ANNALS OF THE FACULTY OF ENGINEERING HUNEDOARA
2004, Tome II, Fascicole 3

SOLVING THE TRANSIENT 2-DIMENSIONAL HEAT DIFFUSION EQUATION USING THE MATLAB PROGRAMM

RAŢIU Sorin, KISS Imre, ALEXA Vasile

UNIVERSITY POLITEHNICA TIMISOARA
FACULTY OF ENGINEERING HUNEDOARA

ABSTRACT

In this study we are introducing one approach for solving the partial differential equation, which describes transient 2-dimensional heat conduction. Fourier series methods are used to solve the problem. In our case, heat conduction equation is describing the heating up of a parallelepiped infinite region in an experimental oven.

KEY WORDS:
heat transfer, non-linear differential
equation with partial derivatives, Fourier series

1. Introduction

On heating some material in the furnace, it is of particular importance to solve the basic problem of heat conduction, problem that can be reduced to determining heat distribution in the body, according to coordinates and time. The relationship between temperature variation in space and in time is expressed by the differential equation of heat conduction.

Finding an analytical solution to the general differential equation of 3-dimensional heat conduction under transient conditions is a hard task. This is why, in order to analytically solve it, one has to resort to some univocity conditions, by taking into consideration bodies with a simple geometry. Reference literature gives solutions to some strictly particular problems, such as: the case of one-dimensional conduction (plane wall or plate of infinite width and length and finite thickness, cylinder of infinite length, sphere, semi-infinite bodies, etc.). This article introduces a method for analytically solving the differential equation of 2-dimensional heat conduction, under transient conditions, making use of the classical method of Fourier series. The solving of the analytical model is made by means of the mathematical software MATLAB.

2. The Analytical Model

In order to construct the analytical model we resorted to a series of simplifying hypotheses, such as:

- ⇨ the thermal and physical properties of the heated material, namely: density - ρ [kg/m^3], specific heat - c [J/(kg·grd)] and heat conductivity λ [W/(m·grd)] are considered to be constant with respect to temperature;
- ⇨ the heating is being achieved evenly, the time variation of the furnace being strictly emulated by the exterior surfaces of the heated material, which implies a very high value of the global coefficient of thermal transfer from the burned gases to the free surface of the metal;
- ⇨ the effect of thermal radiation is not taken into consideration;
- ⇨ we considered that inside the heated material there are no heat-generating sources;
- ⇨ temperature distribution inside the material is even from the beginning of the heating process, all the points inside the field of analysis having at the initial moment the temperature of 20 ^{o}C.

Under these conditions, the differential equation of heat conduction is:

$$\frac{\partial t}{\partial \tau} = \frac{\lambda}{\rho \cdot c} \cdot \left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} \right) \qquad (1)$$

The initial condition is given by the relation below:

$$t\Big|_{\tau=0} = F(x, y) \qquad (2)$$

in this case F(x,y) = 20.

The frontier conditions are represented as a group thus:

$$t\Big|_{\substack{x=\pm L \\ y=\pm B}} = \varphi(\tau) \qquad (3)$$

Relation (3) assigns to the points belonging to the lateral surfaces of the field a temperature variation described by function $\varphi(\tau)$:

$$\varphi(\tau) = 2 \cdot 10^{-11} \cdot \tau^3 - 2 \cdot 10^{-6} \cdot \tau^2 + 0{,}0649 \cdot \tau + 20 \qquad (4)$$

The particular solution of this problem is given by the following relation:

$$
\begin{aligned}
t = \varphi(\tau) + \frac{1}{LB}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\cos\delta_l \frac{x}{L}\cos\delta_m \frac{y}{B} e^{-\beta_3 a\tau}\,\cdot \\
\int_{-L}^{+L}\int_{-B}^{+B}\left[F(x,y)-\varphi(0)\right]\cos\delta_l \frac{x}{L}\cos\delta_m \frac{y}{B}dxdy - \\
-\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{8(-1)^{l+m+1}}{\delta_l\delta_m}\cos\delta_l \frac{x}{L}\cos\delta_m \frac{y}{B} e^{-\beta_3 a\tau}\int_0^{\tau} e^{\beta_3 a\tau}\varphi'(\tau)d\tau
\end{aligned} \tag{5}
$$

where:

$$\delta_l = \frac{2l-1}{2}\pi, \qquad l = 1,2,3... \tag{6}$$

$$\delta_m = \frac{2m-1}{2}\pi, \quad m = 1,2,3... \tag{7}$$

$$\beta_3 = \frac{\delta_l^2}{L^2} + \frac{\delta_m^2}{B^2} \tag{8}$$

The general solution we obtained (5) allows partial solutions for two dimensions, under any temperature variation of the exterior surface, expressed by function φ and under any initial condition, expressed by function F.

3. Results

Considering the symmetry of the heating, the calculation has been done for a field equivalent to one quarter of a horizontal cross section of the bloom, for different values of coordinate z with respect tot the base.

The total duration of the heating was established at 10 h, the analytical calculation being done with a time pitch of 600 s.

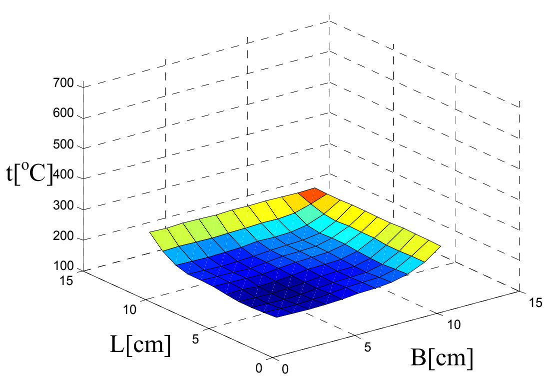

Fig. 1. Thermal field at step no.8

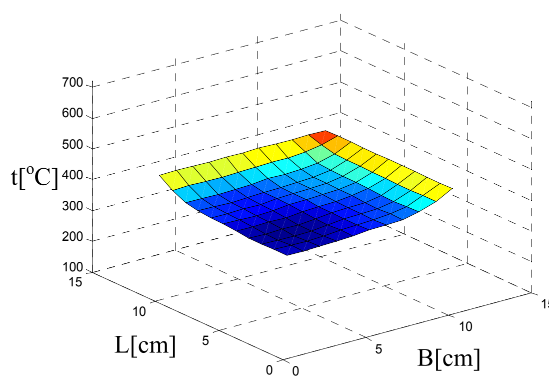

Fig.2 Thermal field at step no.16

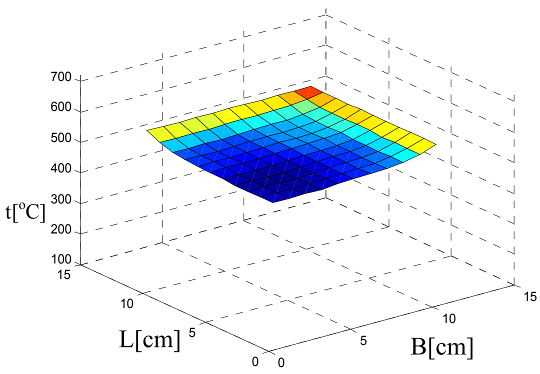

Fig.3 Thermal field at step no.24

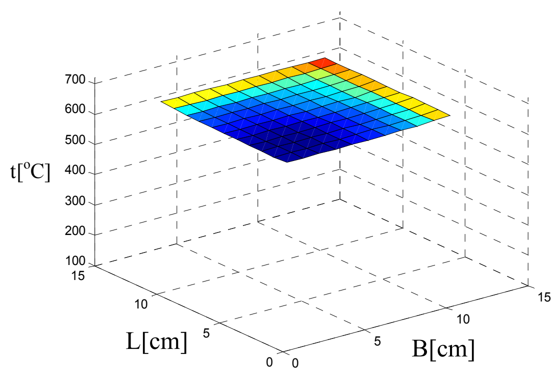

Fig.4 Thermal field at step no.32

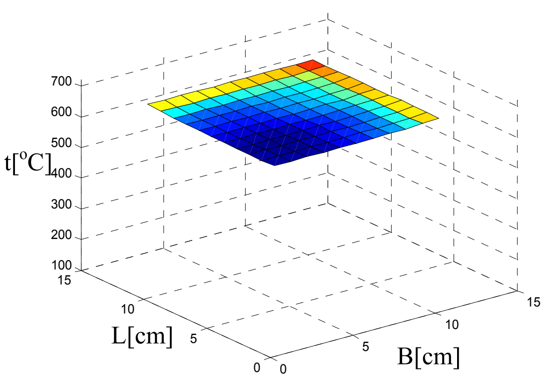

Fig.5 Thermal field at step no.40

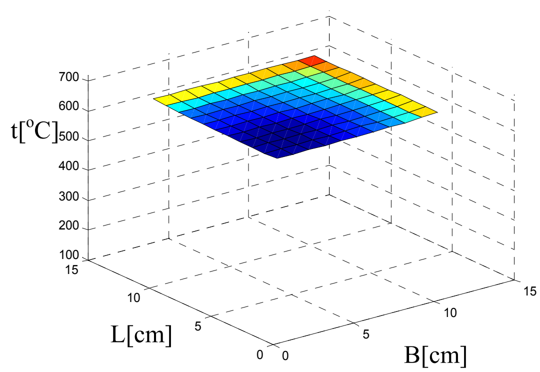

Fig.6 Thermal field at step no.48

4. Conclusions

As one can notice from the figures above, the analytical solution of the problem of 2-dimensional transient heat conduction implies taking into consideration some simplifying hypotheses that lead to relatively great errors, with respect to the experimentally obtained values.

Bibliography

[1.] Lienhard, J.H.IV, Lienhard, J.H.V - *A Heat Transfer Textbook*, Third Edition, Phlogiston Press, Cambridge, Massachusetts;

[2.] Raţiu, S., ş.a. - *The solving of a non-linear problem of thermal transfer by a particular analytical method*, 7-th International Symposium of Interdisciplinary Regional Research, Hungary-Romania-Yugoslavia, Hunedoara, 25-26 September 2003;