

## MATHEMATICAL ANALYSIS OF AIR POLLUTION FROM STACKS USING BRIGGS METHOD

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**Abstract:** In order to determine the pollutants concentration emitted by point stationary sources (stacks), the effective height of the emission must be known. Among the formulas presented in the literature for the calculation of the plume rise, Briggs equations are the most used. In this study Briggs method is used to estimate plume rise above a stationary point source (stack) considering the influence of climatic factors (wind speed and atmospheric air temperature) and effluent characteristics (temperature and velocity at stack exit). Results are presented for neutral atmospheric stability class depending on distance from the stack. This study presents an analysis of air dispersion of a gaseous effluent emitted through a stack having a height of 220 m and 4 m in diameter. The maximum height at which plume centerline rises above stack is estimated using Briggs equations. Also, is considered the influence of several factors which are given further. All estimations are presented for neutral atmospheric stability class in function of the distance from the point source up to 3000 meters, at 500 m intervals.

**Keywords:** Briggs method, buoyant plume rise, stack, emissions, air pollution

### 1. INTRODUCTION

In order to determine the pollutants concentration emitted by point stationary sources (stacks), the effective height of the emission must be known [1]. Source effective height is usually defined as the sum of the stack constructed height ( $H_c$ ) and the plume centerline rise ( $\Delta h$ ) due to buoyancy and momentum fluxes [1, 2]. So, the stack effective height is given by the relation:  $H = H_c + \Delta h$ . In figure 1 [1] is shown a schematic representation of bent-over (a) and vertical (b) plumes also indicating plume rise,  $\Delta h$ .

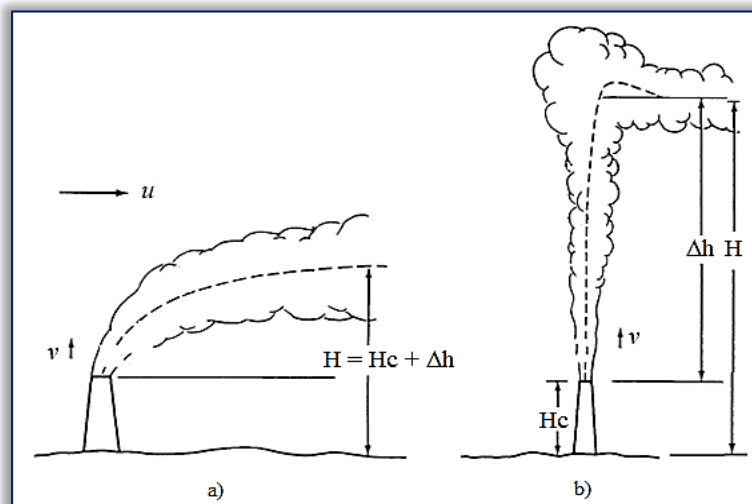


Figure 1. Schematic representation of bent-over (a) and vertical (b) plumes [1]. The plume trajectory in atmosphere is influenced by: stack characteristics, climatic conditions and effluent properties [1-3]. Buoyancy of a plume is given by effluent temperature being greater than that of surrounding air, while plume momentum is given by effluent velocity at stack exit greater than wind speed [4].

Among the formulas presented in the literature for the calculation of the plume rise [2-5], Briggs equations are the most used. In order to calculate plume rise  $\Delta h$  using Briggs equations for unstable or neutral atmospheric stability classes, are required the following quantities [2-8]:

a) Buoyancy flux parameter ( $F_b$ ) and momentum flux parameter ( $F_m$ ):

$$F_b = g \cdot v \cdot d^2 \cdot \frac{T_s - T_a}{4 \cdot T_s} \left( \frac{m^4}{s^3} \right) \quad (1)$$

$$F_m = v^2 \cdot d^2 \cdot \frac{T_a}{4 \cdot T_s} \left( \frac{m^4}{s^2} \right) \quad (2)$$

b) Crossover temperature difference  $(\Delta T)_c$  which indicate if the plume is buoyant or momentum dominated:

$$(\Delta T)_c = 0.0297 \cdot T_s \cdot \frac{v^{\frac{1}{3}}}{d^{\frac{2}{3}}} \text{ (K) , for } F_b < 55 \frac{m^4}{s^3} \quad (3)$$

$$(\Delta T)_c = 0.00575 \cdot T_s \cdot \frac{v^{\frac{2}{3}}}{d^{\frac{1}{3}}} \text{ (K) , for } F_b \geq 55 \frac{m^4}{s^3} \quad (4)$$

if  $\Delta T = T_s - T_a \geq (\Delta T)_c$  the plume rise is buoyant dominated and if  $\Delta T = T_s - T_a < (\Delta T)_c$  is momentum dominated.

c) Downwind distance to final plume rise (i.e. distance factor),  $x_f$ :

$$x_f = 49 \cdot F_b^{\frac{5}{8}} \text{ (m) , for } F_b < 55 \frac{m^4}{s^3} \quad (5)$$

$$x_f = 119 \cdot F_b^{\frac{2}{5}} \text{ (m) , for } F_b \geq 55 \frac{m^4}{s^3} \quad (6)$$

d) Plume rise,  $\Delta h$  :

$$\Delta h = 1.6 \cdot F_b^{\frac{1}{3}} \cdot \frac{x^{\frac{2}{3}}}{u} \text{ (m) , for } x < x_f \quad (7)$$

$$\Delta h = 1.6 \cdot F_b^{\frac{1}{3}} \cdot \frac{x_f^{\frac{2}{3}}}{u} \text{ (m) , for } x \geq x_f \quad (8)$$

Other symbols in equations (1-8) are:  $g = 9,81 \text{ m/s}^2$  - acceleration due to gravity,  $v \text{ (m/s)}$  - effluent velocity at stack exit,  $d \text{ (m)}$  - stack inside diameter,  $T_s \text{ (K)}$  - effluent temperature at stack exit,  $T_a \text{ (K)}$  - atmospheric air temperature,  $x \text{ (m)}$  - downwind distance from stack,  $u \text{ (m/s)}$  - wind speed.

This study presents an analysis of air dispersion of a gaseous effluent emitted through a stack having a height of 220 m and 4 m in diameter. The maximum height at which plume centerline rises above stack is estimated using Briggs equations. Also, is considered the influence of several factors which are given further. All estimations are presented for neutral atmospheric stability class in function of the distance from the point source up to 3000 meters, at 500 m intervals.

## 2. EVALUATION OF PLUME RISE

For analyzing the effect of velocity and temperature of both surrounding air and effluent on plume rise, the following values were considered:

- ≡ wind speed at stack exit:  $u = (5, 15, 20) \text{ m/s}$ ;
- ≡ atmospheric air temperature:  $t_a = (5, 10, 20)^\circ\text{C}$ ;
- ≡ effluent velocity at stack exit:  $v = (20, 25, 30) \text{ m/s}$ .
- ≡ effluent temperature at stack exit:  $t_s = (150, 200, 250)^\circ\text{C}$ .

### —The influence of wind speed

For evaluation of the influence of given above values for the wind speed,  $u$ , on plume rise other used parameters are:  $t_a = 10^\circ\text{C}$  (air temperature),  $v = 25 \text{ m/s}$  (effluent velocity) and  $t_s = 200^\circ\text{C}$  (effluent temperature). With these parameters the buoyant flux and the downwind distance to final plume rise, i.e. distance factor, were calculated with relations (1) and (6) respectively and given in Table 1.

Table 1. Calculated buoyant flux and distance factor in function of wind speed

Wind speed, $u \text{ (m/s)}$	Buoyant flux, $F_b \left( \frac{m^4}{s^3} \right)$	Distance factor, $x_f \text{ (m)}$
5	393.93	1299.32
15		
20		

Next, the crossover temperature difference which indicate if the plume is buoyant or momentum dominated is calculated with relation (4) for  $F_b \geq 55 \frac{m^4}{s^3}$ :  $(\Delta T)_c = 0.00575 \cdot 473 \cdot \frac{25^{\frac{2}{3}}}{4^{\frac{1}{3}}} = 14.4 \text{ K}$ . As the condition  $\Delta T = 190 \text{ K} \geq (\Delta T)_c$  is fulfilled, results that plume rise is buoyant dominated. So, plume rise may be calculated with relation (7) when  $x < x_f$  and with relation (8) when  $x \geq x_f$ . In figure



2 are given the results in graphical form, i.e. plume rise versus downwind distance from stack, in function of three values of wind speed: 5 m/s, 15 m/s and 20 m/s. From Figure 2 it is observed that plume rise increases with downwind distance up to the calculated value of  $x_f$  (table 1) from where it remains constant, but decreases with increasing wind speed. As the effluent is carried by the air current, the wind movement in horizontal direction at high speed prevents the plume from rising. Thus, if the wind blows at a speed of 20 m/s, the maximum height of plume rise is about 70 m, at a speed of 15 m/s the plume rises at about 93 m, while at a speed of 5 m/s the pollutant plume rises to about 280 m above stack.

—The influence of atmospheric air temperature

Further is analyzed the influence of air temperature considering three values: 5, 10 and 20 °C. Other used parameters for plume rise calculation are:  $u = 15$  m/s (wind speed),  $v = 25$  m/s (effluent velocity) and  $t_s = 200$  °C (effluent temperature).

With these variable parameters the buoyant flux and the downwind distance to final plume rise, i.e. distance factor, were calculated with relations (1) and (6) respectively and given in Table 2.

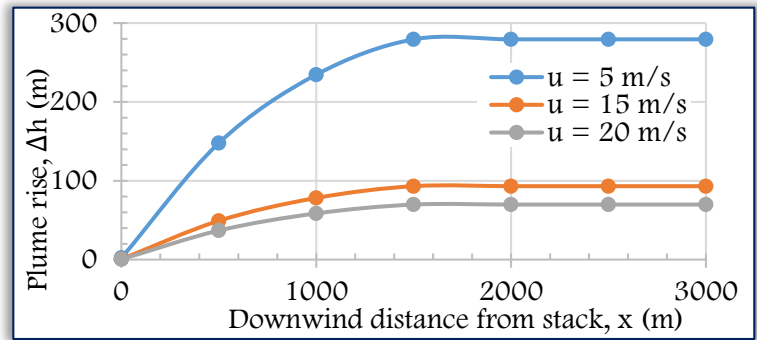


Figure 2. Plume rise versus downwind distance from stack, in function of wind speed

Table 2. Calculated buoyant flux and distance factor in function of air temperature

Air temperature, $t_a$ (°C)	Buoyant flux, $F_b$ ( $\frac{m^4}{s^3}$ )	Distance factor, $x_f$ (m)
5	404.30	1312.89
10	393.93	1299.32
20	373.20	1271.52

From obtained results it is observed that both buoyant flux and distance factor values decrease with increasing air temperature (see table 2). In figure 3 are given the results of plume rise versus downwind distance from stack, in function of air temperature, determined with relations (7) and (8) respectively. From figure 3 it may be observed that for the chosen range between (5÷20)°C, the atmospheric air temperature does not have a major influence on plume rise values which are all estimated at about 90-95 m.

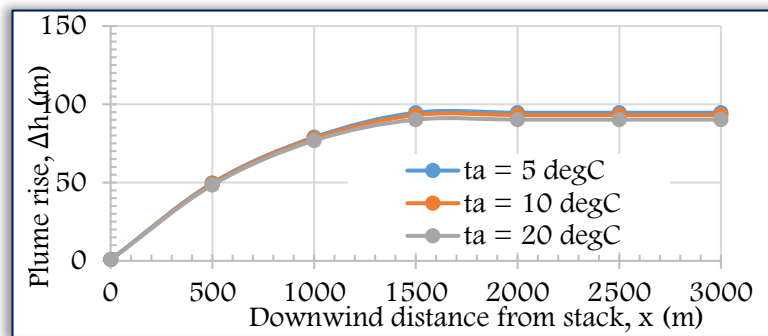


Figure 3. Plume rise versus downwind distance from stack, in function of air temperature

—The influence of pollutant velocity at stack exit

Considering effluent velocity as the variable parameter (which will take the values: 20 m/s, 25 m/s and 30 m/s), fixed parameters are:  $u = 15$  m/s (wind speed),  $t_a = 10$  °C (air temperature) and  $t_s = 200$  °C (effluent temperature). In table 3 are given the buoyant flux and distance factor in function of effluent velocity estimated with relations (1) and (6) respectively. Increasing the effluent velocity, both  $F_b$  and  $x_f$  increase.

Table 3. Calculated buoyant flux and distance factor in function of effluent velocity

Effluent velocity at stack exit, $v$ (m/s)	Buoyant flux, $F_b$ ( $\frac{m^4}{s^3}$ )	Distance factor, $x_f$ (m)
20	315.14	1188.37
25	393.93	1299.32
30	472.72	1397.62

Figure 4 shows plume rise versus downwind distance from stack, in function of effluent velocity. It is observed that an increase in the velocity of the emitted gas causes an increase in the plume rise. Pollutant emission with a speed of 20 m/s may cause the plume centerline to rise above stack by about 862 m, and the emission with a speed of 30 m/s, raises the pollutant plume by about 1161 m.

— **The influence of effluent temperature at stack exit**

Next, plume rise was estimated considering the following initial data:  $t_s = (150, 200, 250)^\circ\text{C}$  (effluent temperature),  $u = 15 \text{ m/s}$  (wind speed),  $t_a = 10^\circ\text{C}$  (air temperature) and  $v = 25 \text{ m/s}$  (effluent velocity). With these data were estimated the buoyant flux and distance factor given in table 4 and plume rise variation with downwind distance shown in figure 5.

Table 4. Calculated buoyant flux and distance factor in function of gas temperature

Effluent temperature at stack exit, $t_s$ ( $^\circ\text{C}$ )	Buoyant flux, $F_b$ ( $\frac{\text{m}^4}{\text{s}^3}$ )	Distance factor, $x_f$ (m)
150	324.56	1202.45
200	393.93	1299.32
250	450.04	1370.44

As expected, buoyant flux value increase with effluent temperature and also the distance from stack where plume centerline is maximum (table 4). Also, an effluent temperature difference of  $100^\circ\text{C}$  causes an increase of about 20 m of pollutant plume rise (figure 5).

**3. CONCLUSIONS**

Plume rise value is an important parameter to determine effective stack high in order to evaluate the pollutant concentration at ground level [1-3]. The Briggs method used in this case study permitted estimation of buoyant plume rise, i.e. the height at which the plume centerline attends maximum value. The influence of four factors was considered, namely: velocity and temperature of both surrounding air end effluent.

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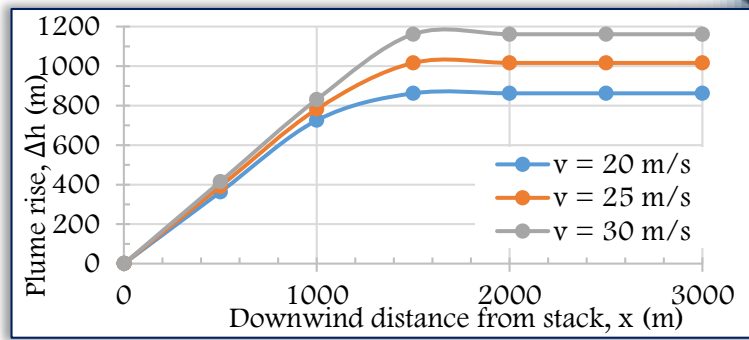


Figure 4. Plume rise versus downwind distance from stack, in function of effluent velocity

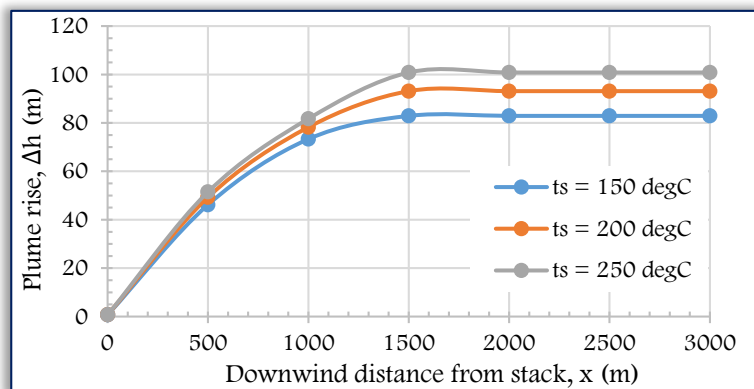


Figure 5. Plume rise versus downwind distance from stack, in function of effluent temperature