[1.] K. L. VERMA

NONLOCAL WAVE SURFACES IN ELASTIC SOLIDS

[1.] Department of Mathematics, Dean Basic and Applied Sciences, Career Point University Hamirpur, INDIA

Abstract: Based on the nonlocal theory of elasticity, some interesting wave-surface features are studied for an elastic material. In nonlocal theory longitudinal and transverse waves remain in pure modes, become dispersive but non-attenuating and influenced by non-locality parameter, whereas theses waves are non-dispersive in its counterpart classical continuum mechanics theory (local theory). It is found that slowness surfaces consists of three concentric spheres, function of non-locality parameter, one with radius equal to the compressional wave slowness, and two, which are coincident, with radius equal to the shear wave slowness. It is observed that phase and group wave velocities are affected only when non-locality parameter is greater than or equal to 1×10^{-3} for polycrystalline 304 stainless steel. All waves are in pure mode. It is found that longitudinal wave (c_ℓ) is equal 1.834 transverse waves $\left(c_t\right)$. Thus, the longitudinal and transverse wave's surfaces cannot cross in the isotropic-solid case. Phase and Group velocities (m/s), polar diagram of phase velocity (m/s) and slowness surfaces for a typical isotropic material polycrystalline 304 stainless steel are also represented graphically.
Keywords: Nonlocal Elasticity, Wave Surface, Slowness, Dispersive, Isotropic, Phase and Group velocities

1. INTRODUCTION

With the initiation of new technologies, the continuous reduction of dimension of the devices from micro- to nano-scale, the consequence of long-range inter-atomic and intermolecular interrelated forces on the properties of such materials is quite significant and cannot be neglected. In classical continuum mechanics theory, elastic waves are mechanical waves propagate in an elastic medium as a consequence of forces connected with volume and shear deformations of medium elements, where the response of continuum is strictly limited to local actions only and the relevant theory is called *local theory*. In contrast to this local theory, of zero-range internal interactions, the modern *nonlocal* continuum mechanics developed and postulates that the local state at a point is influenced by the action of all particles of the body near that point ([1]-[2]). For investigative small-scale structures, most comprehensively reported theory, is the nonlocal elasticity theory, initiated and developed by [3] and [4].

Since the conventional continuum mechanics, which is scale free or size-independent, and its application to long wave limit in relation to the atomic theory is not capable to capture the small nanoscale size effect [5]-[7] when studying with nanostructures. Therefore, the study of length scales between atoms, surface stress properties, grain size, etc., which are correlated with nanostructures must be established to any classical continuum models.

1. Khurana and Tomar [8] have studied the propagation of Rayleigh type surface waves in nonlocal micropolar elastic solid half-space. Dilbag Singh *et al.*, [9] studied waves in nonlocal elastic solid with voids. Recently, Kaur et al. [10] have studied the propagation of Rayleigh-type surface waves in nonlocal elastic solid half-space with voids.
2. Slowness is defined as the inverse of velocity. Slowness surfaces, by means of Christoffel equation for the wave propagation in elastic media, displays many interesting features as in [11]-[13] extra an additional interface in order to imitate a finite thickness layer and endeavored to solve the simplest case of wave interaction with it. Studies of elastic waves in such simple and mostly isotropic systems are widely available in the books [14]-[17]. Verma [24] studied the thermoelastic slowness surfaces in anisotropic media with thermal relaxation.

Owing to the newness of the nonlocal theory, the aspects of wave quantities required in constructing wave fields propagating elastic media are calculated as a function of the slowness vector or of its direction called the wave normal. In this paper some interesting wave-surface features in based on nonlocal elasticity are described. It is found that slowness surfaces consists of three concentric spheres, one with radius equal to the compressional wave slowness, and two, which are coincident, with radius equal to the shear wave slowness are function of nonlocality parameter. All waves are in pure mode. Mechanical stability requires that c_ℓ exceed $1.834c_t$. Thus, the ℓ and t surfaces cannot cross in the isotropic-solid case. Phase and Group velocities (m/s), polar diagram of phase velocity (m/s) and slowness surfaces for a typical isotropic material polycrystalline 304 stainless steel are studied and also represented graphically.

2. LINEAR THEORY OF NONLOCAL ELASTICITY

Unlike the classical theory, the nonlocal stress tensor at a point depends not only on the strain at that point but also on the strain at all other points within the continuum. Consequently, in the constitutive relation of nonlocal elasticity theory Hooke's law (for local theory) is replaced by an integration which governs the nonlocal material behavior and for a linear, homogeneous elastic solids, the nonlocal dynamic equations of motion have the form [3]-[4]:

$$t_{ij,j} + f_i = \rho \ddot{u}_i \tag{1}$$

where ρ and f_i are the mass density and the body forces, respectively; u_i is the displacement vector; and t_{ij} is the stress tensor within the nonlocal body. The stress at a reference point is assumed to be a function of the strain field in the theory of nonlocal elasticity, at any point in the body, the nonlocal stress t_{ij} and classical (i.e., local) stress σ_{ij} have the following relation:

$$t_{ij}(x) = \int_V \alpha\left(|x' - x|, \tau\right) \sigma_{ij}(x') dV(x') \tag{2}$$

where

$$\tau = \frac{e_o a}{\ell} = \frac{\varepsilon}{\ell} \tag{3}$$

e_o :is a material constant; a : is an internal characteristic length such as lattice parameter or granular distance, ℓ ; is an external characteristic length; x : is a reference point in the body;
σ_{ij} :is the local or classical stress tensor; and $\alpha\left(|x' - x|, \tau\right)$: is a kernel function.

Further properties of the properties of the nonlocal kernel $\alpha\left(|x' - x|, \tau\right)$ are: $\alpha\left(|x' - x|, \tau\right)$ has maximum at $|x' - x|$; $\alpha\left(|x' - x|, \tau\right)$ tends rapidly to zero as $\alpha\left(|x' - x|, \tau\right)$ increases; $\alpha\left(|x' - x|, \tau\right)$ is a continuous function of $\alpha\left(|x' - x|, \tau\right)$; $\alpha\left(|x' - x|, \tau\right)$ is a delta-sequence, tending to the Dirac δ -function as $\tau \to 0$; $\lim_{\tau \to 0} \alpha\left(|x' - x|, \tau\right) = \delta |x' - x|$ .that is, nonlocal dependence becomes local in this passage to the limit;

$$\int_V \alpha\left(|x' - x|, \tau\right) dV(x') = 1 \tag{4}$$

Eringen [4] has studied the properties of the functions $\alpha\left(|x' - x|, \tau\right)$ and proposed some specific expressions for these kernels, which are in good agreement with the results of the theory of a crystal lattice. For example, in the three-dimensional space one can choose the following kernel:

$$\alpha\left(|x' - x|, \tau\right) = \frac{1}{8(\pi\eta)^{\frac{3}{2}}} \exp\left(\frac{|x' - x|^2}{4\eta}\right) \tag{5}$$

Where $\eta = a^2/(4k)^2$, a is the lattice parameter; k is the corresponding constant, which can be determined either by experiment or by applying the results of the theory of the atomic lattice.

When $\alpha\left(|x' - x|, \tau\right)$ takes on a Green function of a linear differential operator Ω i.e

$$\Omega \alpha\left(|x' - x|, \tau\right) = \delta\left(|x' - x|\right) \tag{6}$$

the nonlocal constitutive relation in Eq. (2) is transformed into the following relation

$$\Omega t_{ij} = \sigma_{ij} \tag{7}$$

and the integro-partial differential Eq. (1) is correspondingly reduced to the partial differential equation

$$t_{ij,j} + \Omega(f_i - \rho \ddot{u}_i) = 0 \tag{8}$$

By matching the dispersion curves with lattice models, Eringen [4] proposed a nonlocal model with the linear differential operator Ω defined by

$$\Omega = \left(1 - \tau^2 \ell^2 \nabla^2\right) \tag{9}$$

Therefore, the nonlocal constitutive relation can be simplified as

$$\left(1 - \tau^2 \ell^2 \nabla^2\right) t_{ij} = \sigma_{ij} \tag{10}$$

where t_{ij} is the stress tensor within the nonlocal body and σ_{ij} is the local or classical stress tensor. Therefore, the constitutive relations with this kernel function may be simplified to

$$\left(1-\left(e_0 a\right)^2 \nabla^2\right)\tau_{ij} = C_{ijkl} e_{kl} \tag{11}$$

and the and the governing equation can be modified to

$$\sigma_{ij,j} = \rho\left(1-\left(e_0 a\right)^2 \nabla^2\right)\ddot{u}_i \tag{12}$$

For simplicity and to avoid solving integro-partial differential equations, the non-local elasticity model, defined by the relations has been widely adopted for tackling various problems of linear elasticity and micro-/nanostructural mechanics.

3. ANALYSIS

In the nonlocal theory of elasticity, elasto-dynamical equation (12) describing the inertial forces can be written with (the displacement) as

$$C_{ijkl}\frac{\partial^2 u_k}{\partial x_j \partial x_l} = \rho\left(1-\varepsilon^2\nabla^2\right)\frac{\partial^2 u_i}{\partial t^2}, \tag{13}$$

where $\varepsilon\left(=e_0 a\right)$ is the nonlocality parameter wherein a is an internal characteristic length (lattice parameter, granular size or molecular diameters) and e_0 is a constant appropriate to each material.

Specializing the above equations (12) for an isotropic elastic solid medium, the local stress tensor σ_{kl} obeys

$$\sigma_{11} = C_{11}e_{11} + C_{12}e_{22} + C_{12}e_{33},\ \sigma_{22} = C_{12}e_{11} + C_{11}e_{22} + C_{12}e_{33}$$

$$\sigma_{33} = C_{12}e_{11} + C_{12}e_{22} + C_{11}e_{33},\ \sigma_{23} = 2C_{44}e_{23},\ \sigma_{13} = 2C_{44}e_{13},\ \sigma_{12} = 2C_{44}e_{12} \tag{14}$$

where C_{11}, C_{12} and C_{44} are the elastic constants of classical elasticity, which can be given in terms of Young's modulus E and Poisson's ratio υ as

$$C_{11} = \lambda + 2\mu = \frac{E\left(1-\upsilon\right)}{1-\upsilon-2\upsilon^2},\ C_{12} = \lambda = \frac{\upsilon E}{1-\upsilon-2\upsilon^2} \text{ and } C_{44} = \mu = \frac{C_{11}-C_{12}}{2}.$$

where λ and μ are Lame's parameter radjusting the model to match some reliable results by experiments or other theories. The strain tensor e_{kl} for isotropic elastic medium is defined as

$$e_{kl} = \frac{1}{2}\left(u_{k,l} + u_{l,k}\right).$$

The displacement of plane wave can be described by any harmonic form as a function of time (e.g. Fedorov, [25],

$$u_j = U_j \exp[i\xi(\mathbf{n.x} - ct)] \tag{15}$$

where ξ is the wave number, c is the phase velocity ($= \omega/\xi$), ω is the circular frequency, U_j are the constants related to the amplitudes of displacement, n_k (k = 1, 2, 3) are the components of the unit vector giving the direction of propagation. Inserting equation (14) into equations (13), generates the Christoffel equation of the form ([18]-[20]) in nonlocal elasticity, we have

$$\Pi_{ik} U_k = \mathbf{0} \tag{16}$$

where

$$\Pi_{ik} = \Gamma_{ik} - \rho\left(1+\varepsilon^2\xi^2\right)c^2\delta_{ik} \tag{17}$$

Equations (15) can be rewritten in the matrix form as

$$\left[\mathbf{\Gamma} - V^2\mathbf{I}\right]\mathbf{U} = \mathbf{0} \tag{18}$$

Where $\mathbf{I}$ is an identity matrix of order 3 and $V^2 = \rho\left(1+\varepsilon^2\xi^2\right)c^2$, in which δ_{ik} is the Kronecker delta, and Γ_{ik} are the Christofied stiffness as follows:

$$\Gamma_{ik} = \Gamma_{ki} = C_{ijkl} n_j n_l = \bar{\Gamma} \tag{19}$$

Here, n is the unit vector in the slowness direction; summation over repeated indices is implied. The Christoffel equation (2) describes a standard eigenvalue V^2 eigenvector (U) problem for the matrix $\bar{\Gamma}$, with the eigenvalues are determined by

$$\det\left(\Pi_{ik}\right) = 0 \tag{20}$$

Equations (19) simplifies to

$$\begin{vmatrix} \Gamma_{11}-\rho\left(1+\varepsilon^2\xi^2\right)c^2 & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{12} & \Gamma_{22}-\rho\left(1+\varepsilon^2\xi^2\right)c^2 & \Gamma_{23} \\ \Gamma_{13} & \Gamma_{23} & \Gamma_{33}-\rho\left(1+\varepsilon^2\xi^2\right)c^2 \end{vmatrix}=0 \tag{21}$$

implies

$$\left[\rho\left(1+\varepsilon^2\xi^2\right)c^2\right]^3+A\left[\rho\left(1+\varepsilon^2\xi^2\right)c^2\right]^2+B\left[\rho\left(1+\varepsilon^2\xi^2\right)c^2\right]+C=0$$

$$V^3+AV^2+BV+C=0 \tag{22}$$

where $A=C_{11}+2C_{44}$, $B=-(2C_{11}+C_{44})C_{44}$, $C=C_{11}C_{44}^2$.

For an isotropic media equation (22) reduces to

$$\left[C_{44}-\rho\left(1+\varepsilon^2\xi^2\right)c^2\right]^2\left[C_{11}-\rho\left(1+\varepsilon^2\xi^2\right)c^2\right]=0$$

$$c_1=\sqrt{\frac{C_{11}}{\rho\left(1+\varepsilon^2\xi^2\right)}},\ \text{and}\ c_{2,3}=\sqrt{\frac{C_{44}}{\rho\left(1+\varepsilon^2\xi^2\right)}}. \tag{23}$$

Equation (22) shows that although longitudinal and transverse waves remain in pure modes in nonlocal elasticity, become dispersive but non-attenuating and are influenced by non-locality parameter, while theses waves are non-dispersive and non-attenuating in its counterpart local theory of elasticity. It can be seen that the transverse wave in nonlocal elastic solid travels slower than that of longitudinal wave even in the presence non-locality parameter likewise as in case of classical continuum mechanics. When the nonlocality parameter $\varepsilon\left(=e_0a\right)=0$, then $c_1=\sqrt{\frac{C_{11}}{\rho}}$, $c_{2,3}=\sqrt{\frac{C_{44}}{\rho}}$. which are wave velocities of longitudinal and transverse wave in the classical continuum mechanics theory, (local theory) which becomes non-dispersive non-attenuating.

4. PHASE AND GROUP VELOCITIES

The phase vectors depict the direction of the phase velocity whereas group vectors depict the direction of the group velocity. Based on the definition of phase velocity $c_p=\frac{\omega}{k}$, we can replace $\omega=k.c_p$, then we obtain group velocity as

$$c_g=\frac{\partial\omega}{\partial k}=\frac{\partial\left(k.c_p\right)}{\partial k}=c_p+k\frac{\partial\left(c_p\right)}{\partial k}. \tag{24}$$

From equation (23) we observed that longitudinal and transverse waves remain in pure modes in nonlocal elasticity and become dispersive and are influenced by non-locality parameter. Waves are propagating in a dispersive medium; they will have different velocities (Table 1. and Figure 1.) and thus the superposed wave will have a phase velocity c_p that is different from its group velocity c_g. From the Figure 1. $c_p>c_g$, this is the normal dispersion.

In case of classical continuum mechanics when the nonlocality parameter $\varepsilon\left(=e_0a\right)=0$, subsequently $c_p=c_g=18199.4$ (m/s) (longitudinal wave) and $c_p=c_g=9913.56$ (m/s) (transverse wave) for typical isotropic material: polycrystalline 304 stainless steel. Thus waves are non-dispersive in its counterpart local theory of elasticity.

Table 1. Variation of phase and group velocity Speed (m/s) of longitudinal and transverse waves with nonlocality parameter.

Nonlocality parameter	$c_\ell=\sqrt{\frac{C_{11}}{\rho\left(1+\varepsilon^2\xi^2\right)}}$	$c_{s_1,s_2}=\sqrt{\frac{C_{44}}{\rho\left(1+\varepsilon^2\xi^2\right)}}.$	Group velocity $c_{\ell(g)}=\frac{\partial\omega}{\partial k}$, $\omega=ck$	Group velocity $c_{s_1(g),s_2(g)}=\frac{\partial\omega}{\partial k}$
$\varepsilon\le 1\times10^{-3}$	18199.39	9913.55, 9920.78	18199.39	9913.54, 9920.77
$\varepsilon=0.01$	18198.49	9913.06, 9920.29	18196.67	9912.07, 9919.29
$\varepsilon=0.05$	18198.49	9901.19, 9908.4	18131.37	9876.5, 9983.69
$\varepsilon=0.1$	18109.08	9864.36, 9971.55	17929.78	9766.69, 9773.81
$\varepsilon=0.5$	16278.04	8866.96, 8873.42	13022.43	7093.43, 7098.73
$\varepsilon=1$	12868.92	7009.94, 7015.05	6434.46,	3504.97, 3507.53
(Very large ε)	$c_\ell\to0$	$c_{s_1,s_2}\to0$	$c_{\ell(g)}\to0$	$c_{s_1(g),s_2(g)}\to0$

5. SLOWNESS SURFACE

Slowness surface has an important physical significance as a summarizing graphical representation of the variation of both types of velocity with respect to direction of the slowness vector and is used as a visual aid to explaining. Slowness surface are two-dimensional entities in three-dimensional space.

Equation (18) is a cubic characteristic polynomial equation with respect to V^2 and hence has three eigenvalues corresponding to a longitudinal wave, and two transverse waves. Moreover, the velocity of the longitudinal wave is always greater than those of the transverse, waves. Hence the largest eigenvalue of equation (18) corresponds to longitudinal wave propagation, is uniquely defined, and the slowness sheet is the innermost one and is away from the other two which are coincident are function of nonlocality parameter. Further the polarization vector of the longitudinal wave, known as the *l*-eigenvector, is tangent to the wave front normal and the polarization vectors of the transverse waves, known as the *t*-eigenvectors, are normal to *l*-eigenvector, with the three eigenvectors forming an orthogonal system.

6. NUMERICAL DISCUSSION

For numerical computations, typical isotropic material: polycrystalline 304 stainless steel is chosen with elastic constants [23] $C_{11} = 216, \quad C_{12} = 106GPa; \quad \rho$=7.88g/cm^3. In Table.1 phase and group wave velocities for longitudinal, shear are computed and their variation with the nonolocality parameter (ε) is also demonstrated.

It is observed that when $\varepsilon \leq 1\times10^{-3}$, all of these wave velocities are not influenced and has the same value as in the case of classical theory of elasticity whereas with the increase of nonolocality parameter (ε) thsee wave velocities decreases (Table 1 and Figure 1). The two surfaces for the two transverse waves touch or overlap completely and at these locations, the same phase and group velocities are identical though their polarization vectors are perpendicular to one another. It shows that phase and group wave velocities are affected only when non-locality parameter is greater than or equal to 1×10^{-3} for polycrystalline 304 stainless steel. All waves are in pure mode.

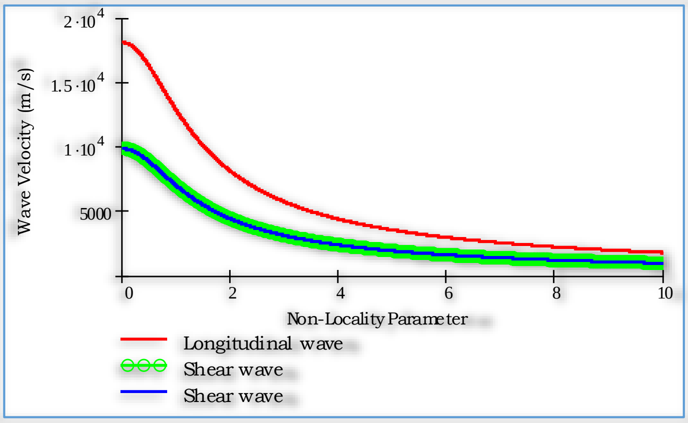


Figure 1.Wave velocity (m/s) for an isotropic material polycrystalline 304 stainless steel with nonolocality parameter (ε)

In Figure 2 phase and group velocities (m/s) curves are exhibited for material polycrystalline 304 stainless steel are with nonolocality parameter (ε), in which group velocity remain less than phase velocity for all values of nonolocality parameter (ε) and thus behavior normal in nonlocal theory of elasticity.

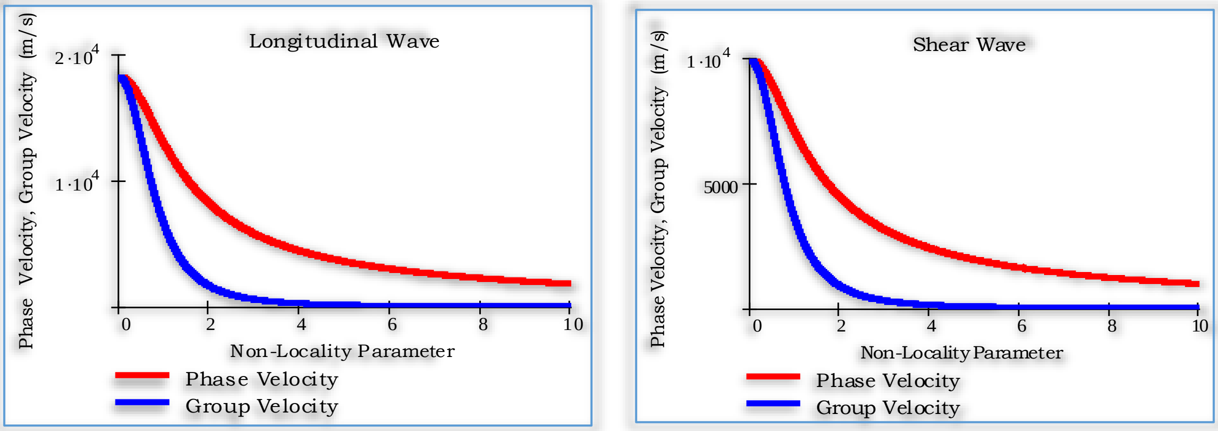
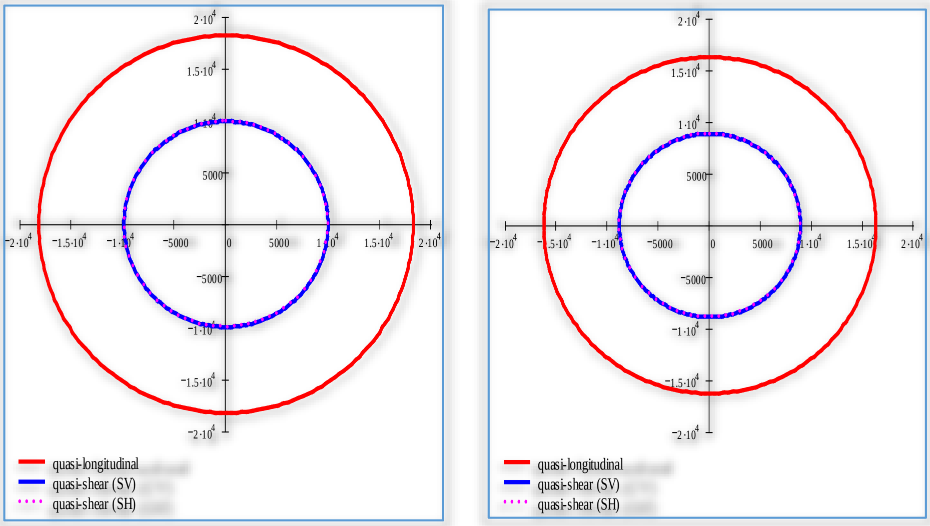


Figure 2. Phase and Group velocities (m/s) for an isotropic material polycrystalline 304 stainless steel with nonolocality parameter (ε)

In Figure 3(*a*) and Figure 3(*b*) polar diagram of phase velocity (m/s) for an isotropic material polycrystalline 304 stainless steel with nonolocal parameter ε = .001 and ε = .5 (*b*). It is observed that slowness surfaces consists of three concentric spheres, one with radius equal to the compressional wave slowness, and two, which are coincident, with radius equal to the shear wave slowness are function of nonlocality parameter. All waves are in pure mode. Mechanical stability requires that c_ℓ exceed $1.834c_t$. Thus, the ℓ and t surfaces cannot cross in the isotropic-solid case.



(a) (b)

Figure 3. Polar diagram of phase velocity (m/s) for an isotropic material polycrystalline 304 stainless steel with nonolocal parameter $\varepsilon = .001$(a) and $\varepsilon = .5$ (b) .

In Figure 4(a) and Figure 4(b) slowness surface for an isotropic material polycrystalline 304 stainless steel with nonolocal parameter $\varepsilon = .001$and $\varepsilon = .5$.

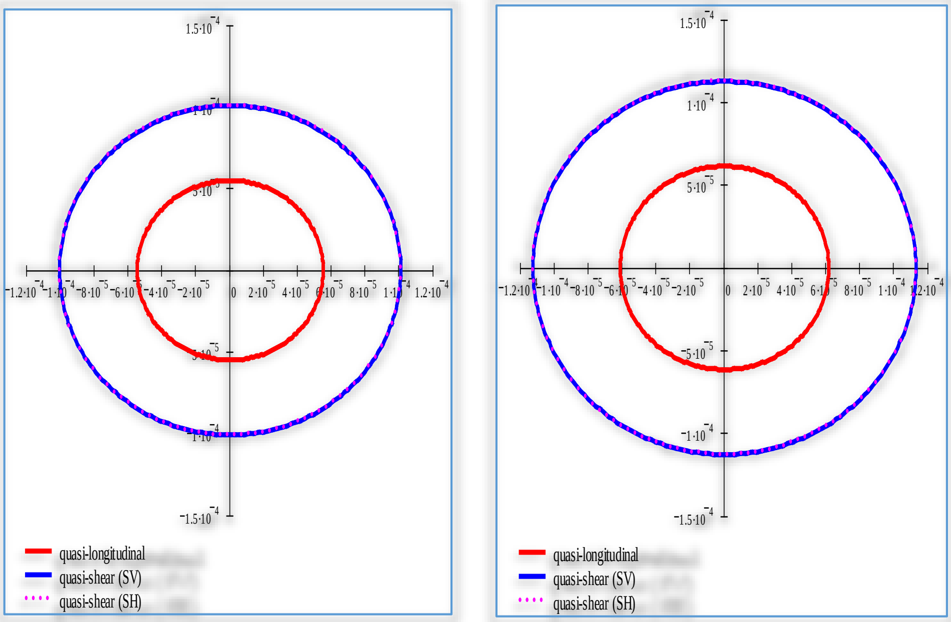


Figure 4. Slowness surface for an isotropic material polycrystalline 304 stainless steel with nonolocal parameter $\varepsilon = .001$(c) and $\varepsilon = .5$(d)

7. CONCLUSIONS

In this study, nonlocal continuum theory for wave surfaces in an isotropic elastic media based on Eringen's nonlocal elasticity theory is investigated. Accordingly this theory assumes that the stress at a specified point to be a functional of the strain field at each point in the solid. It permitted us to consider the small-scale effects that become important when dealing with micro- and nanostructure formation. On the basis this theory, the three slowness sheets are spheres and depends on the nonolocality parameter, with the outer two spheres (corresponding to the two shear waves) being coincident for an isotropic media; correspondingly, any slowness section consists of three circles, with the outer two coincident. It is also observed that longitudinal and transverse waves remain in pure modes in nonlocal elasticity and become dispersive and are influenced by non-locality parameter whereas theses waves are non-dispersive in its counterpart classical continuum mechanics theory (local theory). It turns out that the energy velocity (or group velocity) function of the nonolocality parameter associated with a point on a slowness sheet has the direction of the normal direction to the sheet at that point. Results obtained are useful where the continuous reduction of dimension of the devices from micro- to nano-scale are considered. Nonlocal continuum theory is also quite significant where long-range inter-atomic and intermolecular interrelated forces on the properties of such materials are considerable and cannot be neglected.

Acknowledgements:
Author is thankful to the Professor S.K. Tomar of Panjab University, Chandigrah under his guidance work was initiated when the author was visiting the Department of Mathematics, Center for Advanced Study in Mathematics, Panjab University, Chandigrah under the visiting scientists scheme of University Grants Commission, New Delhi, INDIA.

References

[1] Krone E. "Elasticity theory of materials with long-range cohesive forces", International Journal of Solids and Structures, 3 (1967), pp. 731-742
[2] Eringen A.C. "Linear theory of nonlocal elasticity and dispersion of plane waves", International Journal of Engineering Science, 10 (1972), pp. 425-435
[3] Eringen, A. C. "Nonlocal polar elastic continua". Int J Eng Sci, 10(1), (1972), pp. 1–16 .
[4] Eringen A. C. "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", J Appl Phys, 54(9): (1983), pp. 4703–4710
[5] Gurtin M. E. and Murdoch A. "A continuum theory of elastic material surfaces", Archive for Rational Mechanics Analysis, vol. 57 (4), pp. 291-323, 1975.
[6] Gleiter, H. Nanocristalline materials," Progress in Materials Science", vol. 33, pp. 223-315, 1989.
[7] Lim, C. W., Li, Z. R., and He, L. H,. "Size dependent, non-uniform elastic field inside a nano-scale spherical inclusion due to interface stress", International Journal of Solids and Structures, vol. 43 (17), pp. 5055- 5065, 2006.
[8] Khurana, A. & Tomar, S. K., "Rayleigh-type waves in nonlocal micropolar solid half-space", Ultrasonics 2016
[9] Kaur, G., Singh, D. & Tomar, S.K., Rayleigh-type wave in a nonlocal elastic solid with voids, European Journal of Mechanics / A Solids)
[10] Singh, D., Kaur, G. & Tomar S. K. "Waves in nonlocal elastic solid with voids", .J. Elast (2017), 128, 85-114
[11] Musgrave, M.J.P. "Crystal Acoustics. Holden-Day, San Francisco". (reprint: Acoustical Society of America, New York (1970). .
[12] Auld, B. A. Acoustic waves and fields in solids, vol. 1, New York, NY: Wiley (1973).
[13] Love, A. E. H. Some Problems of Geodynamics. University Press, Cambridge, (1927). (reprint: Dover, New York 1967).
[14] Ewing, W. M., Jardetzky, W. S., and Press, F. Elastic Waves in Layered Media. McGraw-Hill, New York, (1957).
[15] Achenbach, J. D. "Wave Propagation in Elastic Solids", North-Holland, Amsterdam. (1975)..
[16] Ting, T. C. T. "Anisotropic Elasticity: Theory and Applications". Oxford University Press, New York (1996)..
[17] Graff, K. F. Wave Motion in Elastic Solids, Oxford University Press, New York, NY, USA, (1975).
[18] Auld, B.A. "*Acoustic Fields and Waves in Solids*", 2nd ed., Krieger, Malabar, (1990).
[19] Červený, V. *Seismic Ray Theory*, Cambridge University Press, Cambridge, 713 pp, (2001).
[20] Mandal, B., Reflection and transmission properties of elastic waves on a plane interface for general anisotropic media, *J. Acoust. Soc. Am.* 70, (1991), pp. 1106–1118.
[21] Mensch, T., and P. Rasolofosaon. Elastic-wave velocities in anisotropic media of arbitrary symmetry-generalization of Thomsen's parameters α, β and γ, *Geophys. J. Int.*128, (1997), pp. 43–64.
[22] Musgrave, M.P.J., *Crystal Acoustics*, Holden-Day, San Francisco, 1970.
[23] Ledbetter, H., Frederick, N. and Austin M., "Elastic-constant variability in stainless-steel 304" , J. Appl. Phys., 51, 305, 1980.
[24] Verma K. L., Thermoelastic slowness surfaces in anisotropic media with thermal relaxation, Lat. Am. j. solids struct., 11(12), 2014, pp.2227-2240
[25] Fedorov F.I. Theory of elastic waves in crystals, Plenum Press, 1968

ISSN 1584 – 2665 (printed version); ISSN 2601 – 2332 (online); ISSN-L 1584 – 2665