ANNALS OF THE FACULTY OF ENGINEERING HUNEDOARA
2004, Tome II, Fascicole 1

USING OF KALMAN FILTRATION IN AUTOMATICS

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Abstract:

This paper presents the principals of Kalman filtration. For a better understanding of the Kalman filtration it is presented an analyses of the stochastic estimation of processes that are affected by noise and the relation of this to the Kalman filtering. The command signal for an automatic regulation system can be of a low power and affected by noise. For this particularly case it is presented a simulation program that calculates an estimation of the command signal, using the Kalman filtering.

***Keywords**:*
Kalman filtration, stochastic estimation, ideal signal, noise, automation

1. INTRODUCTION

The filtration operation is met in an inevitable way in the signal treatment technique, because they have to go through the transmission channels, as some functional blocks, depending on the operation that has to be met of them.

To filtrate a signal x(t) through f(t), means to make the product:

$$x(t)\cdot f(t) \tag{1}$$

After the filtration, we want to know if the signal spectrum is influenced by this operation. If we apply the Plancherel theorem:

$$x_f(t) = x(t) \otimes f(t) \leftrightarrow X(j\nu)\cdot F(j\nu) \tag{2}$$

results from, that the spectrum of the filtered signal $x_f(t)$ is affected by the filtration operation.

In case that we consider the frequency representation, the filtration operation consist of blocking or allowed passing, totally or partial, of the spectrum lines through the cuadripol filter (figure 1).

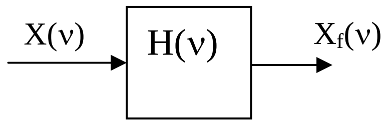

Fig.1 *Functional filtration block*

Using the numerical signal process, in different application domains of filtration, the obtained results are more precisely, for example in signal identification. In averages that are not apriori known by the designer, the adaptive filters have a lot of applications. Adaptive filters are characterized by their possibility to modify the filter parameters, to optimize some characteristics, based on a recursive algorithm.

2. **STOCHASTIC ESTIMATION**

While there are many applications – specific approaches to "computing" (estimating) an unknown state from a set of process measurements, many of these methods do not inherently take into consideration the typically noisy nature of the measurements. This noise is typically statistical in nature, or can be effectively modeled as such, which leads to stochastic methods for addressing the problem.

Consider a dynamic process described by an n-th order difference equation of the form:

$$y_{i+1} = a_{0,i} y_i + \ldots + a_{n-1,i} y_{i-n+1} + u_i , i \geq 0 , \tag{3}$$

where $\{u_i\}$ is a zero-mean (statistically) white (spectrally) random noise process with autocorrelation

$$E\left(u_i , u_j\right) = R_u = Q_i \delta_{ij} , \tag{4}$$

and initial values $\{y_0 , y_{-1}, \ldots, y_{-n+1}\}$ are zero-mean random variables with a known nxn covariance matrix:

$$P_0 = E\left(y_{-j} , y_{-k}\right), j, k \in \{0, n-1\} \tag{5}$$

We consider that the noise is statistically independent from the process to be estimated. Under some other basic conditions, the difference equation (3) can be re-written as:

$$\vec{x}_{i+1} \equiv \begin{bmatrix} y_{i+1} \\ y_i \\ y_{i-1} \\ \ldots \\ y_{i-n+2} \end{bmatrix} = \underbrace{\begin{bmatrix} a_0 & a_1 & \ldots & a_{n-2} & a_{n-1} \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y_i \\ y_{i-1} \\ y_{i-2} \\ \ldots \\ y_{i-n+1} \end{bmatrix}}_{\vec{x}_i} + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ \ldots \\ 1 \end{bmatrix}}_{G} u_i \tag{6}$$

which leads, in that form, to the state-space model:

$$\vec{x}_{i+1} = A\vec{x}_i + Gu_i \tag{7}$$

$$\vec{y}_i = \begin{bmatrix}1 & 0 & \dots & 0\end{bmatrix}\vec{x}_i \tag{8}$$

or the more general form:

$$\vec{x}_{i+1} = A\vec{x}_i + Gu_i \tag{9}$$

$$\vec{y}_i = H_i\vec{x}_i \tag{10}$$

Equation (9) represent the way a new state $\vec{x}_{I+1}$ is modeled as a linear combination of both the previous sate $\vec{x}_i$, and some process noise u_i. Equation (10) describes the way the process measurements or observations $\vec{y}_i$ are derived from the internal state $\vec{x}_i$. These two equations are often referred to respectively as the process model and the measurement model, and they serve as the basis for all linear estimation methods, such as the Kalman filter.

3. THE KALMAN FILTER

Within the significant toolbox of mathematical tools that can be used for stochastic estimation from noisy measurements, is known as the Kalman filter.

The Kalman filter is essentially a set of mathematical equations that implements a predictor-corrector type estimator. That is a optimal operation in sense that it minimizes the estimated error covariance. The Kalman filter is frequently applied, especially in the domains of autonomous and assisted regulation.

The Kalman filter addresses the general problem of trying to estimate the state $x \in R^n$ of a discrete time controlled process that is governed by the linear stochastic difference equation:

$$x_k = Ax_{k-1} + Bu_k + w_{k-1} \tag{11}$$

with a measurement $z \in R^m$ *that is*

$$z_k = Hx_k + v_k \tag{12}$$

The random variables w_k and v_k represent the process and measurement noise. They are assumed to be independent of each other, white and with normal probability distributions.

The nxn matrix A in the difference equation (11) relates the state at the previous time step k-1 to the state at the current state k, in absence of the process noise.

It will be defined a value of x to be a priori state estimate at step k, $\hat{x}^{-}{}_{k} \in R^{n}$. This value gives us knowledge of the process prior step k, and $\hat{x}_k \in R^n$ defines a posteriori state estimate at step k given measurement z_k. We can then define a priori and a posteriori estimate errors as:

$$e_k^- \equiv x_k - \hat{x}_k^- \qquad (13)$$

and

$$e_k = x_k - \hat{x}_k \qquad (14)$$

The a priori estimate covariance is:

$$P_k^- = E\left[e_k^- e_k^T\right], \qquad (15)$$

and the a posteriori estimate error covariance is:

$$P_k = E\left[e_k e_k^T\right]. \qquad (16)$$

In deriving the equations for the Kalman filter, we begin with the goal of finding an equation that computes an a posteriori state estimate $\hat{x}_k$, as a linear combination of an a priori estimate $\hat{x}_k^-$ and a difference between an actual measurement z_k and a measurement prediction $H\hat{x}_k^-$, as shown below in equation. The justification for the next equation is given in the probabilistic origins of the filter.

$$\hat{x}_k = \hat{x}_k^- + K\left(z_k - H\hat{x}_k^-\right) \qquad (17)$$

The difference $\left(z_k - H \cdot \hat{x}_k^-\right)$ in equation (17) is called the residual or the measurement innovation. The residual reflects the discrepancy between the predicted measurement $H \cdot \hat{x}_k^-$ and the actual measurement z_k. A residual of zero means that the two are in complete agreement.

The nxm matrix K in equation (17) is chosen to be the gain that minimizes the a posteriori error covariance equation (16).

The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in form of measurements. As such, the equation for the Kalman filter falls into two groups: time update equations and measurement equation. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step.

The measurement update equations are responsible for the feedback – i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate.

The specific equations for time and measurement update are presented in the following table:

Tabel 1

Discrete Kalman filter time update equations	*Discrete Kalman filter measurement update equations*
$\hat{x}_k^- = A \cdot \hat{x}_{k-1} + Bu_k$	$K_k = P_k^- H^T \left(HP_k^- H^T + R \right)^{-1}$
$P_k^- = AP_{k-1}A^T + Q$	$\hat{x}_k = \hat{x}_k^- + K_k \left(z_k - H\hat{x}_k^- \right)$
	$P_k = \left(1 - K_k H \right) P_k^-$

We notice how the time update equations project the state and covariance estimates forward from time step k-1 to step k.

After each time and measurement update pair the process is repeated with the previous a posteriori estimates used to project the new a priori estimates. This recursive nature is one of the most used features of the Kalman filter – it makes practical implementation of the filter more feasible then, for example, an implementation of a Wiener filter which is designed to operate on all the data directly for each estimate. The Kalman filter estimates the actual value depending on the previous measured values.

Figure 2 presents a complete picture of the operations effectuated by the filter, based on table 1:

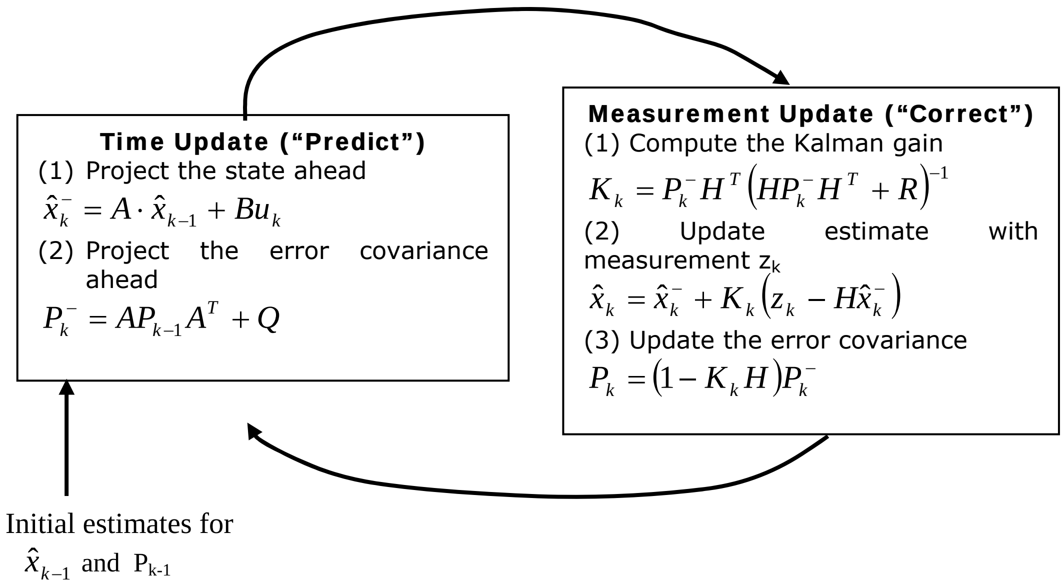

Fig.2 *Complete picture of the operation of the Kalman filter*

4. Parameter estimation of a control signal, affected by noise, for an automatic regulation system

One of the main reasons for a automatic regulation system to function in the correct direction for that it was implemented, it's necessary that the control signal parameters of the system shouldn't be changed by the specifically environment noises where it acts.

Through the implementation of the Kalman filter equation in MatLab, the user will have the possibility to modify a series of filter parameters, gain, number of iterations, the characteristics for the analyzed signal and that for the disturb signal.

The analyzed signal in that application is a step signal affected by an random, Gaussian white noise.

Figure 3 presents the obtained results throughout the simulation: the ideal signal form, not affected by noise (-), the disturber signal (--) and the estimate signal (0), obtained after the application of the predict correction Kalman filter algorithm.

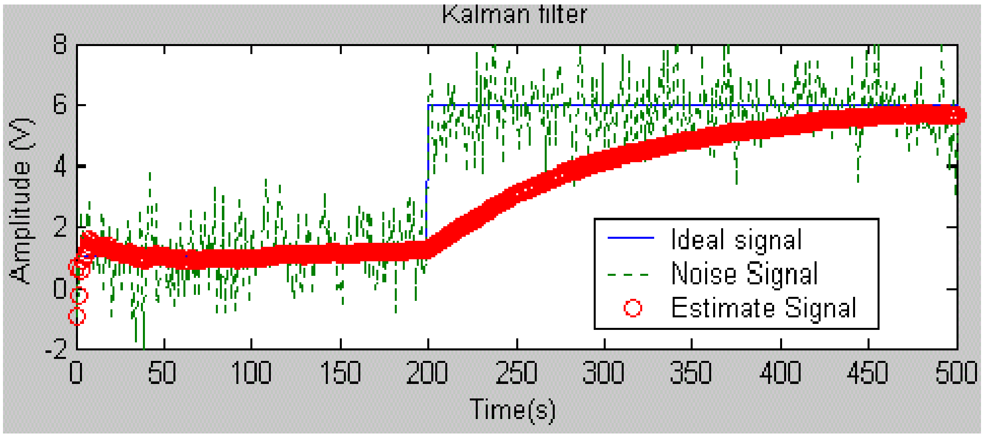

Fig.3 *Ideal signal, noise signal, estimate signal*

5. CONCLUSION

From the presented in this paper we can see an large applicability for the Kalman filter in the automation domain, under a large and fast extension of the digital leading devices in obtaining better general quality indicative.

6. BIBLIOGRAPHY

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