[1.]Dapkupar NONGRUM, [2.]Rajkishur MUDOI

ASER OF RQAM WITH MRC RECEIVER OVER CORRELATED RAYLEIGH FADING CHANNELS

[1-2.] Department of Electronics & Communication Engineering, North-Eastern Hill University, Shillong, INDIA

Abstract: In this paper, the average symbol error rate (ASER) with rectangular quadrature amplitude modulation (RQAM) scheme has been analyzed over Rayleigh fading channels considering exponential correlation among the input branches of an MRC receiver. Exponential correlation is noticed by the antennas placed in a linear array at the receiver. The effect of correlation on the ASER for various levels of fading parameter as well as with different diversity order has been observed.
Keywords: ASER, MRC, RQAM, Rayleigh fading

1. INTRODUCTION

Quadrature amplitude modulation (QAM) is a very useful modulation scheme for the high spectral efficiency in wireless communications. QAM scheme cab be classified as rectangular QAM (RQAM), cross QAM (XQAM) and square QAM (SQAM). RQAM is a common scheme, since it has SQAM, QPSK, BPSK, orthogonal binary frequency shift keying (BFSK) and multilevel amplitude shift keying modulation schemes as certain cases [1]. Rayleigh fading is useful in heavily built up urban environments. Rayleigh fading is applicable when there is no propagation along a line of sight between the transmitter and receiver. Rayleigh fading is a suitable model when there are many objects in the environment to scatter the radio signal before it appears at the receiver.

The performance for RQAM scheme under different fading environments, have been reported in several studies. In [2], an ASER expression was developed for cooperative diversity systems with RQAM scheme over Rayleigh fading channels. The ASER expression is derived for RQAM technique under Nakagami- m fading conditions in [3][4]. In [5], ASER analysis for the RQAM is expressed for MRC receiver over L independent and identically distributed Nakagami- m fading conditions.

We give an ASER expression for an MRC receiver for RQAM scheme over exponentially correlated Rayleigh fading channels. We apply the PDF based approach to obtain the expression of ASER.

2. CHANNEL MODEL DESCRIPTION

We consider an MRC receiver to improve the quality of the downlink information. In the MRC receiver, the received signals from all diversity antennas are co-phased, multiplied by a weight factor proportional to the branch SNR and add together to maximize the output SNR. The instantaneous output SNR γ of the MRC receiver is given by [6]

$$\gamma = \sum_{l=1}^{L} \gamma_l \,. \tag{1}$$

where, γ_l is the instantaneous SNR of l^{th} branch and L is the total number of diversity branches in the MRC receiver. The receiving antennas are placed in a linear array due to space constraints, hence exponential correlation is observed among the received signals. The correlation coefficient between i^{th} and j^{th} received antenna is defined as [6] [7]

$$\rho_{ij} = \rho^{|i-j|}; i, j = 1, 2, \ldots, L \,. \tag{2}$$

Performing RV transformation the PDF of output SNR of an MRC receiver can be given as [7]

$$f_\gamma(\gamma) = \frac{1}{\Gamma\left(\frac{L^2}{r}\right)\left(\frac{r\bar{\gamma}}{L}\right)^{\frac{L^2}{r}}} \gamma^{\frac{L^2}{r}-1} \exp\left(\frac{-L\gamma}{r\bar{\gamma}}\right), \tag{3}$$

where, $\bar{\gamma}$ is the average input SNR in each branch of MRC receiver and $r = L + \frac{2\rho}{1-\rho}\left(L - \frac{1-\rho^L}{1-\rho}\right)$, ρ is the correlation among the input branches of MRC receivers and $\Gamma(.)$ is the Gamma function.

3. ASER ANALYSIS

The ASER, P_e is defined as

$$P_e = \int_0^\infty P_e(e|\gamma)\, f_\gamma(\gamma)\, d\gamma \,. \tag{4}$$

where $P_e(e|\gamma)$ is the conditional error probability for AWGN channels and it is given for M-ary RQAM as [8]

$$P_e(e|\gamma) = 2\omega Q(a\sqrt{\gamma}) + 2\xi Q(b\sqrt{\gamma}) - 4\omega\xi Q(a\sqrt{\gamma})Q(b\sqrt{\gamma}) \,. \tag{5}$$

where, $M = M_I \times M_Q$; $\omega = 1-\left(\frac{1}{M_I}\right)$; $\xi = 1-\left(\frac{1}{M_Q}\right)$; $\mathcal{Q}(.)$ is the Gaussian $\mathcal{Q}$-function; $\beta = \frac{d_Q}{d_I}$; and $a = \sqrt{\frac{6}{(M_I^2-1)+(M_Q^2-1)\beta^2}}$; $b = \beta a$; d_Q and d_I are the quadrature and in-phase decision distance, respectively.

Using exponential approximation of Gaussian Q-function, i.e., $Q(z) \approx \frac{1}{12}e^{-\frac{z^2}{2}} + \frac{1}{4}e^{-\frac{2z^2}{3}}$ [9] in (5), $p(e|\gamma)$ is rewritten as[10]

$$\begin{aligned} P_e(e|\gamma) = {} & 2\omega\left[\frac{1}{12}e^{\frac{-a^2\gamma}{2}} + \frac{1}{4}e^{\frac{-2a^2\gamma}{3}}\right] + 2\xi\left[\frac{1}{12}e^{\frac{-b^2\gamma}{2}} + \frac{1}{4}e^{\frac{-2b^2\gamma}{3}}\right] \\ & -4\omega\xi\left[\frac{1}{144}e^{-\left(\frac{a^2+b^2}{2}\right)\gamma} + \frac{1}{48}e^{-\left(\frac{a^2}{2}+\frac{2b^2}{3}\right)\gamma} + \frac{1}{48}e^{-\left(\frac{2a^2}{3}+\frac{b^2}{2}\right)\gamma} + \frac{1}{16}e^{-\left(\frac{2a^2+2b^2}{3}\right)\gamma}\right]. \end{aligned} \tag{6}$$

Substituting equations (3) and (6) into (4), we have

$$\begin{aligned} P_e = {} & \frac{2}{\Gamma\left(\frac{L^2}{r}\right)\left(\frac{r\bar{\gamma}}{L}\right)^{\frac{L^2}{r}}} \Bigg\{ \underbrace{\int_0^\infty \omega\gamma^{\frac{L^2}{r}-1} \exp\left(\frac{-L\gamma}{r\bar{\gamma}}\right)\left[\frac{1}{12}e^{\frac{-a^2\gamma}{2}} + \frac{1}{4}e^{\frac{-2a^2\gamma}{3}}\right]d\gamma}_{\Delta_1} + \underbrace{\int_0^\infty \xi\gamma^{\frac{L^2}{r}-1} \exp\left(\frac{-L\gamma}{r\bar{\gamma}}\right)\left[\frac{1}{12}e^{\frac{-b^2\gamma}{2}} + \frac{1}{4}e^{\frac{-2b^2\gamma}{3}}\right]d\gamma}_{\Delta_2} \\ & -2\left[\underbrace{\int_0^\infty \omega\xi\gamma^{\frac{L^2}{r}-1} \exp\left(\frac{-L\gamma}{r\bar{\gamma}}\right)\left[\frac{1}{144}e^{-\left(\frac{a^2+b^2}{2}\right)\gamma} + \frac{1}{48}e^{-\left(\frac{a^2}{2}+\frac{2b^2}{3}\right)\gamma} + \frac{1}{48}e^{-\left(\frac{2a^2}{3}+\frac{b^2}{2}\right)\gamma} + \frac{1}{16}e^{-\left(\frac{2a^2+2b^2}{3}\right)\gamma}\right]d\gamma}_{\Delta_3}\right]\Bigg\} \end{aligned} \tag{7}$$

The ASER expression in (7) has three integral terms and putting $B = \frac{2}{\Gamma\left(\frac{L^2}{r}\right)\left(\frac{r\bar{\gamma}}{L}\right)^{\frac{L^2}{r}}}$, it can be shown as,

$$p_e = v_1 + v_2 - v_3 \,. \tag{8}$$

where, $v_1 = B\Delta_1$; $v_2 = B\Delta_2$, $v_3 = B\Delta_3$ and from (8),

$$v_1 = \frac{2\omega}{\Gamma\left(\frac{L^2}{r}\right)\left(\frac{r\bar{\gamma}}{L}\right)^{\frac{L^2}{r}}} \int_0^\infty \gamma^{\frac{L^2}{r}-1} \exp\left(\frac{-L\gamma}{r\bar{\gamma}}\right)\left[\frac{1}{12}e^{\frac{-a^2\gamma}{2}} + \frac{1}{4}e^{\frac{-2a^2\gamma}{3}}\right]d\gamma \,. \tag{9}$$

Applying [11, (3.381.4)] and after simplification, one can express (9) as,

$$v_1 = \omega\left[\frac{1}{6\left(1+\frac{a^2r\bar{\gamma}}{2L}\right)^{\frac{L^2}{r}}} + \frac{1}{2\left(1+\frac{2a^2r\bar{\gamma}}{3L}\right)^{\frac{L^2}{r}}}\right]. \tag{10}$$

Similarly the derivations for v_2 and v_3 can be shown as,

$$v_2 = \xi\left[\frac{1}{6\left(1+\frac{b^2r\bar{\gamma}}{2L}\right)^{\frac{L^2}{r}}} + \frac{1}{2\left(1+\frac{2b^2r\bar{\gamma}}{3L}\right)^{\frac{L^2}{r}}}\right], \tag{11}$$

and,

$$v_3 = \omega\xi\left[\frac{1}{36\left(1+\frac{r\bar{\gamma}}{L}\left(\frac{a^2+b^2}{2}\right)\right)^{\frac{L^2}{r}}}+\frac{1}{24\left(1+\frac{r\bar{\gamma}}{L}\left(\frac{a^2}{2}+\frac{2b^2}{3}\right)\right)^{\frac{L^2}{r}}}+\frac{1}{24\left(1+\frac{r\bar{\gamma}}{L}\left(\frac{2a^2}{3}+\frac{b^2}{2}\right)\right)^{\frac{L^2}{r}}}+\frac{1}{4\left(1+\frac{r\bar{\gamma}}{L}\left(\frac{2a^2+2b^2}{3}\right)\right)^{\frac{L^2}{r}}}\right] \quad (12)$$

4. RESULTS WITH DISCUSSIONS

The mathematical expression of ASER derived in section 3, is numerically evaluated. The results are plotted for different values of L the diversity order, correlation coefficient ρ and different values of β. In Figure 1, the ASER versus average SNR is presented for 4×2 RQAM and 8×4 RQAM schemes with different L values. In this figures, $\rho = 0.5$ and $\beta = 1$ are kept constant. It is seen from the Figure 1, that with the increase in the values of received antennas L at the MRC receiver, the ASER performance improves. In Figure 2, the ASER performance versus Average SNR is plotted for certain values of $\beta = 1$, $L = 4$ and varying values of correlation coefficient ρ. It is observed that with the increase in the correlation coefficient ρ, the ASER performance of the receiver degrades for both 4×2 RQAM and 8×4 RQAM.

In Figure 3, the ASER performance versus Average SNR is plotted for 4×2 RQAM and 8×4 RQAM schemes by changing β. From the Figure 3, it is seen that the ASER performance outperforms for $\beta = 1$ than the ASER performance with $\beta = 0.5$ and $\beta = 2$. In Figure 3, $\rho = 0.5$ and $L = 4$ are constant values. It can be noted from the Figures that the ASER performance of 4×2 RQAM scheme outperforms 8×4 RQAM scheme. Thus, as expected the ASER performance improves with increase in diversity order and decreases with increase in antenna correlation.

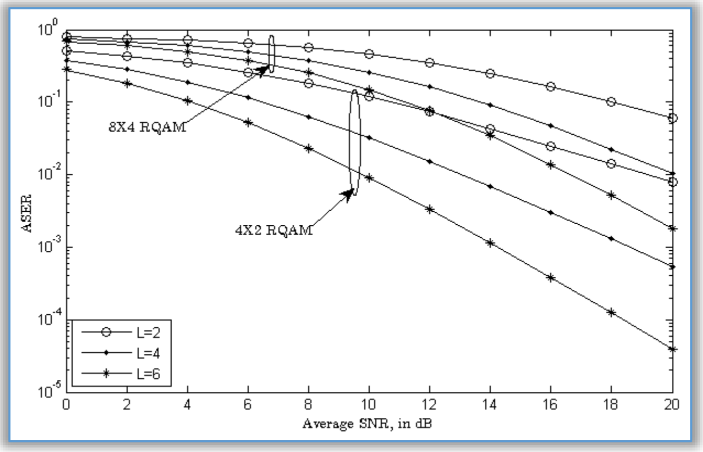

Figure 1: ASER vs. Average SNR ($\bar{\gamma}$) of 4×2 RQAM and 8×4 RQAM with $\rho = 0.5$ and $\beta = 1$.

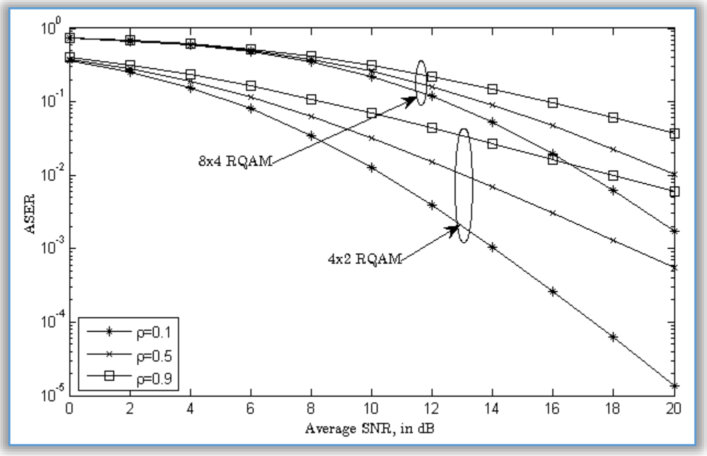

Figure 2: ASER vs. Average SNR ($\bar{\gamma}$) of 4×2 RQAM and 8×4 RQAM with $\beta = 1$ and $L = 4$.

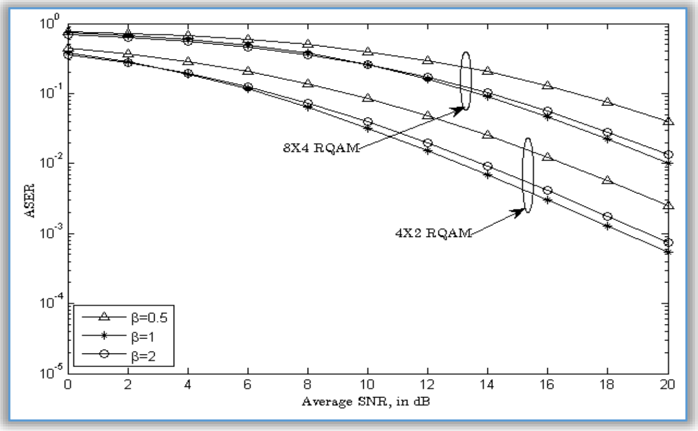

Figure 3: ASER vs. Average SNR ($\bar{\gamma}$) of 4×2 RQAM and 8×4 RQAM with $\rho = 0.5$ and $L = 4$.

5. CONCLUSIONS

In the paper, we examine the ASER performance of an L-branch MRC receiver system with RQAM technique, under Rayleigh fading channels. Closed form formulation is derived for ASER, which is obtainable in mathematical software. The evaluated results of the expression have been plotted to illustrate the effect of correlation and number of received antennas i.e., diversity branches on the receiver performance.

References

[1] Proakis JG. Digital Communications. 4th ed. New York: McGraw-Hill; 2001.

[2] D. Dixit, and P. R. Sahu, "Symbol error rate of rectangular QAM with best relay selection in cooperative systems over Rayleigh fading channels," IEEE Communications Letters, Vol.16, No.4, pp.466-469, 2012.

[3] N. Kumar, and V. Bhatia, "Exact ASER analysis of rectangular QAM in two-way relaying networks over Nakagami- m fading channels," IEEE Wireless Communications Letters, Vol.5, No.5, 2016, pp.548-551.

[4] G. K. Karagiannidis, "On the symbol error probability of general order rectangular QAM in Nakagami-m fading," IEEE Communications Letters, Vol.10, No.11, 2006, pp.745-747.

[5] X. Lei, P. Fan, and L. Hao, "Exact symbol error probability of general order rectangular QAM with MRC diversity reception over Nakagami- m fading channels," IEEE Commun. Lett., Vol. 11, No. 12, pp. 958-960, Dec. 2007.

[6] M. K. Simon and M.-S. Alouini, Digital Communications over Fading Channels, 2nd ed., Wiley-Interscience. John Wiley & Sons, Inc., 2005.

[7] V. A. Aalo, "Performance of maximal-ratio diversity systems in a correlated Nakagami-fading environment," IEEE Trans. Commun., Vol.43, No. 8, pp. 2360-2369, Aug. 1995.

[8] N. C. Beaulieu, "A useful integral for wireless communication theory and its application to rectangular signaling constellation error rates," IEEE Trans. Commun., Vol. 54, No. 5, pp. 802-805, May 2006.

[9] M. Chiani, D. Dardari, and M. K. Simon, "New exponential bounds and approximations for the computation of error probability in fading channels,'' IEEE Trans. Wireless Commun., Vol. 2, No. 4, pp. 840-845, Jul. 2003.

[10] M. Bilim and N. Kapucu, "Average Symbol Error Rate Analysis of QAM Schemes Over Millimeter Wave Fluctuating Two-Ray Fading Channels", IEEE Access, vol. 7, pp. 105746-105754, July, 2019.

[11] S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, Products, 6th ed. New York, NY, USA: Academic, 2000.

ISSN 1584 – 2665 (printed version); ISSN 2601 – 2332 (online); ISSN-L 1584 – 2665