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THE APPLICATION OF GRIFFITH'S THEORY IN ROCK FRACTURE

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ABSTRACT

The transposition of the fracture theories from the material resistance, developed for metals, showed more inconveniences than advantages in the case of rocks. So, Coulomb's theory, Coulomb – Navier or that of Mohr proved not to correspond to the experimental data.

Griffith' theory, applicable to brittle materials proved to be the most nearer to the experimental results. The paper presents the bi-dimensional model based on the existence of the crack in a thin plate and on Inglis' solution on the problem of efforts distribution around the cracks. It also shows the changes brought by Mc Clintock and Walsch, Brace, a.o.

The theory allows the description of the efforts/loading necessary to the development of cracks or their ramification.

In the end of the paper the main elements of concordances and deviations, respectively resulted between Griffith' modified theory and the experimental results for some rocks.

KEY WORDS:
rock fracture, Griffith theory

1. Introduction

The paper [6] presents the peculiarities of the rocks, which differentiate them from the metals. They make difficult up to impossible the application of fracture theories specific to metals. The structure of the rocks, the scale effect, the level of loading, the stress field, the cracking, the porosity, the anisotropy a.s.o., have to be considered for a complete approach. That is the reason why it will take more time to consolidate an integral concept.

The nearest to the experimental results was proved to be the application of Griffith's theory specific for brittle materials. The paper develops the application of Griffith's theory for the rocks, the exemplification of the fracture criterion and the developments proposed by researchers who embraced this theory.

2. Griffith's theory for the brittle materials

A A. Griffith in the paper published in 1921 in Phil. Trans. Roy, Soc. London, theoretically substantiated the brittle materials fracture supposing that they contain in their structure cracks having an aleatory orientation. At the ends of these cracks stress concentrations produce, which lead to the crack propagation and finally to the macroscopic fracture of the material.

The problem is dealt with on energetic basis: the external loading is locally transferred in the interior of the material, in the defect zone where the fracture develops, So, is made the passing from the global interpretation of resistance – characteristic to the classical resistance criteria as that of Coulombs, Coulomb-Navier or Mohr – to a local interpretation specific to the area where the fracture is developing, particularly the defect zone [3], [4].

The concept is exemplified in the clearest way in figure 1, which sketches a unitary thickness strip which contains an ellipse shape crack, and the big axis is 2c long, perpendicular on the direction of the traction effort. At the ends of the ellipse effort concentrations are produced σ^{max}, which lead to the crack propagation, For example, the maximum effort determined by Inglis according to [5] is:

$$\sigma^{max} = 2.\sigma_o.(c/\rho)^{1/2} \qquad (1)$$

where σ_o is the average value of the effort, and ρ is the curve radium from the ends of the ellipse.

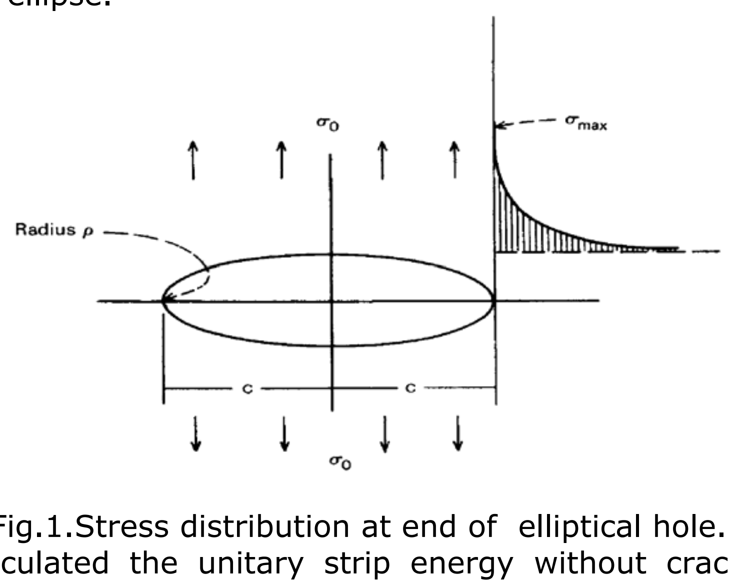

Fig.1.Stress distribution at end of elliptical hole.

Griffith calculated the unitary strip energy without crack and with crack. Due to the presence of the crack the strip energy is reduced with the value:

$$W = W_e - W_s = \pi.c^2.\ \sigma_o^2/E - 4\ c.T \qquad (2)$$

where:

$W_e = \pi . c^2 . \sigma_o^2 / E$ represents the deformation elastic energy released after the cracking,
$W_s = 4c.\ T$ represents the dissipated energy to win the resistance at the crack propagation, with T – the surface energy, a material constant. (Complete demonstrations are to be found in [3] and [4]).

The beginning of the crack propagation takes place when the energy derivative a function of c is annulled, namely $\delta W/\delta c = 0$, which leads to the Griffith's criterion under one of the following forms:

$\sigma_o^2 = 2.E.T/ \pi.c = T_0^2$ according to [5], with T_o the traction resistance (3)

$\sigma_{o.}\ (c)^{1/2} = (2.E.T/\pi)^{1/2}$ according to [4] (4)

$\sigma_o = (2.E.T / \pi.c)^{1/2}$ according to [3]. (5)

Researches on the surface tension values made by Brace and Walsh proved to be difficult and not exact [5]. These difficulties were passed over by Hoek and Inglish who determined a fracture criterion in the terms of the resistance to the uniaxial loading.

3. Development of Griffith's theory: Hoek, Bieniawski, Inglis, Mc Clintock and Walsh

A plane plate is considered of unitary thickness and an ellipse form crack in the middle with the axis inclined against the main effort $\boldsymbol{\sigma_1}$ as in figure 2.

The normal efforts σ_x and those tangential τ_{xy}, at the limit of the element (the dotted rectangular), are according to the theory of the plane loading (ex. according to [1] page 351-352 or [2] page 203-205) given by the relations:

$$2\,\sigma_x = (\sigma_1 + \sigma_3) - (\sigma_1 - \sigma_3).\cos 2\psi \quad (6)$$

$$2\,\tau_{xy} = (\sigma_1 - \sigma_3).\sin 2\psi \quad (7)$$

valid for the axis x. The components after the direction y do not influence the concentration of efforts and so they are neglected.

Hoek and Bieniawski analysed the plane state of tensions on the ellipse contour. Inglish introduced elliptical coordinates and after a series of mathematical transformations, series development and the neglecting of the two degree terms or more established the relation:

$$(\sigma_1 - \sigma_3)^2 / (\sigma_1 + \sigma_3) = 8\,T_0 \quad (8)$$

which represents the Griffith's fracture criterion after the direction y introduced by the uniaxial traction resistance. If $\sigma_3 = 0$, then $\sigma_1 = C_o$ is the uniaxial compression resistance and there results:

$$C_0 = 8\,T_0 \qquad (9)$$

In this mode the Griffith's criterion for brittle materials, which contain established length microcracks, stipulates that the compression fracture efforts have to be exactly eight times bigger than the tensile effort. Analysing the experimental data for more rocks taken from [7], result the data from table 1. These data show that for the majority of rocks the compression resistance exceeds 10-15 times or more the traction resistance

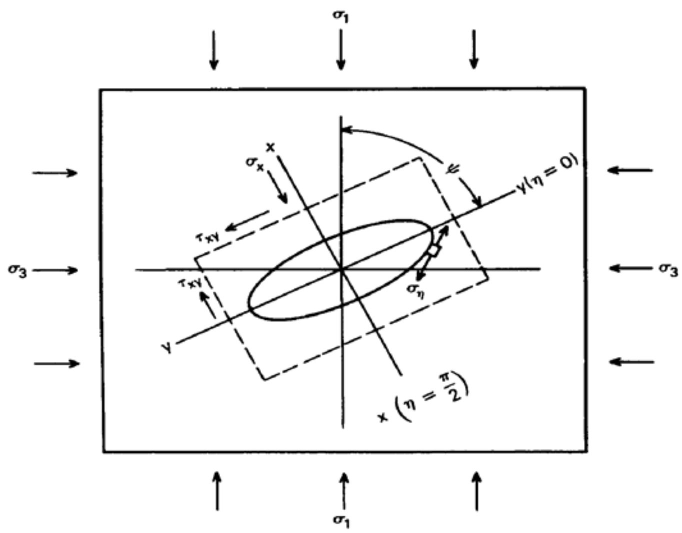

Fig.2.Stress acting on an elliptical crack inclined at an angle ψ to σ_1 .

TABLE 1

	Rock	Compression resistance	Traction resistance	C_0 / T_0
Measure unit	-	C_0 , [MPa]	T_0 , [MPa]	-
1	Doman limestone	60,3- 196,6	5- 13,3	12,06- 14,78
2	Moldova-Nouă banatit (granodiorite)	26,8- 64,7	4,8- 8,8	5,58 – 7,35
3	Anina marly limestone	69,0- 131,8	8,6	8,02 – 15,32
4	Praid halite	21,95	1,46	15,03
5	Ponor Anina millstone grit	14,7-36,6	0,8 –2,2	18,37 –16,37

Mc Clintock and Walsh modified Griffith's theory considering that within the compression cracks close and frictions produce on their surface. The fracture is produced when:

$$\sigma_1.[(1+\mu^2)^{1/2} - \mu] - \sigma_3.[(1+\mu^2)^{1/2} + \mu] = 4.T_0.(1+\sigma_c/T_0)^{1/2} - 2.\mu.\sigma_c \quad (10)$$

where: μ is the friction coefficient on the crack surface, and

σ_c is the effort necessary to close the crack (perpendicular on it).

It is specified that μ the friction coefficient is not identical with that in the Mohr's fracture theory, although these magnitudes can be dependent. Brace showed that σ_c is very small and can be neglected. So, the relation (10) becomes:

$$\sigma_1.[\ (\ 1 + \mu^2\ )^{1/2} - \mu\ ] - \sigma_3.[\ (\ 1 + \mu^2\ )^{1/2} + \mu\ ] = 4.T_0 \qquad (\ 11\ )$$

If $\sigma_1 = C_o$, at $\sigma_3 = 0$ the pure tensile takes place and there results:

$$C_0\ /\ T_0 = 4\ /\ .[\ (\ 1 + \mu^2\ )^{1/2} - \mu\ ] \qquad (\ 12\ )$$

As $\mu = 1$, there results $C_o/T_o = 10$ which is the closest to the experiment.

In the end we remember that Griffith's theory is based on the bidimensional model which in its turn is based on the crack existence in the thin plate, and around the crack the behaviour is an elastic one. It is based on Inglish's theory regarding the distribution of efforts around an elliptic shape hole. The crack is a limit case when the small axis tends to zero.

4. Conclusion

The fracture mechanics problem for the rocks is focused on the establishment of the links between the application of loading on bodies and the development of cracks, together with their orientation up to the complete fracture. There are a lot of external factors, which complicate the process: the humidity, the pressure in pores, temperature a.o. The internal factors have not to be forgotten: the very complicated structure of the rocks, the dimensions of the bodies in nature and those engineers work with a.o.

Griffith's theory is based on an internal fracture mechanism: the existence of some cracks and the concentration of loading at the end of cracks, which determine their propagation. This theory establishes the compression resistance eight times bigger than that developing in traction, which is quite different of the experiment. The modification of the theory, which is based on the cracks closure and the appearance of friction increases the ratio to 10.

The more recent interpretation of Griffith's theory supposes that it is valid only to establish the beginning of the crack propagation and it does not describe the complete fracture. The experience really showed that the initiation of the crack propagation started much before the fracture. On these basis it is considered that the ratio C_o/T_o = 8-10 is quite close with the initiation of the fracture.

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