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QUASI-STATIC THERMAL STRESS ANALYSIS OF A MODERATELY THICK DISK WITH TIME-DEPENDENT HEAT SOURCE

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Abstract: This paper analyzes a quasi-static thermal stress of moderately thick disk with time-dependent heat source impulse**.** The lower and upper faces of the plate is kept at zero temperature, while the inner and outer circular curved surface is thermally insulated. The Laplace transform has been found convenient for obtaining the solution of particular distribution of temperature, which satisfies the time-dependent heat conduction equation. Thermoelastic Airy's function and harmonic potential functions are used to obtain the displacement components and its associated stresses. The results are obtained in a form in terms of Bessel's function. The results for temperature, displacement, and stresses have been computed numerically considering special functions and illustrated graphically.
Keywords: heat conduction, moderately thick plate, thermal stress, Laplace transform

1. INTRODUCTION

Many studies have appeared in the literature about the transient response of the plate and its associated thermal stresses. The following references are chosen because they include mathematical methods for deriving the response formulas contained in present description. In classical papers, Nowacki [1] has determined steady-state thermal stresses in a thick circular plate subjected to an axisymmetric temperature distribuion on the upper face with zero temperature on the lower face and the circular edge. Further Roy Choudhuri [2] has succeeded in determining the quasi-static thermal stresses in a circular plate subjected to transient temperature along the circumference of a circle over the upper face with lower at zero temperture and the fixed circular edge thermally insulated. Nasser [3, 4] proposed the concept of heat sources in generalized thermoelasticity and applied to a thick plate problem. In all aforementioned investigations an axisymmetrically heated thick plate has been considered and its results are more useful in engineering problems. Recently, Kulkarni and Deshmukh [5] present paper deals with the determination of a quasi-static thermal stresses in a thick circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature and the fixed circular edge thermally insulated using Lapalce method.

Similarly, Kulkarni and Deshmukh [6] has also determinated the transient heat conduction and thermal stresses of a thick annular disc subjected to arbitrary heat flux on the upper and lower surfaces where as the fixed circular edges are at zero temperature, using integral transform technique. Varghese and Lalsingh [7] has determined the thermo-elastic stress in a thick disc due to interior heat generation within the solid, under thermal boundary condition, which is subjected to arbitrary initial temperature on the upper and lower face at zero temperature, and the fixed circular edges with additional sectional heat supply, using unconventional integral transform. Okumura [8] has analysed the thermal-bending stresses in an annular sector by the theory of moderately thick plates. Saidi and Baferani [9] investigated the thermal buckling of simply supported moderately thick functionally graded annular sector plate using first order shear deformation plate theory and obtained the solution by using an analytical method. Hu et al. [10] presented a set of high order partial differential equations for elastic rectangular thick plate with four edges completely clamped support base on Mindlin theory and solved by the double finite integral transform method. Saheb and Aruna [11] developed a simple and efficient coupled displacement field method to study the buckling load parameters of the simply supported moderately thick rectangular plates. Genckal et al. [12] studied the dynamic responses of composite plates on elastic foundation subjected to impact and moving loads by using Galerkin's method, then the equations are solved by using the Runge-Kutta method. However, they did not consider a thermoelastic problem in which a source is generated on the basis of a linear function of temperature that satisfies the time-dependent heat conduction equation.

Based on previous literature on moderately-thickness annular disks, the authors observed that no analytical procedure was established in view of the generation of internal heat sources in the body. Due to the lack of research in this area, the author was motivated to conduct this research. Theoretical calculations have been studied using dimensional parameters, while the graphical calculations are carried out using dimensionless parameters. The success of this novel research lies in new mathematical procedures that have adopted a simpler optimized design approach in terms of material utilization and technical performance, particularly in determining the thermoelastic behavior of moderately-thickness disks engaged as the foundation of pressure container, furnaces, etc.

2. FORMULATION OF THE PROBLEM

We consider a moderately thick annular plate occupying the space $D:\{(r,z)\in R^2:\ a<r<b,\ 0<z<\ell\}$ under unsteady-state temperature field with a time-dependent internal heat sources within it.

— Unsteady-state temperature field

The governing equation for heat conduction in isotropic solids is given as follows

$$\left\{\nabla^2 - \frac{1}{\kappa}\frac{\partial}{\partial t}\right\} T(r,z,t) = -q_0\,\ell\,\Phi(t)[H(r-r_0) - H(r-r_1)],\ r_0 < r < r_1 \tag{1}$$

in which $T = T(r,z,t)$ is the temperature change,

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

and

$$\Phi(t) = \begin{cases} (\Phi_0 / \mathrm{T}_0)\,t, 0 \le t \le t_0 \\ \Phi_0, \qquad\qquad t > t_0 \end{cases} \tag{2}$$

subjected to the boundary conditions

$$T\big|_{t=0} = T\big|_{z=0} = T\big|_{z=\ell} = \frac{\partial T}{\partial r}\bigg|_{r=a} = \frac{\partial T}{\partial r}\bigg|_{r=b} = 0 \tag{3}$$

— Displacement and stress field

Let u_r and u_z be components of displacement is expressed as

$$2Gu_r = \frac{\partial}{\partial r}\left[r\frac{\partial \phi_1}{\partial r} + z\frac{\partial \phi_3}{\partial z} - 4(1-\upsilon)\phi_1\right] + \frac{\partial \chi}{\partial r},$$
$$2Gu_z = \frac{\partial}{\partial z}\left[r\frac{\partial \phi_1}{\partial r} + z\frac{\partial \phi_3}{\partial z} - 4(1-\upsilon)\phi_3\right] + \frac{\partial \chi}{\partial z} \tag{4}$$

in which G and υ denote the shear modulus and Poisson's ration, respectively, and

$$\nabla^2\phi_1 = 0,\ \nabla^2\phi_3 = 0,\ \nabla^2\chi = \frac{\alpha E}{1-\upsilon}\tau. \tag{5}$$

where ϕ_1, ϕ_3 are harmonic functions, χ is a thermoelastic potential function, $\tau = T - T_i$ is temperature change with T_i as initial temperature, respectively.

The components of stress taking account of heat is as follows

$$\sigma_{rr} = 2G\left(\varepsilon_{rr} + \frac{\upsilon}{1-2\upsilon}e\right) - \frac{\alpha E\tau}{1-2\upsilon},\ \sigma_{\theta\theta} = 2G\left(\varepsilon_{\theta\theta} + \frac{\upsilon}{1-2\upsilon}e\right) - \frac{\alpha E\tau}{1-2\upsilon},$$
$$\sigma_{zz} = 2G\left(\varepsilon_{zz} + \frac{\upsilon}{1-2\upsilon}e\right) - \frac{\alpha E\tau}{1-2\upsilon},\ \sigma_{rz} = 2G\varepsilon_{rz} \tag{6}$$

in which

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \varepsilon_{\theta\theta} = \frac{u_r}{r}, \varepsilon_{zz} = \frac{\partial u_z}{\partial z}, \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)$$
$$e = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \tag{7}$$

where $\sigma_{rr},..,\sigma_{rz}$ are the components of stresses, $\varepsilon_{rr},..,\varepsilon_{rz}$ are the components of strain, α represent the coefficient of linear thermal expansion and E is Young's modulus and e is the cubical dilation, respectively.

For the traction-free surfaces the stress functions

$$\sigma_{rz}\big|_{z=0} = \sigma_{zz}\big|_{z=0} = \sigma_{rz}\big|_{z=\ell} = \sigma_{zz}\big|_{z=\ell} = 0 \tag{8}$$

The equations (1) to (8) constitute the mathematical formulation of the problem.

3. SOLUTION TO THE PROBLEM

— Solution for the temperature distribution

Applying the Laplace transformation in Eqs. (1) and (4)

$$\left\{\nabla^2 - \frac{p}{\kappa}\right\}\overline{T}(r,z,p) = -\frac{q_0\Phi_0\ell}{p^2 t_0}(1 - \mathrm{e}^{-pt_0})[H(r-r_0) - H(r-r_1)] \tag{9}$$

$$\overline{T}\big|_{z=0} = \overline{T}\big|_{z=\ell} = \frac{\partial \overline{T}}{\partial r}\bigg|_{r=a} = \frac{\partial \overline{T}}{\partial r}\bigg|_{r=b} = 0 \tag{10}$$

in which the symbol (¯) represents a function in the transformed domain and $\overline{T}(r,z,p)$ is the transformed function of $T(r,z,t)$ with Laplace parameter as p.

Now applying the finite Fourier sine transform in Eqs. (9) one obtains

$$\left\{\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\beta_m-\frac{p}{\kappa}\right\}\overline{\overline{T}}(r,m,p)=-\frac{q_0\Phi_0\ell}{p^2 t_0}(1-e^{-pt_0})[H(r-r_0)-H(r-r_1)] \tag{11}$$

$$\left.\frac{\partial\overline{\overline{T}}}{\partial r}\right|_{r=a}=0,\ \left.\frac{\partial\overline{\overline{T}}}{\partial r}\right|_{r=b}=0 \tag{12}$$

where $\overline{\overline{T}}(r,m,p)$ is the transform of $\overline{T}(r,z,p)$ with respect to nucleus is $\sin(m\pi z/\ell)$.

To obtain the expression for the temperature $\overline{\overline{T}}(r,m,p)$ of Eqs. (11)-(12), we assume

$$\overline{\overline{T}}(r,m,p)=\sum_{n=1}^{\infty}A_n J_0(\alpha_n r) \tag{13}$$

Assume

$$H(r-r_0)-H(r-r_1)=f(r)=\sum_{n=1}^{\infty}B_n J_0(\alpha_n r) \tag{14}$$

Putting Eq. (13) and (14) in Eq. (11), one obtains

$$A_n=\frac{\kappa q_0\ell(1-e^{-pt_0})}{t_0 p^2(p+\gamma_{mn})}B_n \tag{15}$$

in which $\gamma_{mn}=\beta_m^2\kappa+\alpha_n^2\kappa$.

Taking the theory of Bessel function on Eq. (15), one obtains

$$B_n\int_a^b r[J_0(\alpha_n r)]^2dr=\int_a^b r\,f(r)J_0(\alpha_n r)dr=\int_{r_0}^{r_1} rJ_0(\alpha_n r)dr \tag{16}$$

since $f(r)=\begin{cases}1 \text{ for } r_0<r<r_1\\ 0 \text{ for } a<r<r_0 \text{ and } r_1<r<b\end{cases}$

Hence

$$B_n=\frac{2}{\alpha_n}\left\{\frac{r_1J_1(\alpha_n r_1)-r_0J_1(\alpha_n r_0)}{b^2[J_0(\alpha_n b)]^2-a^2[J_0(\alpha_n a)]^2}\right\} \tag{17}$$

Using Eqs. (13), (15), (17) we get,

$$\overline{\overline{T}}(r,m,p)=\frac{2\kappa q_0\ell}{t_0}\sum_{n=1}^{\infty}\frac{(1-e^{-pt_0})}{p^2(p+\gamma_{mn})\alpha_n}\left\{\frac{r_1J_1(\alpha_n r_1)-r_0J_1(\alpha_n r_0)}{b^2[J_0(\alpha_n b)]^2-a^2[J_0(\alpha_n a)]^2}\right\}J_0(\alpha_n r) \tag{18}$$

Now applying the inverse Fourier sine transform to Eq. (18), one yield

$$\overline{T}(r,z,p)=\frac{4\kappa q_0}{t_0}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(1-e^{-pt_0})}{p^2(p+\gamma_{mn})\alpha_n}\left\{\frac{r_1J_1(\alpha_n r_1)-r_0J_1(\alpha_n r_0)}{b^2[J_0(\alpha_n b)]^2-a^2[J_0(\alpha_n a)]^2}\right\}\times J_0(\alpha_n r)\sin(m\pi z/\ell) \tag{19}$$

Applying the Laplace inversion theorems on Eq. (19), one yield

$$\tau=T(r,z,t)=\frac{4\kappa q_0}{t_0}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\alpha_n}\left\{\frac{r_1J_1(\alpha_n r_1)-r_0J_1(\alpha_n r_0)}{b^2[J_0(\alpha_n b)]^2-a^2[J_0(\alpha_n a)]^2}\right\}J_0(\alpha_n r)\sin\left(\frac{m\pi z}{\ell}\right)\times\{\Phi_n(r,z,t)-\Phi_n(r,z,t-t_0)H(t-t_0)\} \tag{20}$$

where:

$$L^{-1}\left\{\frac{1-e^{-t_0p}}{p^2[p+\gamma_{mn}]}\right\}=L^{-1}\left\{\frac{1-e^{-t_0p}}{\gamma_{mn}}\left[\frac{1}{p^2}-\frac{1}{\gamma_{mn}}\left(\frac{1}{p}-\frac{1}{p+\gamma_{mn}}\right)\right]\right\}=\Phi_n(r,z,t)-\Phi_n(r,z,t-t_0)H(t-t_0)$$

in which

$$\Phi_n(r,z,t)=\frac{1}{\gamma_{mn}}\left(t-\frac{1-e^{-\gamma_{mn}t}}{\gamma_{mn}}\right)$$

— Solution for potential functions and thermal stresses

Now assume ϕ_1,ϕ_3 are harmonic functions and χ is a thermoelastic potential function that satisfies Eq. (5) as

$$\chi(r,z,t)=\frac{\alpha E}{1-\upsilon}\frac{4\kappa q_0}{\ell\kappa t_0}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\gamma_{mn}}{\alpha_n}\left\{\frac{r_1J_1(\alpha_n r_1)-r_0J_1(\alpha_n r_0)}{b^2[J_0(\alpha_n b)]^2-a^2[J_0(\alpha_n a)]^2}\right\}J_0(\alpha_n r)\sin\left(\frac{m\pi z}{\ell}\right)\times\{\Phi_n(r,z,t)-\Phi_n(r,z,t-t_0)H(t-t_0)\} \tag{21}$$

$$\phi_1(r,z,t)=\frac{\alpha E}{1-\upsilon}\frac{4\kappa q_0}{\ell\kappa t_0}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\gamma_{mn}}{\alpha_n}\left\{\frac{r_1 J_1(\alpha_n r_1)-r_0 J_1(\alpha_n r_0)}{b^2[J_0(\alpha_n b)]^2-a^2[J_0(\alpha_n a)]^2}\right\} \times\{C_1 J_0(\alpha_n r)+D_1(\alpha_n r)J_1(\alpha_n r)\}\sin(m\pi z/\ell) \times\{\Phi_n(r,z,t)-\Phi_n(r,z,t-t_0)H(t-t_0)\} \tag{22}$$

$$\phi_3(r,z,t)=\frac{\alpha E}{1-\upsilon}\frac{4\kappa q_0}{\ell\kappa t_0}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\gamma_{mn}}{\alpha_n}\left\{\frac{r_1 J_1(\alpha_n r_1)-r_0 J_1(\alpha_n r_0)}{b^2[J_0(\alpha_n b)]^2-a^2[J_0(\alpha_n a)]^2}\right\} \times\{E_1 J_0(\alpha_n r)+F_1(\alpha_n r)J_1(\alpha_n r)\}\sin(m\pi z/\ell) \times\{\Phi_n(r,z,t)-\Phi_n(r,z,t-t_0)H(t-t_0)\} \tag{23}$$

Using Eqs. (21)-(23) in Eq. (4), one gets the required displacement components. The components of stress given in Eq. (6) can be obtained using displacement components and strain components given in Eq. (7). Finally C_1, D_1, E_1 and F_1 are the unknown constants which can to be determined using traction free conditions given in Eq. (8) as

$$X_1C_1+Y_1D_1+Z_1E_1+T_1F_1=R_1,\ X_2C_1+Y_2D_1+Z_2E_1+T_2F_1=R_2,\quad X_3C_1+Y_3D_1+Z_3E_1+T_3F_1=R_3,\ X_4C_1+Y_4D_1+Z_4E_1+T_4F_1=R_4 \tag{24}$$

where,

$$X_1=-a_1J_1-a_2J_{0,2}+a_3,\ Y_1=(a_1J_{0,2})/2+J_0(a_4-a_3+a_5)-a_6J_{1,13}$$

$$Z_1=-a_1J_1+2\beta\alpha_nJ_1a_3-\alpha_n^2J_2,\ T_1=-a_7J_2+(a_1/2-a_8)J_{0,2},$$

$$R_1=[a_1J_1(a)+J_0(a)/2-a_9][a],$$

$$X_2=-a_1J_1-2\Im_{12}(a_4-a_5)-a_2J_{0,2}C_L+aS_L\alpha_n^2J_{1,13}/2+\Im_{11}[-1+2(1-\nu)-3aa_4+aa_5]-\Im_{13},$$

$$Y_2=(a_1C_L/2+a_4S_L-a_5S_L\alpha_n)J_{0,2}+[-a_2C_L/\alpha_n+S_L\alpha_n+2(1-\nu)S_L\alpha_n+6\Im_{14}+a_3]J_{1,13}+(1/4+\Im_{14})J_{2,24},$$

$$Z_2=J_1[a_{10}S_L+\Im_{15}-C_L(a_1+La_1/a-2a_8)]-(\alpha_n+\Im_{16})2L\Im_{16},$$

$$T_2=\Im_{16}(2\Im_{14}+a_4-aa_5)-\Im_{15}J_{0,2}-(1/2+\Im_{14})K,$$

$$R_2=a\{a_1J_1C_L+2a_4\Im_{12}+(1+4\Im_{14})K1+a_8(C_L)'+(1/a)[4\Im_{14}(J_0S_L)']+(1+4\Im_{14})2(J_1S_L\alpha_n)'+(1+4\Im_{14})(J_1S_L)''\},$$

$$X_3=2\alpha_n\beta[J_{1r}(12-16\nu)-2r\alpha_n(J_{0r}-J_{2r})],$$

$$Y_3=\alpha_n[(-J_{0r}+J_{2r})\beta(12-16\nu)+2r\beta\alpha_n(3J_{1r}-J_{3r})],$$

$$Z_3=8\alpha_n\beta J_{1r},\ T_3=4\alpha_n\beta(-J_{0r}+J_{2r}),$$

$$R_3=8[-\Im_{11}\beta J_{1r}-(1/2)\Im_{11}J_{0r}+(\Im_{11}\beta J_{0r})'][r],$$

$$X_4=2\alpha_n\beta[J_{1r}C_L(12-16\nu)-(2r\alpha_n\beta C_L-4S_L\alpha_n)(J_{0r}-J_{2r})+rS_L\alpha_n^2(J_{1r}-J_{3r})],$$

$$Y_4=\alpha_n[(-J_{0r}+J_{2r})\beta C_L(12-16\nu)+2r\beta\alpha_nC_L(3J_{1r}-J_{3r})+4S_L\alpha_n(3J_{1r}-J_{3r})+rS_L\alpha_n^2(3J_{0r}-4J_{2r}+J_{4r})],$$

$$Z_4=4\alpha_n[2\beta J_{1r}(C_L-\beta LS_L)+L\beta\alpha_nC_L(J_{0r}-J_{2r})-4S_L\alpha_n((1-\nu)J_{0r}+(1-\nu)J_{2r})],$$

$$T_4=2\alpha_n[(2L\beta^2S_L+\beta C_L)(J_{0r}-J_{2r})+L\beta\alpha_nC_L(3J_{1r}-J_{3r})-12\alpha_nC_L],$$

$$R_4=8[-\Im_{11}\beta J_{1r}-(1/2)\Im_{11}J_{0r}+(\Im_{11}\beta J_{0r})'][r],$$

$$a_1=\beta\nu\alpha_n/(1-2\nu),\ a_2=a\beta\nu\alpha_n^2/[2(1-2\nu)],\ a_3=2(1-\nu)\nu/(1-2\nu),a_4=\nu/[2a(1-2\nu)],$$

$$a_5=[2(1-\nu)\nu]/[a(1-2\nu)],a_6=a\beta\nu\alpha_n^2,a_7=\alpha_n/[2a(1-2\nu)],a_8=2\beta\nu(1-\nu)\alpha_n/(1-2\nu),$$

$$a_9=\beta\nu J_0'/(1-2\nu),a_{10}=\alpha_n/(1-2\nu),S_L=\sin(L\beta),C_L=\cos(L\beta),$$

$$\Im_{11}=J_{0,2}S_L\alpha_n^2,\Im_{12}=J_1S_L\alpha_n,\Im_{13}=a\nu\alpha_n^2S_LJ_{1,13}/[2(1-2\nu)],$$

$$\Im_{14}=\nu/[4(1-2\nu)],\Im_{15}=L\beta^2\nu\alpha_nS_L/(1-2\nu),\Im_{16}=\beta J_{0,2}C_L\alpha_n,$$

$$J_{0,2}=J_0(a\alpha_n)-J_2(a\alpha_n),J_{2,4}=J_2(a\alpha_n)-J_4(a\alpha_n),$$

$$J_{1,3}=J_1(a\alpha_n)-J_3(a\alpha_n),J_{1,13}=J_1(a\alpha_n)+[J_1(a\alpha_n)-J_3(a\alpha_n)]$$

$$J_{2,24}=-[J_0(a\alpha_n)-J_2(a\alpha_n)]-(1/2)[J_2(a\alpha_n)-J_4(a\alpha_n)],$$

$$J_{(0,2)r}=J_0(r\alpha_n)-J_2(r\alpha_n)$$

Using Cramer's Rule, one obtains constants as

$$C1=\frac{\Delta_2}{\Delta_1};D1=\frac{\Delta_3}{\Delta_1};E1=\frac{\Delta_4}{\Delta_1};F1=\frac{\Delta_5}{\Delta_1} \tag{25}$$

where

$$\Delta_1 = \begin{vmatrix} X1 & Y1 & Z1 & T1 \\ X2 & Y2 & Z2 & T2 \\ X3 & Y3 & Z3 & T3 \\ X4 & Y4 & Z4 & T4 \end{vmatrix}, \Delta_2 = \begin{vmatrix} R1 & Y1 & Z1 & T1 \\ R2 & Y2 & Z2 & T2 \\ R3 & Y3 & Z3 & T3 \\ R4 & Y4 & Z4 & T4 \end{vmatrix}, \Delta_3 = \begin{vmatrix} X1 & R1 & Z1 & T1 \\ X2 & R2 & Z2 & T2 \\ X3 & R3 & Z3 & T3 \\ X4 & R4 & Z4 & T4 \end{vmatrix}, \Delta_4 = \begin{vmatrix} X1 & Y1 & R1 & T1 \\ X2 & Y2 & R2 & T2 \\ X3 & Y3 & R3 & T3 \\ X4 & Y4 & R4 & T4 \end{vmatrix}, \Delta_5 = \begin{vmatrix} X1 & Y1 & Z1 & R1 \\ X2 & Y2 & Z2 & R2 \\ X3 & Y3 & Z3 & R3 \\ X4 & Y4 & Z4 & R4 \end{vmatrix}$$

The resulting equation of stresses can be obtained by substituting the unknown constant obtained from Eq. (25). The equations of stresses are rather lengthy, and the same has been omitted here for the sake of brevity but have been considered during graphical discussion using MATHEMATICA software.

4. NUMERICAL RESULTS, DISCUSSION AND REMARKS

For the interests of simplicity of calculation, we introduce the following dimensionless values

$$\bar{r} = r/b,\ \bar{z} = z/b,\ \bar{\tau} = \kappa t/b^2, \bar{T} = T/T_0, \quad \bar{u}_i = u_i / E\alpha T_0, \bar{\sigma}_{ij} = \sigma_{ij} / E\alpha T_0\ (i, j = r, \theta) \tag{26}$$

Substituting the value of Eq. (26) in Eqs. (20) and components of stresses, we obtained the expressions for the temperature distribution and thermal stresses respectively for the numerical discussion. The numerical computations have been carried out for moderately thick disk having the thermo-mechanical properties: modulus of elasticity E = 70 GPa, Poisson's ratio υ = 0.35, thermal expansion coefficient $\alpha = 23\times10^{-6}/^0C$, thermal diffusivity $\kappa = 84.18\times10\text{-}6\ m^2s^{-1}$ and thermal conductivity λ = 204.2 $Wm^{-1}K^{-1}$. The physical parameter for the plate as $a = 0.2\,\text{m},\ b = 1\,\text{m},\ \ell = 0.08\,\text{m}$ and $T_0 = 150^0C$. In order to examine the influence of internal heat source on the moderately thick disk plate, the numerical calculations were performed for all the variables, and numerical calculations are depicted in the following figures with the help of MATHEMATICA software.

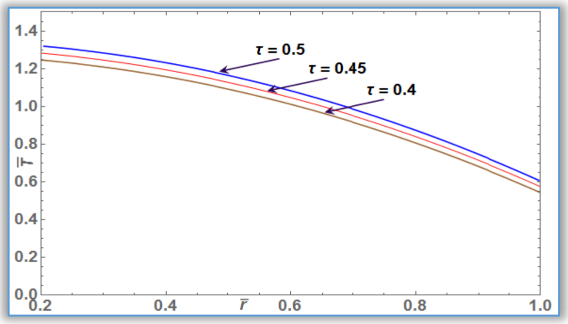

Figure 1: Temperature distribution along $\bar{r}$ -direction for different values of time

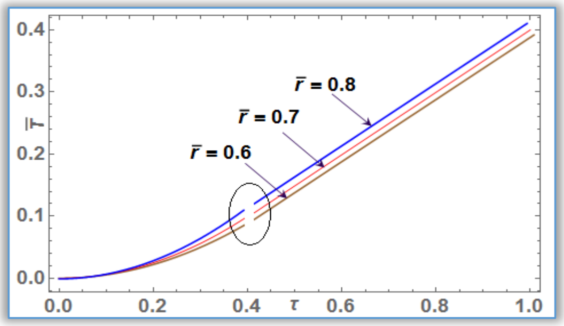

Figure 2: Temperature distribution along time for different values of $\bar{r}$

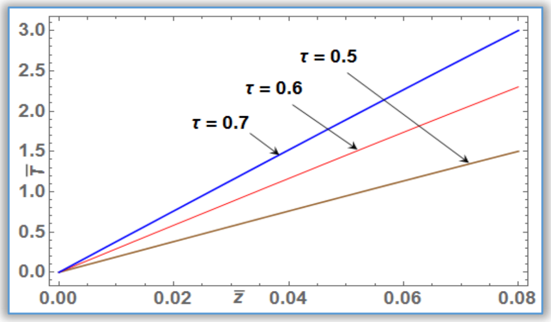

Figure 3: Temperature distribution along $\bar{z}$ -direction for different values of time

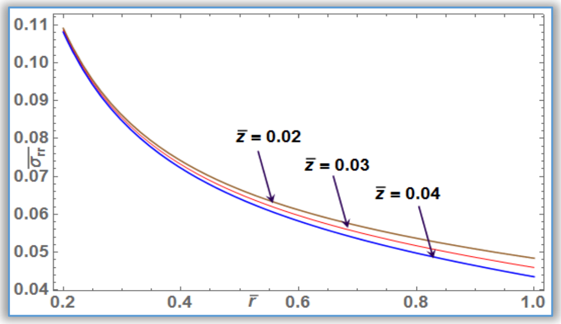

Figure 4: Radial Stress $\bar{\sigma}_{rr}$ along $\bar{r}$ -direction for different values of $\bar{z}$

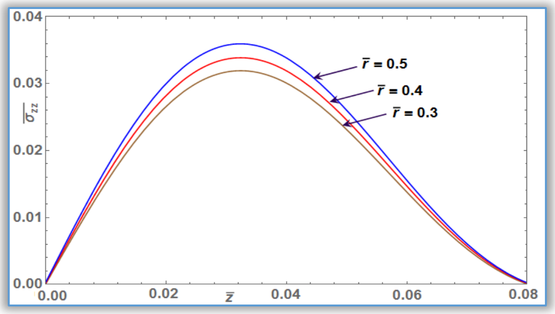

Figure 5: Axial Stress $\bar{\sigma}_{zz}$ along $\bar{z}$ -direction for different values of $\bar{r}$

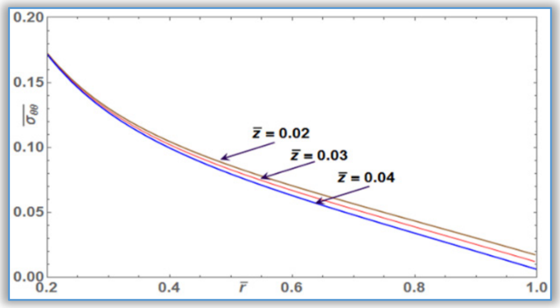

Figure 6: Tangential Stress $\bar{\sigma}_{\theta\theta}$ along $\bar{r}$ -direction for different values of

Plotted Figures 1-6 illustrate the numerical results of temperature distribution and the thermal stresses of the moderately thick disk due to internal heat generation within the solid. From the Figure 1, it is noted that the behaviour of temperature trend is decreasing along redial direction in different value of time due to the available internal heat supply.

Figure 2 shows the temperature distribution with increase of time trend for different radius value. In this figure, temperature increases gradually towards the outer end to the internal heat energy, but a discontinuity in temperature was observed at $\overline{\tau} = 0.4$, and it may be due to the fixed time parameter $\overline{\tau_0} = 0.4$. In the Figure 3, it is observed that the temperature increases linearly along the axial direction for different value of time due to the thickness. Figure 4 explains that the stresses are initially on the higher side which goes on decreasing as along the radial direction at different values of $\overline{z}$. Figure 5 shows the distribution of axial stress along the $\overline{z}$ - direction. It is noted from the figure that the stress along axial direction attains a maximum at centre portion due to the accumulation of heat energy whereas at both boundaries the stress is zero. From Figure 6, it can be concluded that tangential stress will be more at the initial point and gradually decreases along the radial direction.

5. CONCLUSIONS

The proposed analytical solution of transient thermal stress of the moderately thick plate was handled in cylindrical coordinate system with the presence of a source of internal heat. To the author's knowledge, there have been scarce of reports available so far in which sources are generated according to the linear function of the temperature in mediums in the form for moderately thick plate of finite height with Dirichlet type boundary conditions. The temperature is supposed to satisfy the general quasi-static heat conduction equation having ramp-type internal heat source in the plate. By using the Laplace integral transformation technique and classical method, the solution of thermal distribution, thermal stresses are solved. Compared with other methods proposed by the researchers, the integral transformation technology presented here is relatively simple and widely applicable. The results obtained for our research can be described as follows:

- The advantage of this method is its versatility and mathematical ability to handle different types of mechanical and thermal boundary conditions.
- The advantage of this method is its generality and its mathematical power to handle different types of mechanical and thermal boundary conditions.
- The maximum tensile stress shifting from central core to outer region may be due to heat, stress, concentration or available internal heat sources under considered temperature field.

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