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A MATHEMATICAL DESCRIPTION OF HYDRODYNAMIC LUBRICATION OF JOURNAL BEARING

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SUMMARY

Journal bearings represent basic tribology systems and, in that context, a lot of literature is devoted to them.These bearings have a wide practical application because of low cost, simple maintenance and because of some satisfactory dynamic characteristics. Journal bearings of ICE are used to support and lead crankshaft, connecting rod, camshaft, and rocker shaft. Bearings, like pistons, operate mainly in regime of hydrodynamic lubrication. By hydrodynamic lubrication the space between two separate surfaces is filled with lubricant, creating a sufficient (hydrodynamic) pressure to support the large external load. This phenomenon is described by Reynolds partial differential equation. By applying Ritz-method, distribution of pressure of lubricant film for various journal bearings is determined in this paper.

KEY WORDS:

journal bearings, hydrodynamic lubrication, Reynolds partial differential equation, Hamilton variational princip, Ritz method

1. HYDRODYNAMIC LUBRICATION

Journal position in its bearing is directly related to external load. When a bearing is supplied with appropriate lubricant quantity and external load is low ($\varepsilon \sim 0$), journal axis and bearing axis are coaxial. However, when load is growing, the journal is moving inside the bearing in the direction of surface speed of journal.

Thus the journal and the bearing form a wedge-shaped space generating high pressure in it. This type of lubrication is called Hydrodynamic Lubrication (HL) and in practice it results in film thickness of the order of micrometers and generated pressures of the order of Mega Pascal's. This phenomenon was first recognized by British engineer Beauchamp Tower. In his papers “''First Report on Friction Experiments” (1883/1884.) and “Second Report on Friction Experiments” (1885) he reports about the discovery.

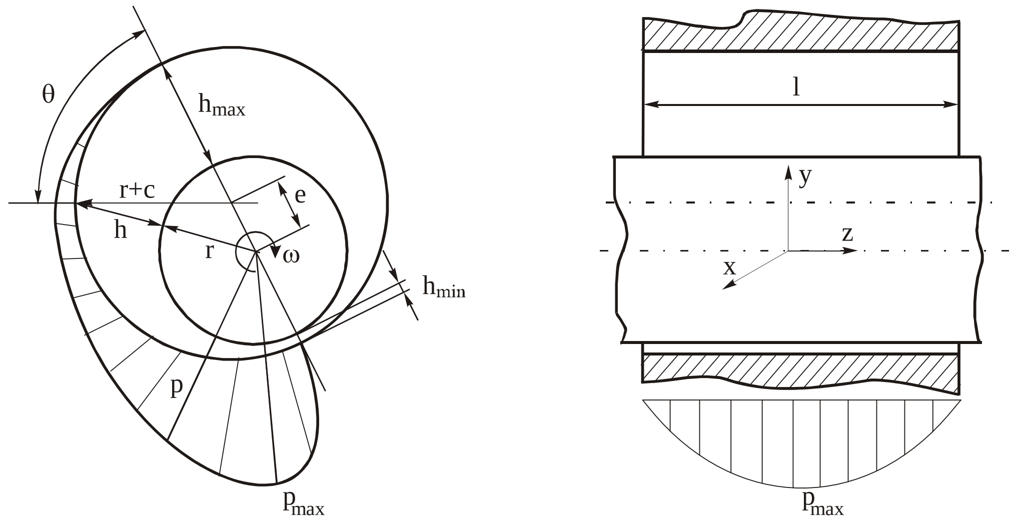

Fig. 1. Plain journal bearing

Theoretical explanation of the phenomena was given by the Osborne Reynolds in the paper “''On the Theory of Lubrication and its Application to Mr. Beauchamp Tower's Experiments, Including an Experimental Determination of the Viscosity of Olive Oil” (1886), [2]. The result of Reynolds’ work is the well-known partial differential equation of elliptic type named the Reynolds equation. The equation has the following form:

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6\eta\ v\frac{dp}{dx} \tag{1}$$

Introducing dimensionless coordinates i.e.

$$\frac{x}{r}=\theta\,,\ \frac{z}{l}=Z\,,\ \frac{p-p_0}{p_0}=P\,,\quad \frac{h}{c}=H=\left(1+\varepsilon\ \cos\theta\right)$$

the equation (1) becomes

$$\frac{\partial}{\partial\theta}\left(H^3\frac{\partial P}{\partial\theta}\right)+\left(\frac{r}{l}\right)^2\frac{\partial}{\partial Z}\left(H^3\frac{\partial P}{\partial Z}\right)=6\frac{\eta\ vr}{p_0c^2}\frac{dH}{d\theta} \tag{2}$$

According to nature of the problem and hypothesis III [6], which says that the lubricant film extents from θ=0 to θ=π, boundary conditions are as follows:

$$P(0,Z)=P(\pi,Z)=0\,,\ \frac{\partial P}{\partial\theta}(\theta_m,Z)=0 \tag{3a}$$

$$P\left(\theta\ ,-\frac{1}{2}\right)=P\left(\theta\ ,+\frac{1}{2}\right)=0\,,\ \frac{\partial P}{\partial Z}(\theta\ ,\ 0)=0 \tag{3b}$$

As it could be seen, the Reynolds equation establishes a relation between slip velocity, geometry of surfaces, fluid properties and the pressure in the lubricant film i.e. load that could be carried by the bearing. The equation (2) is mostly being solved using numerical methods. Kingsbury (1931) determined the pressure distribution of the oil film of finite journal bearings by an experimental electrical analogy.

Christopherson (1941) did the same thing by utilizing the mathematical method of "relaxation". Cameron and Wood have extended the work of Christopherson to show the effect of length-diameter ratio on the output parameters (eccentricity ratio, friction coefficient and attitude angle). Dayton and Simons performed examination of cyclic loaded journal bearing, [2].

Many other authors also solved the equation by using different analytical, numerical and graphical methods (Pinkus i Sternlicht, 1961; Cameron, 1976; Fuller, 1984; Hamrock, 1994; Frene, 1997; Szeri, 1998; Khonsari i Booser, 2001). The analytical methods that allow direct integration, require certain simplification of the equation (1) reducing it into two types: the equation of long bearing (l/r > 1) and the equation of short bearing (r/l > 1). When $l \to \infty$ (practically ... l > 4r), the equation (2) loses the second term and becomes

$$\frac{\partial}{\partial \theta}\left(H^3 \frac{\partial P}{\partial \theta}\right) = 6 \frac{\eta\ vr}{p_0\ c^2} \frac{\partial H}{\partial \theta} \quad (4)$$

Sommerfeld (1904) was the first who found the solution of the equation (4) in the final form [3] using boundary condition (3a). This solution has the following form:

$$P = \Lambda \frac{\varepsilon\ (2 + \varepsilon\ \cos\theta)\sin\theta}{(2 + \varepsilon^2)(1 + \varepsilon\ \cos\theta)^2} \quad (5)$$

In case of short bearing (l < r) the axial pressure gradient is considerably higher than the circumferential pressure gradient, so the first term on the left-hand side of the equation (2) can be neglected i.e.

$$\left(\frac{r}{l}\right)^2 \frac{\partial}{\partial Z}\left(H^3 \frac{\partial P}{\partial Z}\right) = 6 \frac{\eta\ vr}{p_0\ c^2} \frac{\partial H}{\partial \theta} \quad (6)$$

Taking into account boundary conditions (3b), DuBois and Ocvirk (1953) found the solution of the equation (6), which says

$$P = \frac{\Lambda}{2}\left(\frac{l}{r}\right)^2 \left(\frac{1}{4} - Z^2\right) \frac{\varepsilon\ \sin\theta}{(1 + \varepsilon\ \cos\theta)^3} \quad (7)$$

2. THE RITZ METHOD

Contemporary science tends to unification meaning that all its theories could be derived from one principle. The variational principles of mechanics are very close to realization of this idea, above all the Hamilton's variational principle which shows certain advantages of variational conception.

An existing problem is considered to be described variationally if the equations that describe the problem could be derived from Euler-Lagrange's equations, whose Lagrange's function gives the extreme value to a definite integral, known as the action integral, [7], [8]. The boundaries of the integral represent the boundaries of the actual space. Methods for solving a problem based on variational principles of mechanics are called direct methods. The typical representative of the methods is the Ritz method that is directly derived from the Hamilton's variational principle. In this part of the paper the Ritz method is applied to solve the Reynolds equation (2).

Lagrange's function for the Reynolds equation (2) can be written as

$$L = \frac{1}{2}\left(H^3 \frac{\partial P}{\partial \theta}\right)^2 + \frac{1}{2}\lambda^2 H^3\left(\frac{\partial P}{\partial Z}\right)^2 + \Lambda \frac{dH}{d\theta} P \tag{8}$$

where the following dimensionless constans are introduced:

$$\lambda = \frac{r}{l}, \qquad \Lambda = 6\frac{\eta\ \omega\ r^2}{p_0 c^2}.$$

With respect to the boundary conditions (3a, 3b) the action integral has the form:

$$I = \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_0^{\pi} \left\{ \frac{1}{2}(1+\varepsilon\ \cos\theta)^3 \left[\frac{\partial P}{\partial \theta}\right]^2 + \frac{1}{2}\lambda^2 (1+\varepsilon\ \cos\theta)^3 \left[\frac{\partial P}{\partial Z}\right]^2 - \lambda\ \varepsilon\ P \sin\theta \right\} dZ d\theta \tag{9}$$

The solution that satisfies the boundary conditions (3a, 3b) is of the form:

$$P = \left(\frac{1}{4} - Z^2\right)\left(C_1 \sin\theta + C_2 \sin 2\theta + C_3 \sin 3\theta + C_4 \sin 4\theta\right) \tag{10}$$

where C_1, C_2, C_3 and C_4 are the unknown constants which are determined from the action integral after its minimization.

Substituting the expression (10) into the action integral (9), and solving it, the solution has been found that has the following general form:

$$I = I(C_1, C_2, C_3, C_4, \varepsilon, \lambda, \Lambda) \tag{11}$$

According to the Ritz method, the unknown constants C_1, C_2, C_3 and C_4 are determined from the algebraic equations:

$$\frac{\partial I}{\partial C_i} = 0, \qquad i = 1,2,3,4 \tag{12}$$

with the solution

$$C_i = C_i(\varepsilon, \lambda, \Lambda) \tag{13}$$

Hence, the approximate solution of the Reynolds equation (2) in dimensionless form is:

$$P = \left(\frac{1}{4} - Z^2\right)\sum_{i=1}^{4} C_i \sin(i\cdot\theta) \tag{14}$$

which represents pressure distribution in journal bearing.

3. RESULTS

Solving the system of algebraic equations (12) for Λ = const. and different values of parameters ε and λ the results (C_1, C_2, C_3 and C_4) are obtained and shown in the tables T1, T2, T3 and T4. Having found C_1, C_2, C_3 and C_4 the pressure distribution in journal bearing is finaly determined. Its graphic presentation is given in the following diagrams.

T1

	ε	C_1	C_2	C_3	C_4
$\lambda = 0$	0.7	3.7524Λ	-2.2407Λ	1.0323Λ	-0.31182Λ
	0.5	2.5574Λ	-1.0403Λ	0.35613Λ	-0.091125Λ
	0.3	1.5046Λ	-0.34903Λ	0.071199Λ	-0.012387Λ

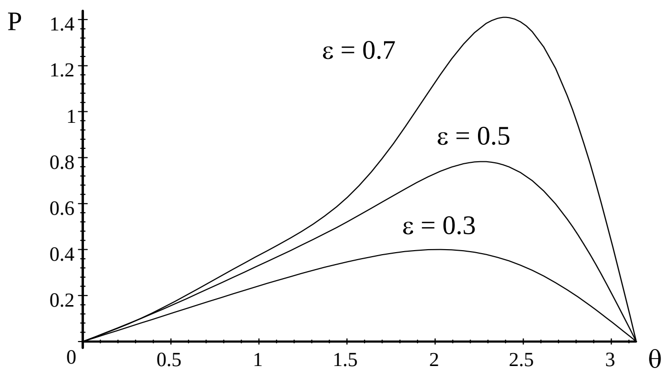

Fig. 2. Dimensionless pressure distribution for $\lambda = 0$, $Z = 0$, Λ = const.

T2

	ε	C_1	C_2	C_3	C_4
$\lambda = 0.25$	0.7	2.7561Λ	-1.8277Λ	0.87477Λ	-0.26937Λ
	0.5	1.7283Λ	-0.79019Λ	0.28269Λ	-0.073976Λ
	0.3	0.98678Λ	-0.29797Λ	0.067668Λ	-0.012455Λ

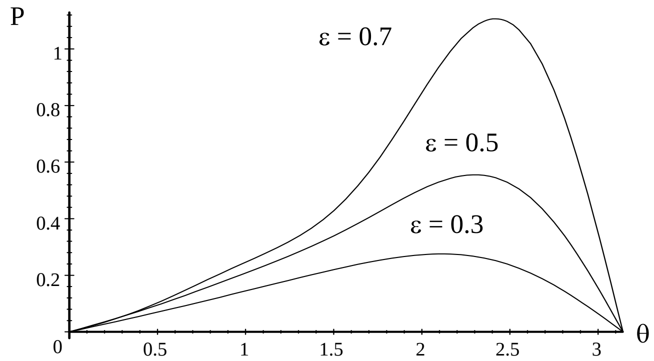

Fig. 3. Dimensionless pressure distribution for λ = 0.25, Z = 0, Λ = const.

T3

	ε	C_1	C_2	C_3	C_4
λ = 0.5	0.7	1.6094Λ	-1.2548Λ	0.64306Λ	-0.20553Λ
	0.5	0.89851Λ	-0.49252Λ	0.19032Λ	-0.051921Λ
	0.3	0.46299Λ	-0.14739Λ	0.034264Λ	-0.0063957Λ

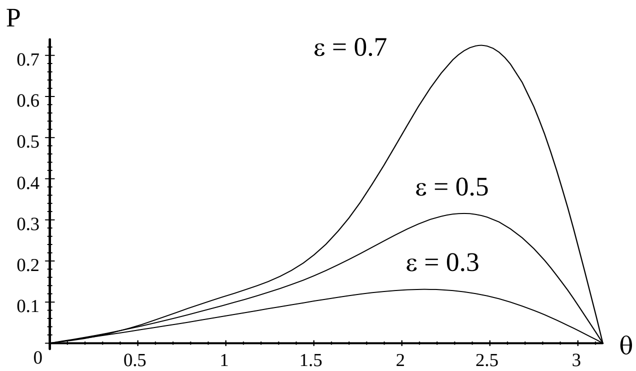

Fig. 4. Dimensionless pressure distribution for λ = 0.5, Z = 0, Λ = const.

T4

	ε	C_1	C_2	C_3	C_4
λ = 0.75	0.7	0.98755Λ	-0.85962Λ	0.46788Λ	-0.15519Λ
	0.5	0.50885Λ	-0.31547Λ	0.13025Λ	-0.036991Λ
	0.3	0.25039Λ	-0.090847Λ	0.022649Λ	-0.0044106Λ

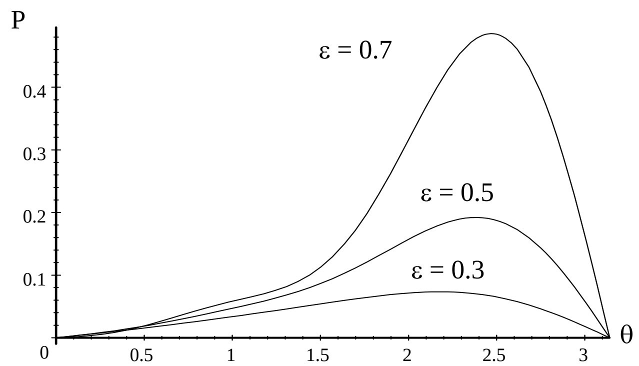

Fig. 5. Dimensionless pressure distribution for λ = 0.75, Z = 0, Λ = const.

T5

	ε	C_1	C_2	C_3	C_4
λ = 1	0.7	0.6527Λ	-0.60718Λ	0.34598Λ	-0.11852Λ
	0.5	0.3193Λ	-0.21296Λ	0.092354Λ	-0.027137Λ
	0.3	0.15285Λ	-0.059888Λ	0.015713Λ	-0.0031692Λ

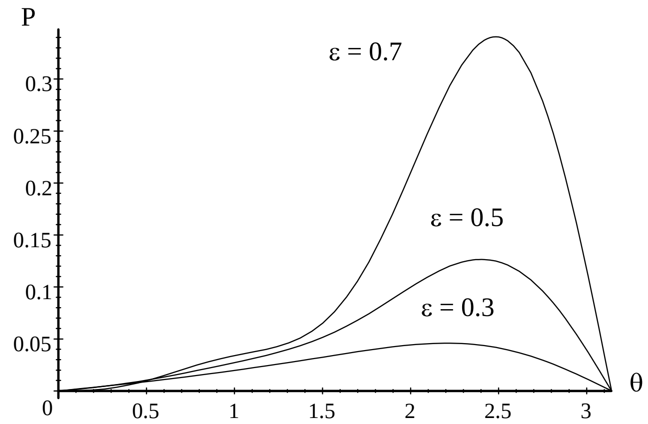

Fig. 5. Dimensionless pressure distribution for λ = 1, Z = 0, Λ = const.

4. CONCLUSION

The Ritz method is widespread in almost any field of applied science. It is efficient in finding approximate solutions by choosing different functions and using the highest possible number of terms of the function. An application of the method in the field of hydrodynamic lubrication is given in the paper.

The pressure distribution in journal bearing, where only its convergent area is supplied with lubricant [6], is determined. By varrying some characteristic parameters their influence on the pressure distribution has been researched.

NOMENCLATURE

D- bearing diameter: D=2d
d- journal diameter: d=2r
Δ- diametral clearance: Δ=D-d
c- radial clearance: c=R-r=Δ/2
l- bearnig length
l/d – length-diameter ratio
ψ – diametral clearance ratio: ψ=Δ/d=c/r
e – eccentricity of journal and bearing axes
ε – eccentricity ratio or attitude: ε=e/c, (0 ≤ ε ≤1):
θ – angle measured from maximum film thickness (0≤θ≤π)
$θ_m$ – angle measured from location of maximum film thickness to location of peak film pressure
h – local fluid film thickness: h = c (1 + ε cosθ), c/r << 1
x, y, z - coordinates, x= rθ
η – absolute viscosity of fluid
n – journal speed
ω- angular velocity of journal

v – surface speed of journal: v=rω
p – local fluid film pressure
p_m – peak pressure in fluid film
p_o – oil pressure at inlet to bearing

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