Factors S_ω, so one will obtain the constitutive elements of the Model Law:

$$\pi_1 = v \cdot b^{-1} = v / b \Leftrightarrow S_v = S_b \tag{1}$$

$$\pi_2 = \frac{E \cdot a \cdot b^2}{L \cdot F} \Leftrightarrow S_F \cdot S_L = S_E \cdot S_a \cdot (S_b)^2 \Rightarrow S_E = \frac{S_F \cdot S_L}{S_a \cdot (S_b)^2}$$

Afterwards, by repeating the above-mentioned analysis without variable g, the version V2 was obtained. In this case, due to the fact, that by eliminating this physically irrelevant variable g, some of the variables haven't had got all dimensions, the changing of the dimensions (N instead of kg and s) was introduced.

Consequently, matrix A was constituted by the independent variables (L, F, a, b) and matrix B remained the same:

		B		A			
		v	E	L	F	a	b
	m_x	0	1	1	0	0	0
	m_y	1	-2	0	0	0	1
V2	m_z	0	-1	0	0	1	0
	N	0	1	0	1	0	0
	π_1	1	0	0	0	0	-1
	π_2	0	1	-1	-1	1	2
		D		C			

It can be observed, that by performing the same calculi, one will obtain the same Model Law (1).

By changing the involved independent variables, similar calculi were performed and the authors obtained several useful versions (V3, V4, V5, V6 and V7) with practically the same (equivalent) Model Laws, presented in the following:

		B		A			
		v	F	E	L	a	b
	m_x	0	0	1	1	0	0
	m_y	1	0	-2	0	0	1
V3	m_z	0	0	-1	0	1	0
	N	0	1	1	0	0	0
	π_1	1	0	0	0	0	-1
	π_2	0	1	-1	1	-1	-2
		D		C			

$$\pi_1 = v \cdot b^{-1} = v / b \Leftrightarrow S_v = S_b \tag{2}$$

$$\pi_2 = \frac{F \cdot L}{E \cdot a \cdot b^2} \Leftrightarrow S_F \cdot S_L = S_E \cdot S_a \cdot (S_b)^2 \Rightarrow S_F = \frac{S_E \cdot S_a \cdot (S_b)^2}{S_L}$$

		B		A			
		v	b	E	L	F	a
	m_x	0	0	1	1	0	0
	m_y	1	1	-2	0	0	0
V4	m_z	0	0	-1	0	0	1
	N	0	0	1	0	1	0
	π_1	1	0	0.5	-0.5	-0.5	0.5
	π_2	0	1	0.5	-0.5	-0.5	0.5
		D		C			

$$\pi_1 = v \cdot \sqrt{\frac{E \cdot a}{L \cdot F}} \Leftrightarrow (S_v)^2 \cdot S_E \cdot S_a = S_L \cdot S_F \Leftrightarrow S_v = \sqrt{\frac{S_L \cdot S_F}{S_E \cdot S_a}}$$

$$\pi_2 = b \cdot \sqrt{\frac{E \cdot a}{L \cdot F}} \Leftrightarrow (S_b)^2 \cdot S_E \cdot S_a = S_L \cdot S_F \Rightarrow S_b = \sqrt{\frac{S_L \cdot S_F}{S_E \cdot S_a}} \tag{3}$$

		B		A			
		v	L	E	b	F	a
	m_x	0	1	1	0	0	0
	m_y	1	0	-2	1	0	0
V5	m_z	0	0	-1	0	0	1
	N	0	0	1	0	1	0
	π_1	1	0	0	-1	0	0
	π_2	0	1	-1	-2	1	-1
		D		C			

$$\pi_1 = v \cdot b^{-1} = v / b \Leftrightarrow S_v = S_b$$

$$\pi_2 = \frac{F \cdot L}{E \cdot b^2 \cdot a} \Leftrightarrow S_F \cdot S_L = S_E \cdot S_a \cdot (S_b)^2 \Rightarrow S_L = \frac{S_E \cdot S_a \cdot (S_b)^2}{S_F} \tag{4}$$

		B		A			
		v	a	E	L	F	b
	m_x	0	0	1	1	0	0
	m_y	1	0	-2	0	0	1
V6	m_z	0	1	-1	0	0	0
	N	0	0	1	0	1	0
	π_1	1	0	0	0	0	-1
	π_2	0	1	1	-1	-1	2
		D		C			

$$\pi_1 = v \cdot b^{-1} = v / b \Leftrightarrow S_v = S_b$$

$$\pi_2 = a \cdot \frac{E \cdot b^2}{L \cdot F} \Leftrightarrow S_F \cdot S_L = S_E \cdot S_a \cdot (S_b)^2 \Rightarrow S_a = \frac{S_F \cdot S_L}{S_E \cdot (S_b)^2} \tag{5}$$

		B		A			
		v	a	F	b	L	E
	m_x	0	0	0	0	1	1
	m_y	1	0	0	1	0	-2
V7	m_z	0	1	0	0	0	-1
	N	0	0	1	0	0	1
	π_1	1	0	0	-1	0	0
	π_2	0	1	-1	2	-1	1
		D		C			

$$\pi_1 = v \cdot b^{-1} = v / b \Leftrightarrow S_v = S_b$$

$$\pi_2 = a \cdot \frac{E \cdot b^2}{L \cdot F} \Leftrightarrow S_F \cdot S_L = S_E \cdot S_a \cdot (S_b)^2 \Rightarrow S_a = \frac{S_F \cdot S_L}{S_E \cdot (S_b)^2} \tag{6}$$

Observations:

- The inclusion of a and b in the Dimensional Set will be equivalent with a priori imposing of the geometrical similarity of the model and the prototype, although in versions V4, V6 and V7 only one of them (a or b) appears as independent variable;
- The variables' succession inside of matrix A, as well as in B, doesn't influence the final Model Law's expression;
- The variables placed in matrix A are independent ones; they can be freely chosen not only for the prototype, but also for the model, too;
- The variables placed in matrix B are considered independent for the prototype and dependent for the model; these last ones (for the model) will be obtained exclusively by applying of the Model Law (they are variables Category II.);
- Consequently, all variables for the prototype, excepting the pursued vertical displacement „v_1", represent variables from Category I. and can be freely chosen or given a priori;
- The pursued vertical displacement „v_1" represents a Category II variable, due to the fact that, based on effective measurements on model of the corresponding displacement „v_2", one will apply the Model Law in order to finally obtain this mentioned vertical displacement „v_1".

Based on the mentioned optimisation procedures, described in the contributions [7; 8] as well as in [12], the authors applied the variables' number reduction based on amalgamation; the variables (a, b) were substituted by the moment of inertia $I_z \equiv I_{zG} = \frac{a \cdot b^3}{12}$ of the cross-section, obtaining the following final Dimensional Set, as well as a single relationship for Model Law, with the identical meaning for the correspondence between the prototype and model:

		B	A			
		v	F	I_{zG}	L	E
V8	m_x	0	0	0	1	1
	m_y	1	0	3	0	-2
	m_z	0	0	1	0	-1
	N	0	1	0	0	1
	π_1	1	1	-1	1	-1
		D	C			

$$\pi_1 = \frac{v \cdot F \cdot L}{E \cdot I_z} \Leftrightarrow S_v \cdot S_F \cdot S_L = S_E \cdot S_{Iz} \Rightarrow S_v = \frac{S_E \cdot S_{Iz}}{S_F \cdot S_L} \tag{7}$$

If we change the order of the variables in matrix A, we will obtain the same Model Law (see Version V9)

		B	A			
		v	F	L	I_{zG}	E
V9	m_x	0	0	1	0	1
	m_y	1	0	0	3	-2
	m_z	0	0	0	1	-1
	N	0	1	0	0	1
	π_1	1	1	1	-1	-1
		D	C			

$$\pi_1 = \frac{v \cdot F \cdot L}{E \cdot I_z} \Leftrightarrow S_v \cdot S_F \cdot S_L = S_E \cdot S_{Iz} \Rightarrow S_v = \frac{S_E \cdot S_{Iz}}{S_F \cdot S_L}$$

Observations:

- In these last cases (V8 and V9) it became possible to choose all variables (F, L, E as well as I_{zG}) freely, both for the prototype and the model; the single condition consists in the Model Law enforcement by the selected values;
- Here (V8 and V9), MDA offers the great facility of a good dimensional similarity between the prototype and model without a geometric similarity, because I_{zG} was an independent variable, that one can choose freely;
- Consequently, in these cases (V8 and V9) the cross-section can be of any type (circular, tubular, rectangular or any other) with the single constrain of the equality of their I_{zG} magnitudes (numerical values).

3. COMPARATIVE CALCULI INVOLVING THE DIMENSIONAL SETS PROPOSED BY THE AUTHOR OF [7: 8]

As a comparison with the authors' above-mentioned own results, the Dimensional Sets as well as the corresponding Model Laws of Professor Thomas SZIRTES, the author of contributions [7; 8], who developed the mentioned MDA method, will be briefly presented in the following.

The diminishing of the dimensions' number was his first attempt: instead of applying the lengths' separation along the main rectangular directions (x,y,z), i.e. m_x, m_y, m_z, a unitary length, m, was considered with the same number of the potential variables (v,E,L,F,a,b). A second order matrix A was obtained, with the corresponding Dimensional Set (Version: TSz 01) and four constitutive elements for the Model law, presented below.

		B				A	
		v	a	L	F	b	E
	m	1	1	1	0	1	-2
	N	0	0	0	1	0	1
TSz 01	π_1	1	0	0	0	-1	0
	π_2	0	1	0	0	-1	0
	π_3	0	0	1	0	-1	0
	π_4	0	0	0	1	-2	-1
		D				C	

In this case the independent variables were (b, E), the rest of them being dependent ones.

$$\begin{gathered} \pi_1 = v / b \Leftrightarrow S_v = S_b \\ \pi_2 = a / b \Leftrightarrow S_a = S_b \\ \pi_3 = L / b \Leftrightarrow S_L = S_b \\ \pi_4 = \frac{F}{b^2 \cdot E} \Leftrightarrow S_F = (S_b)^2 \cdot S_E \end{gathered} \tag{8}$$

One other Dimensional Set was obtained with the matrix A, containing the (L, E) as independent variables (see version TSz 02), having a final equivalent Model Law.

		B				A	
		F	a	b	v	L	E
	m	0	1	1	1	1	-2
	N	1	0	0	0	0	1
TSz 01bis	π_1	1	0	0	0	-2	-1
	π_2	0	1	0	0	-1	0
	π_3	0	0	1	0	-1	0
	π_4	0	0	0	1	-1	0
		D				C	

$$\pi_1 = \frac{F}{L^2 \cdot E} \Leftrightarrow S_F = (S_L)^2 \cdot S_E$$
$$\pi_2 = a / L \Leftrightarrow S_a = S_L$$
$$\pi_3 = b / L \Leftrightarrow S_b = S_L \tag{9}$$
$$\pi_4 = v / L \Leftrightarrow S_v = S_L$$

By mean of the variables' amalgamation, taking the apparition of variables (a, b) only as $\alpha = a \cdot b^3$ in the vertical displacement's expression, these two were substituted by this latter α, obtaining a diminishing of the dimensionless variables with 1, by mean of the Dimensional Set TSz 02, as well as its corresponding Model Law:

		B			A		
		F	α	v	L	E	
	m	0	4	1	1	-2	$\alpha = a \cdot b^3$
	N	1	0	0	0	1	
TSz 02	π_1	1	0	0	-2	-1	
	π_2	0	1	0	-4	0	
	π_3	0	0	1	-1	0	
		D			C		

$$\pi_1 = \frac{F}{L^2 \cdot E} \Leftrightarrow S_F = (S_L)^2 \cdot S_E$$
$$\pi_2 = \alpha / L^4 \Leftrightarrow S_\alpha = (S_L)^4 \tag{10}$$
$$\pi_3 = v / L \Leftrightarrow S_v = S_L$$

Additionally, considering that pairing between (F, L) is always as the maximum bending moment $M_{iz\,max} = F \cdot L$, these two former variables were substituted by this latter one, obtaining the variant TSz 03 for the Dimensional Set, as well as for the Model Law.

		B		A		
		v	$M_{iz\,max} = F \cdot L$	α	E	
	m	1	3	4	-2	
TSz 03	N	0	1	0	1	$M_{iz\,max} = F \cdot L$
	π_1	1	0	-0.25	0	
	π_2	0	1	-1.25	-1	
		D		C		

$$\pi_1 = \frac{v}{\sqrt[4]{\alpha}} \Leftrightarrow S_v = \sqrt[4]{S_\alpha} \tag{11}$$
$$\pi_2 = \frac{M_{iz\,max}}{E \cdot \sqrt[4]{\alpha^5}} \Leftrightarrow S_{M_{iz\,max}} = S_E \cdot (S_\alpha)^{5/4}.$$

4. CONCLUSIONS

- The analyzed variants allow us to observe a large number of options concerning on the variables, which can be included in the Dimensional Set, as well as in the Model Law;
- Depending on the selected variables (both the independent and the dependent ones), one can obtain a diminishing of the constitutive elements of the Model Law, as well as a simplicity of the analysis;
- Even there are several expressions of the Model Law, finally they will offer the same requested „ v_1 " displacement for the prototype;

- In the authors' opinion, the described MDA method in the contributions [7; 8], elaborated by Professor Thomas SZIRTES, represents the most simple, advanced and unitary way in the prototype's behaviour analysis via the model's ones;
- Applying the MDA method, the number of the experimental investigations will be drastically diminished and simplified;
- All experimental measurements will be performed exclusively on the model (which represents an easier manner, in comparison with the prototype's ones), offering also a significant diminishing of the involved costs.

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ISSN 1584 - 2665 (printed version); ISSN 2601 - 2332 (online); ISSN-L 1584 - 2665
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