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RELIABILITY ANALYSIS OF A SINGLY REINFORCED CONCRETE RECTANGULAR BEAM WITH UNCERTAIN PARAMETERS

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Abstract: In this paper, the results of the reliability analysis of a singly reinforced concrete rectangular beam with respect to the limit state of bending, shear and deflection in accordance with the design provisions of BS 8110, Part 1, 1997, are discussed. The design points of the derived limit state functions and the corresponding reliability indices were obtained using a MATLAB program developed based on First Order reliability format. Sensitivity analysis was carried out to ascertain the impact of each random variable on the safety levels of the beam. The results of the reliability analysis showed that the reliability indices generally decreased with increase in load ratio and beam span for beam capacity based on concrete and steel, shear and deflection criterion. The reliability indices also increased with increase in effective depth of the beam, decreased with increase in imposed load acting on the beam, decreased with increase in steel ratio for beam capacity based on concrete and steel and decreased with increase in area per spacing of shear reinforcement of beam for shear criterion, increased in the effective depth of beam, decreased with increase in imposed load and increased with increase in breadth of the beam for deflection criterion. The design was found to be conservative in shear but satisfactory in bending and deflection when compared with the target safety index value of 3.8 for 50-year reference period for Reliability class 2 at ultimate limit state.
Keywords: Reliability analysis, singly reinforced concrete, rectangular beam, design points, sensitivity analysis

1. INTRODUCTION

It is the interest of the engineering community that structures when designed and built should serve the intended purpose, be safe and economical with respect to both construction and maintenance cost (Mosley and Bungay, 1989). The loads that act on structures or structural members are time varying quantities and their real values are not known. The structural safety is dependent on the resistance and load effects. Both the resistance and the load effects exhibit variability and are treated as probabilistic quantities. The use of judgmental safety factors in the design of civil engineering structures cannot guarantee absolute structural safety. The increased fatality rate and damage of properties resulting from collapse of reinforced concrete structures and members in recent times is worrisome (Punch Newspaper, 2016; Taiwo and Afolami, 2011). The failure may have been triggered by uncertainties inherent in the structural design parameters. The application of partial safety factor in the existing structural design codes and standards may lead to inadequate or uneconomical design of structures (Melchers et al., 1999; Sule, 2011; Sule and Benu, 2019; Abubakar, 2006; Ranganathan, 1999). It is therefore a task of great importance to engineers to carry out engineering designs incorporating parameter uncertainties since the achievement of absolute structural safety is impossible in the presence of uncertain parameters. Uncertainties that are associated with the design parameters can only be addressed using probability and statistics (Abejide, 2014). Structural reliability provides a rational framework that quantifies the uncertain parameters that are associated with both the structural resistance and load effects (Goutham and Manjunath, 2016; El-Reedy, 2013).

In this paper, the reliability analysis of a singly reinforced rectangular concrete beam with respect to bending (based on concrete and steel), shear and deflection is carried out based on First Order Reliability procedure. The limit state functions corresponding to the limit states considered were derived in accordance with the design provisions of BS 8110, Part 1-3, 1997. The limit state functions were solved to obtain the design points on the failure surface and their corresponding reliability indices using an optimization algorithm coded in MATLAB language.

2. FORMULATION OF LIMIT STATE FUNCTIONS

The limit state functions were derived in accordance with the design provisions of BS 8110: Part 1-3, 1997, for design of concrete structures. A simply supported singly reinforced concrete rectangular beam under a uniform loading and the assumed stress distribution is considered in the reliability study (Figure 1), where A_s = area of tension reinforcement, b = breadth of beam section, d = effective depth of tension reinforcement, x = depth of neutral axis, F_{cc} = compressive force of concrete, F_{st} = tensile force of steel bar

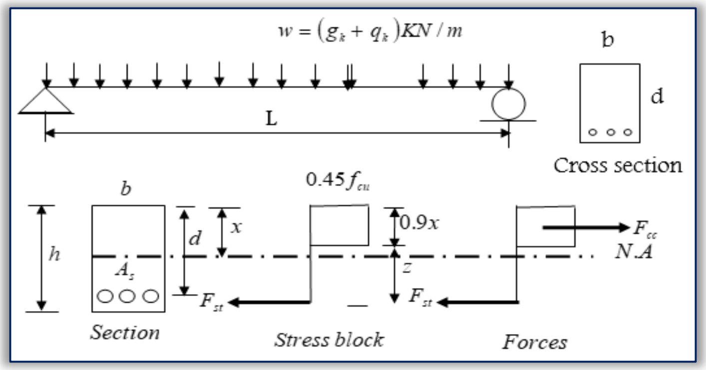

Figure 1: A singly reinforced concrete beam under uniform loading and assumed stress distribution

— **Limit state function for bending**

The failure condition in bending is given by:

$$M_{ult} \le M_{app} \tag{1}$$

Therefore, the limit state function in bending is given by:

$$G(x) = M_{ult} - M_{app} \tag{2}$$

where M_{ult} = ultimate moment of resistance of the beam, M_{app} = applied moment due to applied load on the beam

The maximum bending moment due to applied load is given by:

$$M_{app} = \frac{wL^2}{8} \tag{3}$$

Moment of resistance of a singly reinforced concrete beam based on concrete about the neutral axis of Figure 1 is given by:

$$M_{ult} = 0.156 f_{cu} b d^2 \tag{4}$$

where f_{cu} = compressive strength of concrete, b = width of rectangular beam,

d = effective depth of rectangular beam,

Similarly,

Moment of resistance based on steel about the neutral axis of Figure 1 is given by:

$$M_{ult} = 0.87 f_y A_s Z \tag{5}$$

where f_y = characteristic strength of steel, A_s = area of reinforcing bar, Z = lever arm distance

The limit state function in bending based on concrete capacity is given by:

$$G(x) = 0.156 f_{cu} bd^2 - \frac{(1.4a + 1.6) q_k L^2}{8} \tag{6}$$

Multiplying the first term of equation (6) by A_s and dividing by same and applying $\rho = \frac{A_s}{bd}$ changes equation (6) to:

$$G(x) = 0.156 f_{cu} d \frac{A_s}{\rho} - \frac{(1.4a + 1.6) q_k L^2}{8} \tag{7}$$

The limiting value of lever arm distance Z is given by:

$$Z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d \tag{8}$$

Applying equation (3) and (4), the limit state function for the beam capacity based on steel is given by:

$$G(x) = 0.87f_y A_s Z - \frac{(1.4a + 1.6)q_k L^2}{8} \tag{9}$$

Applying $Z = 0.95d$ from equation (8), equation (9) transforms to:

$$G(x) = 0.8265 f_y A_s d - \frac{(1.4a + 1.6)q_k L^2}{8} \tag{10}$$

The area of tension steel is given by:

$$A_S = \rho bd \tag{11}$$

Applying equation (11), equation (10) changes to:

$$G(x) = 0.8265 f_y \rho bd^2 - \frac{(1.4a + 1.6)q_k L^2}{8} \tag{12}$$

where $\rho =$ steel ratio of the designed beam section

Let the load ratio, a be given by:

$$a = \frac{g_k}{q_k} \tag{13}$$

— **Limit state function for shear**

The area per spacing of stirrup in a reinforced concrete beam is given by:

$$A_s \geq \frac{b(v - \nu_c)S_v}{0.87 f_{yv}} \tag{14}$$

The average shear stress is given by:

$$v = \frac{V}{bd} \tag{15}$$

Applying equation (14) and (15), the shear resistance due to concrete and steel is given by:

$$V = \frac{A_{sv}}{S_v} 0.87 f_y d + \upsilon_C bd \tag{16}$$

The factored applied shear force due to uniform loading is given by:

$$V_{app} = \frac{q_k (1.4a + 1.6)L}{2} \tag{17}$$

The failure condition in shear is given by:

$$V \leq V_{app} \tag{18}$$

Therefore, the limit state function in shear is given by:

$$G(x) = V - V_{app} \tag{19}$$

Applying equation (16) and equation (17), the limit state function in shear is given by:

$$G(x) = A_{sv} 0.87 f_y + \upsilon_C S_v b - \frac{S_v q_k (1.4a + 1.6)L}{2d} \tag{20}$$

According to BS 8110 (1997), the design concrete shear stress, v_c is given by:

$$v_c = \frac{0.79}{\gamma}\left(\frac{100A_s}{bd}\right)^{\frac{1}{3}} * \left(\frac{400}{d}\right)^{\frac{1}{4}} \tag{21}$$

Applying equation (21), equation (20) now changes to:

$$G(x) = A_{sv} 0.87 f_{yv} + \frac{0.79}{\gamma}\left(\frac{100A_S}{bd}\right)^{\frac{1}{3}} * \left(\frac{400}{d}\right)^{\frac{1}{4}} bS_v - \frac{S_V q_k (1.4a + 1.6)L}{2d} \tag{22}$$

— **Limit state function for deflection**

The failure condition in deflection is given by:

$$\delta_{all} \leq \delta_{max} \tag{23}$$

Therefore, the limit state function in deflection is given by:

$$G(x) = \delta_{all} - \delta_{max} \tag{24}$$

The allowable deflection for a singly reinforced concrete beam is given by:

$$\delta_{all} = \text{Basic}\frac{L}{d} * \text{M.F.} \tag{25}$$

$$\delta_{max} = \frac{L}{d} \tag{26}$$

where $\text{Basic}\frac{L}{d}$ = basic span-depth ratio for a simply supported rectangular beam = 20 (BS 8110, part 1, 1997), M.F. = modification factor for tension reinforcement

The modification factor for tension reinforcement in a singly reinforced concrete beam is given by:

$$\text{M.F.} = 0.55 + \frac{477 - f_s}{120\left(\frac{M}{bd^2} + 0.9\right)} \tag{27}$$

where f_s = design service stress in tension reinforcement; $f_s = \frac{5}{8} f_y * \frac{A_s reqd}{A_s prov} * \frac{1}{\beta_b}$

where β_b = percentage moment redistribution = 1 for simply supported beam

According to BS 8110, part 1, the basic span-depth ratio for a simply supported rectangular beam is 20. Therefore, the limit state function for deflection is given by:

$$G(x) = 20 * \left(0.55 + \left(\frac{477 - f_s}{120\left(\frac{M_{app}}{bd^2} + 0.9\right)}\right)\right) - \frac{L}{d} \tag{29}$$

Applying equation (2), (25), (26) and (27), the limit state function in deflection is given by:

$$G(x) = 20 * \left(0.55 + \left(\frac{477 - f_s}{120\left(\frac{(1.4a + 1.6)q_k L^2}{8bd^2} + 0.9\right)}\right)\right) - \frac{L}{d} \tag{30}$$

3. ESTIMATE OF RELIABILITY INDEX

Let the limit state function in the space of input variables $X_1, X_2, ..., X_n$ be given by:

$$g(X_1, X_2, ..., X_n) = 0 \tag{31}$$

Also, the input variables are assumed to be collected in the vector $X = [X_1, X_2, ..., X_n]^T$ with second moment statistics $E(X)$ and $Cov(X, X^T)$.

The normalized random variables $Y_1, Y_2, ..., Y_n$ are introduced by using a one to one linear mapping, $X = L(Y)$ such that $Y = L^{-1}(X)$. The corresponding y space is then defined by the transformation:

$$X = L(Y), \quad Y = L^{-1}(X) \tag{32}$$

where L = cholesky factor of the transformed input vector

Applying equation (32) maps equation (31) into:

$$h(y_1, y_2, ..., y_n) = 0 \tag{33}$$

The function h is defined by:

$$h(y) = g[L(y)] \tag{34}$$

Equation (33) is the limit state equation in normalized coordinate. The mean value of y is the origin and the projection of y on the arbitrary straight line through the origin yields the random variable with a standard deviation of unity.

The distance between the origin and the failure surface in normalized coordinate is the geometric reliability index β. It is given by:

$$\beta = \min\left\langle \sqrt{\sum y_1^{\ 2} + y_2^{\ 2} + \ldots + y_n^{\ 2}} \,\middle|\, h(y_1, y_2, \ldots, y_n) \right\rangle = 0 \qquad (35)$$

In matrix notation, equation (35) can be re-written as:

$$\beta = \min\left\langle \sqrt{y^t y} \,\middle|\, h(y) \right\rangle = 0 \qquad (36)$$

The values of the design variables, $y = (y_1, y_2, \ldots, y_n)^T$ that minimize the reliability index, β subject to $h(y_1, y_2, \ldots, y_n) = 0$ and the corresponding reliability index are found by invoking an optimization algorithm.

4. AN EXAMPLE OF A SINGLY REINFORCED CONCRETE RECTANGULAR BEAM

A singly reinforced concrete rectangular beam of span 7.5 m under uniform loading consisting of dead and imposed load of 12.5KN/m and 8.5KN/m, was designed in accordance with the design provisions of BS 8110: part 1-3, 1997. The safety of the deterministic design of the singly reinforced concrete rectangular beam was investigated using First Order Reliability procedure coded in MATLAB language. The statistics of the basic variables are presented in Table 1.

Table 1: Statistics of basic variables

S/N	Variable	Mean	Standard Deviation	Variation Coefficient	Type of probability distribution
1	Area of tension reinforcement, A_s	1176 mm²	17.64 mm²	0.015	Normal
2	Beam effective depth, d	450 mm	18 mm	0.04	Normal
3	Beam span, L	6000 mm	300 mm	0.05	Normal
4	Load Ratio, α	0.20	-	-	Deterministic
5	Yield strength of steel, f_y	460 N/mm²	23 N/mm²	0.05	Normal
6	Imposed load on beam, q_k	8.5 KN/m	2.55 KN/m	0.30	Gamma
7	With of beam, b	275 mm	11 mm	0.04	Normal
8	Concrete strength, f_{cu}	30 N/mm²	5.4 N/mm²	0.18	Normal
9	Steel ratio, ρ	0.01	0.0016	0.16	Normal
10	Area of link reinforcement, A_{sv}	100 mm²	1.5 mm²	0.015	Normal
11	Spacing of link reinforcement, S_v	150 mm	6 mm	0.04	Normal
12	Yield strength of link reinforcement, f_{yv}	250 N/mm²	12.5 N/mm²	0.05	Normal
13	Area per spacing of link, A_{sv}/S_v	0.667	-	-	Deterministic

Source: Nader and Okasha, 2017; Ranganathan 1990; Abubakar and Aliyu, 2017; Abubakar et al., 2014; Cavaco et al., 2010

5. RESULTS AND DISCUSSION

The reliability indices corresponding to the various failure criteria considered in the study were obtained using a computer program developed in MATLAB language based on First Order reliability estimate. The results obtained from the sensitivity analyses conducted on the random variables are shown in Figures 2 to 16 respectively.

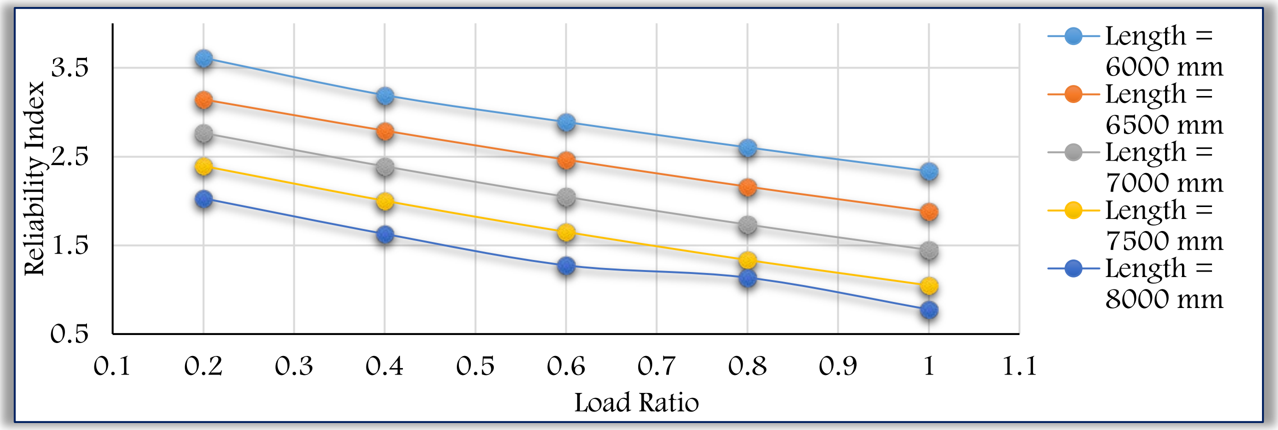

Figure.2: Reliability index against load ratio for varying beam span (beam capacity based on concrete)

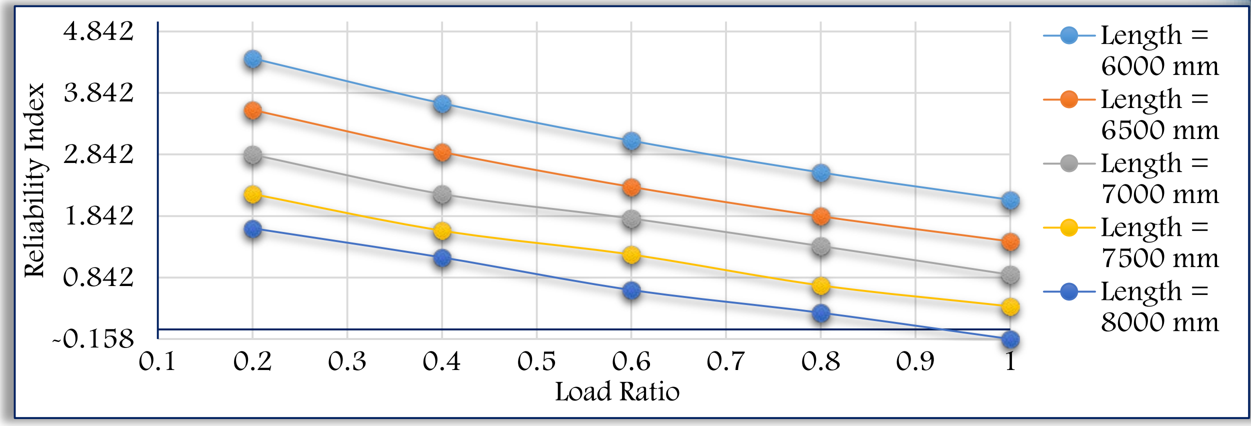

Figure.3: Reliability index against load ratio for varying beam span (beam capacity based on steel)

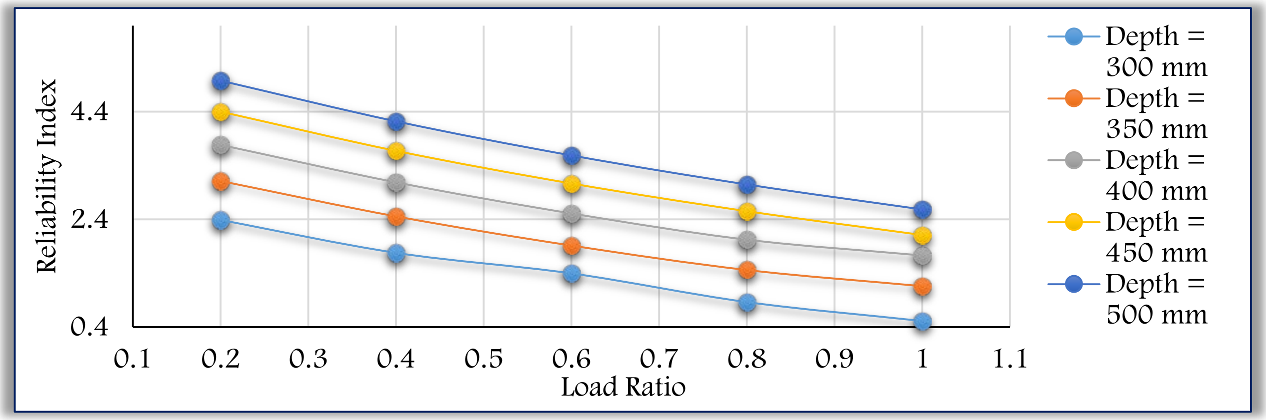

Figure 4: Reliability index against load ratio for varying effective depth of beam (beam capacity based on steel)

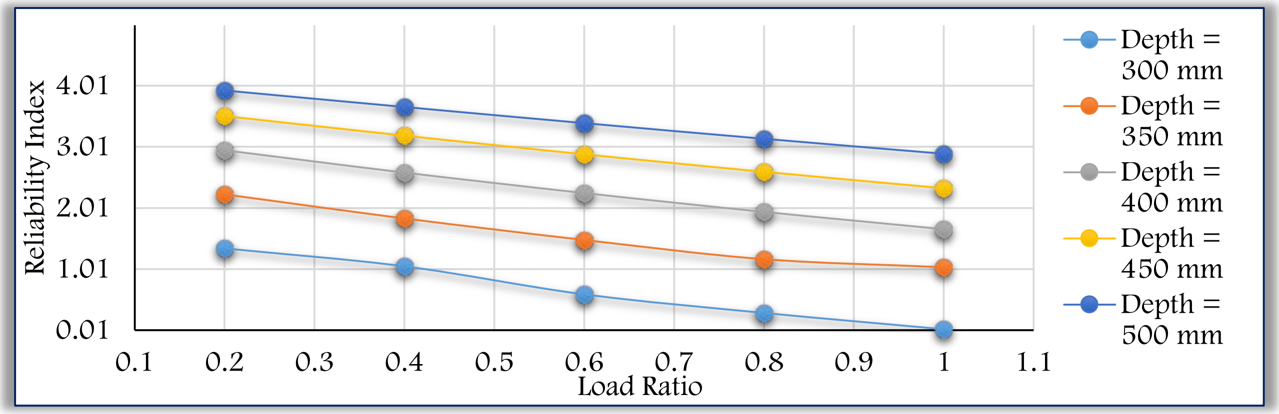

Figure 5: Reliability index against load ratio for varying effective depth of beam (beam capacity based on concrete

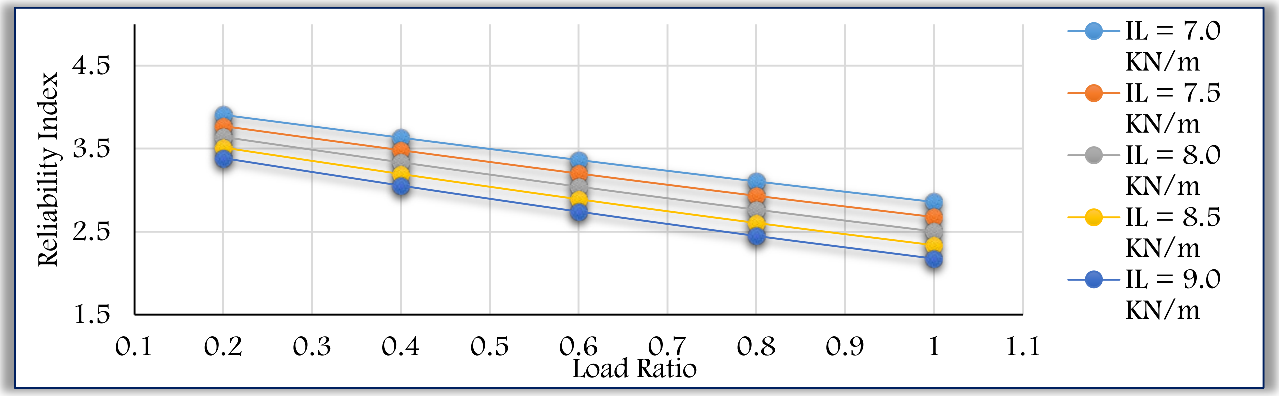

Figure 6: Reliability index against load ratio for varying imposed load (beam capacity based on concrete)

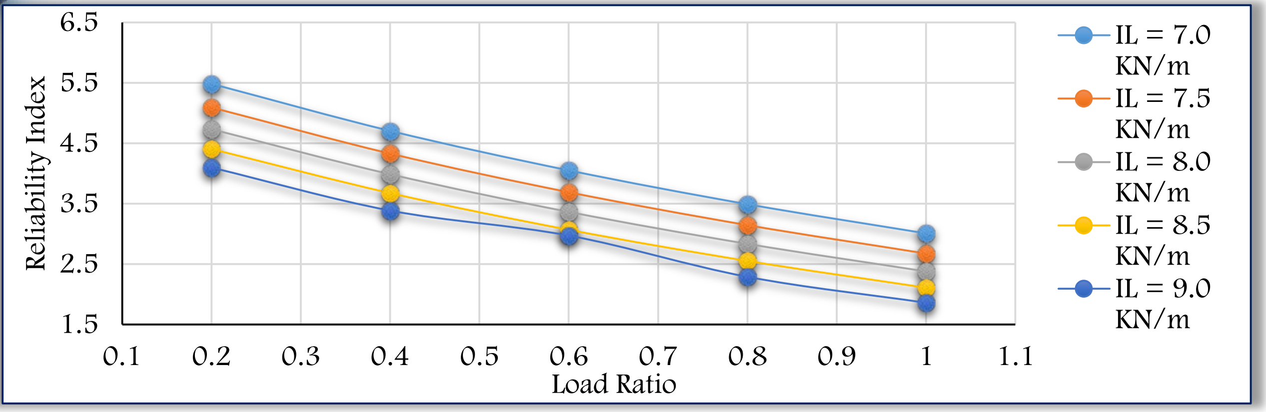

Figure7: Reliability index against load ratio for varying imposed load (beam capacity based on steel)

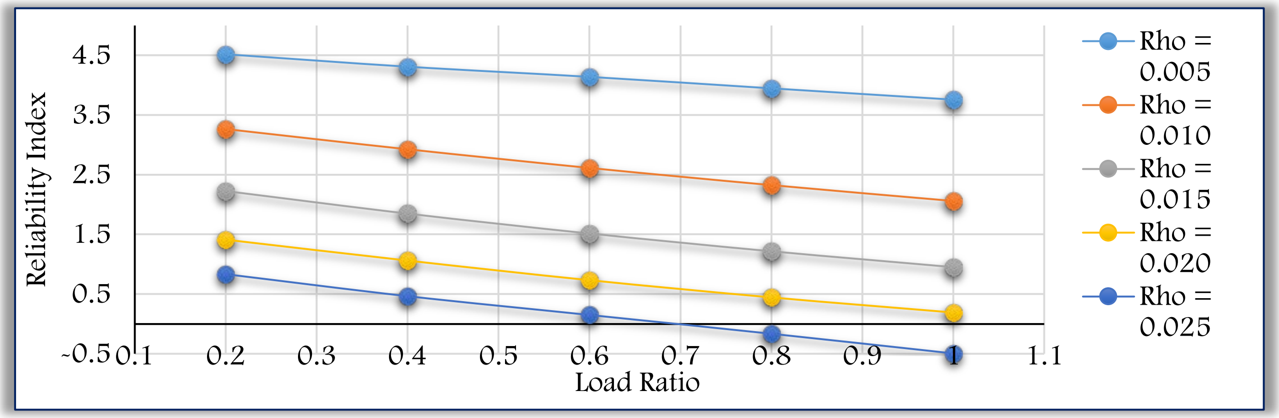

Figure 8: Reliability index against load ratio for varying steel ratio (beam capacity based on concrete)

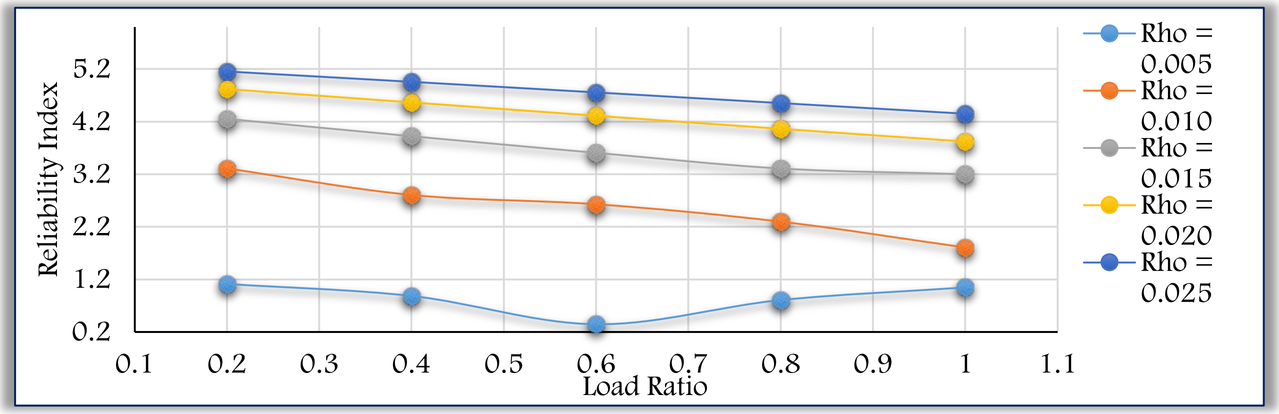

Figure 9: Reliability index against load ratio for varying steel (beam capacity based on steel)

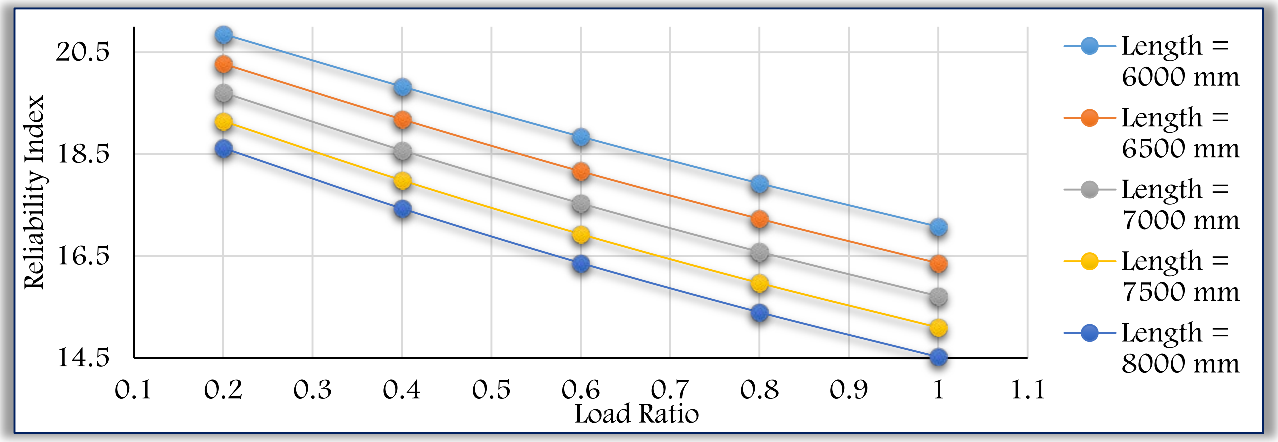

Figure 10: Reliability index against load ratio for varying beam span (Shear criterion)

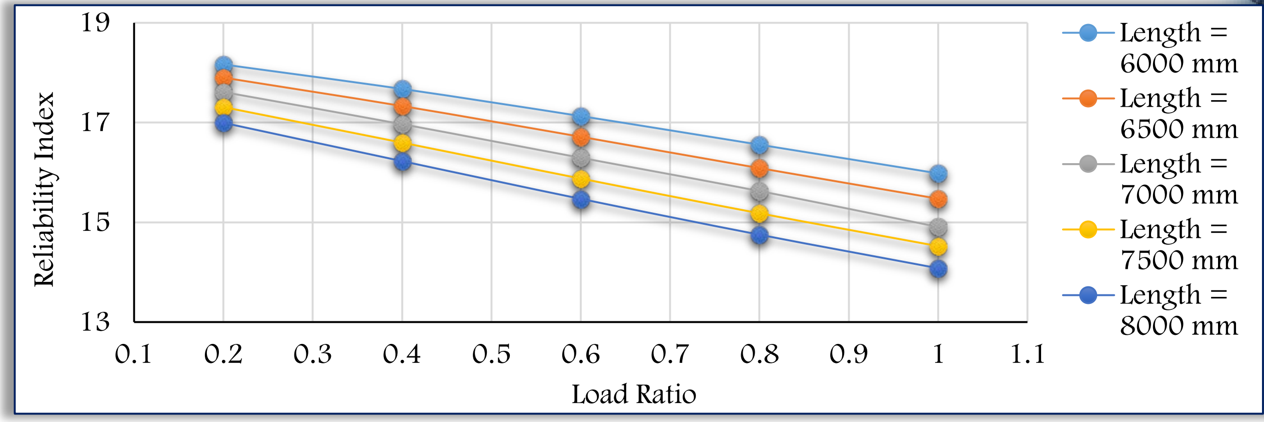

Figure 11: Reliability index against load ratio for varying beam span (Shear criterion)

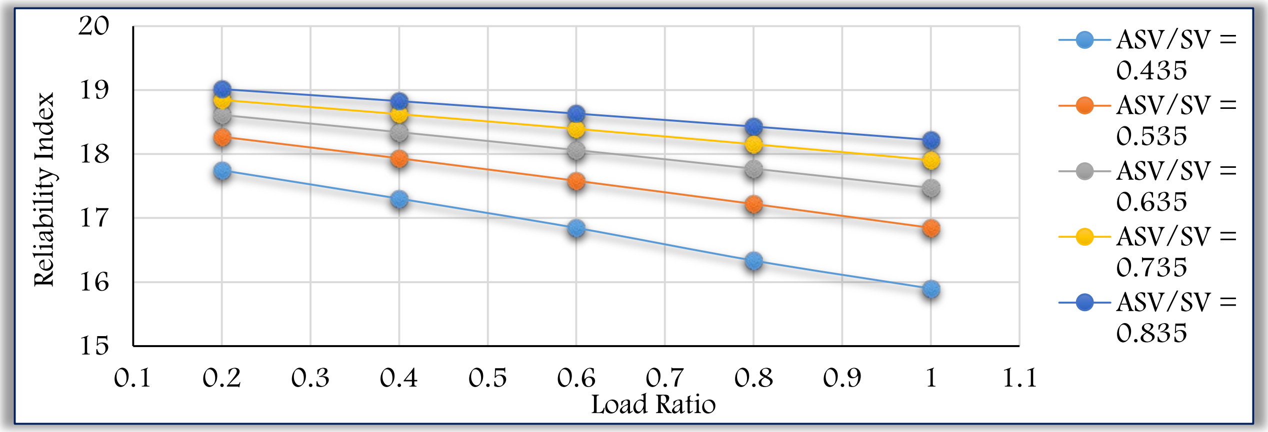

Figure 12: Reliability index against load ratio for varying ASV/SV (Shear criterion)

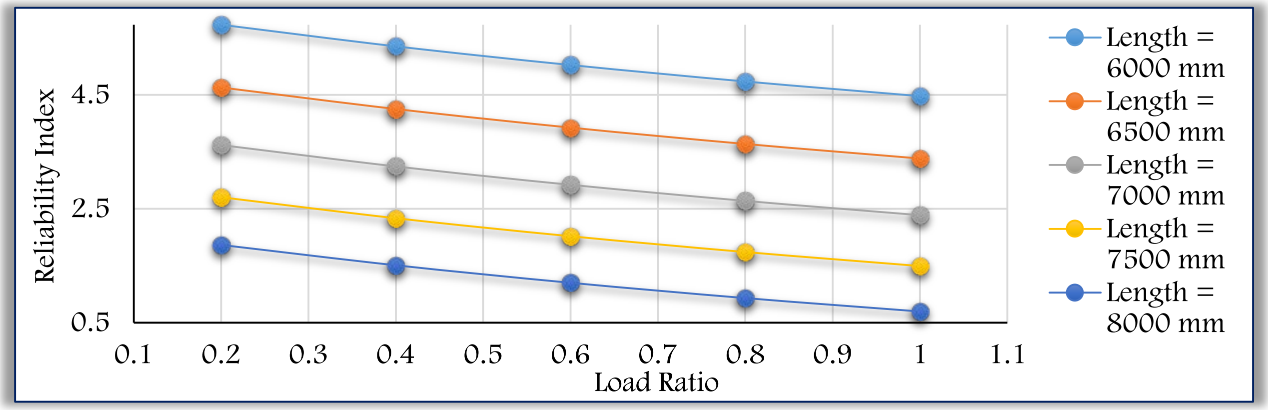

Figure 13: Reliability index against load ratio for varying beam span (Deflection criterion)

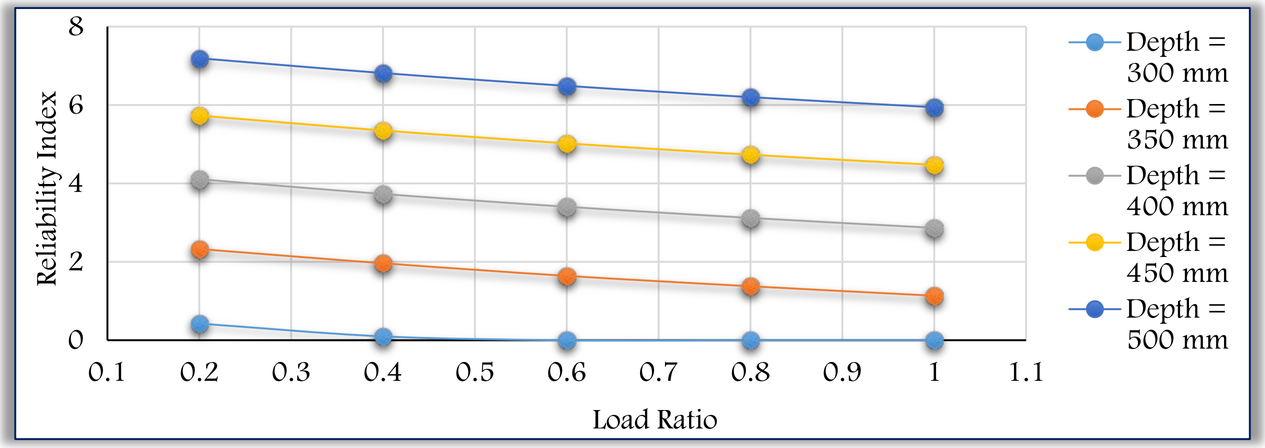

Figure 14: Reliability index against load ratio for varying effective depth of beam (Deflection criterion)

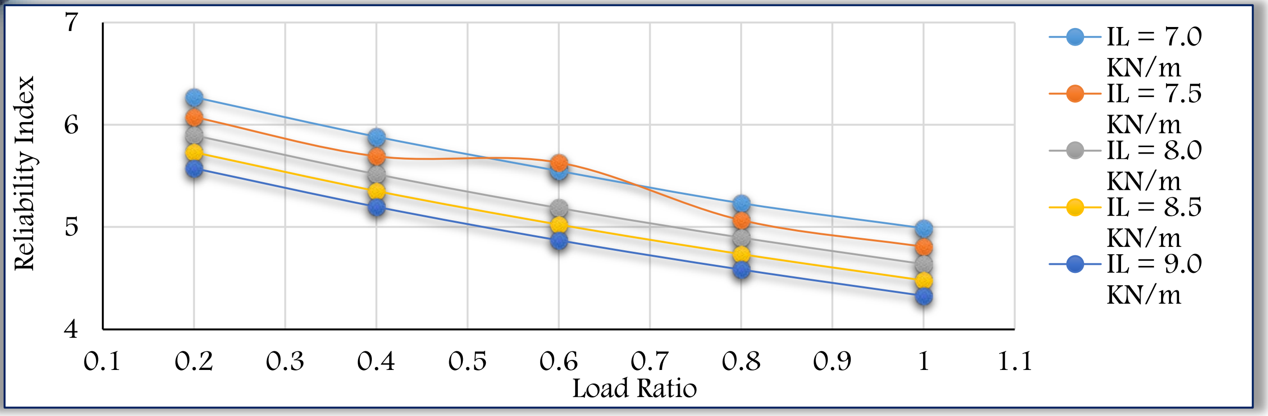

Figure 15: Reliability index against load ratio for varying imposed load (Deflection criterion)

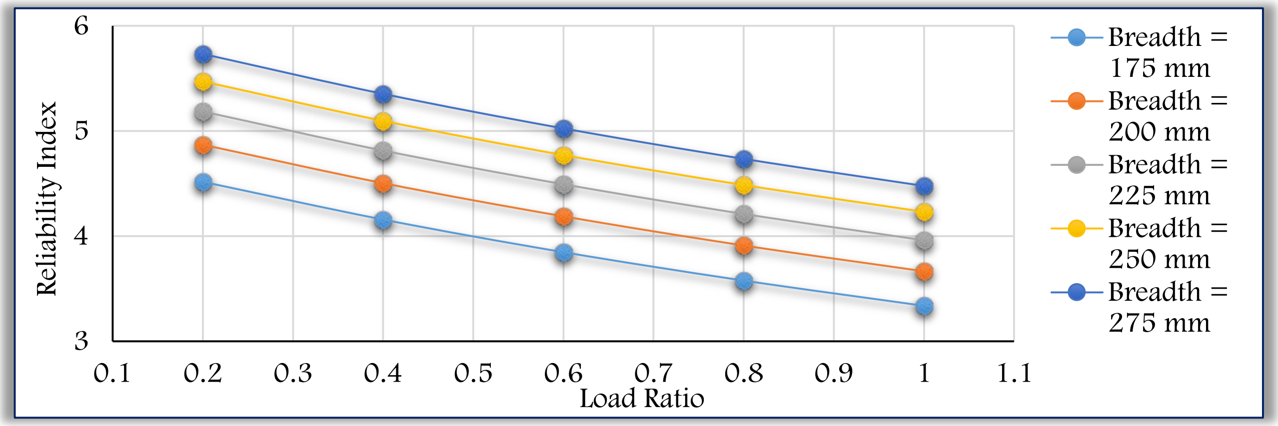

Figure 16: Reliability index against load ratio for varying width of beam (Deflection criterion)

The design points on the failure surface and the corresponding reliability indices for the various failure modes considered were obtained by invoking a computer program written in MATLAB language. Figures 2 to 16 show the relationship between the reliability indices and varied values of the random parameters for bending, shear and deflection criterion. It can be observed from plots that:

— The reliability indices generally decrease with increase in load ratio and beam span for beam capacity based on concrete and steel, shear and deflection criterion. This is because increasing the beam span increased the applied moment on the beam with a resultant reduction in the load carrying capacity of the beam which reduced the reliability levels of the beam.
— Increasing the load ratio and beam span beyond 0.9 and 8m will jeopardize the safety of the beam. The negative value of the reliability index shows that the load ratio value of 0.9 and 8m are not safe (EN 1990, 2002).
— The reliability indices decrease with increase in load ratio and increased with increase in effective depth of the beam for beam capacity based on concrete and steel (Figures 4 and 5). The increase in safety level with increase in effective depth of the beam is due to increased beam stiffness with a resultant increase in the load carrying capacity of the beam. The increased load ratio resulted to reduction in the load carrying capacity of the beam leading to reduced reliability level.
— Reliability indices decrease with increase in imposed load acting on the beam for beam capacity based on concrete and steel (Figures 6 and 7). The decreased safety levels may be attributed to the reduction of load carrying capacity of the beam with increased values of imposed load.
— The reliability indices decrease with increase in steel ratio for beam capacity based on concrete and steel (Figures 8 and 9) respectively. The decreased reliability level with increased value of steel ratio resulted from congestion of steel reinforcement with increased values of steel ratio leading to reduction of the beam load carrying capacity.
— The reliability indices decrease with increase in area per spacing of shear reinforcement of beam considering shear failure criterion (Figure 12).

— The reliability indices increase with increase beam effective depth considering the deflection failure criterion (Figure 14).
— The reliability indices decrease with increase in imposed load acting on beam considering the deflection failure criterion (Figure 15).
— The reliability index increase with increase in breadth of beam considering the deflection failure criterion (Figure 16).

5. CONCLUSION

The results of reliability analysis of a reinforced concrete rectangular beam for varying load ratio, beam span, imposed load, depth, breadth and steel ratio considering beam capacity based on concrete and steel, shear and deflection using First Order reliability procedure have been presented. The results of the reliability analysis showed that reliability indices generally decreased with increase in load ratio and beam span for beam capacity based on concrete and steel, shear and deflection criterion. The reliability indices also increased with increase in effective depth of the beam, decreased with increase in imposed load acting on the beam, decreased with increase in steel ratio for beam capacity based on concrete and steel, decreased with increase in area per spacing of shear reinforcement of beam for shear criterion, increased with increase in the effective depth of beam and increased with increase in breadth of beam for deflection criterion. The design was found to be conservative in shear but satisfactory in bending and deflection when compared with the target safety index value of 3.8 for 50-year reference period for Reliability class 2 at ultimate limit state.

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