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DISTRIBUTION TRUNCATE χ^2 MODELLING WHITH THREE PARAMETERS

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ABSTRACT:
In this article we study the generalization of χ^2 classic distribution densities to possible situations in real cases. The classic distribution densities are truncated, obviously keeping the properties of a density. This new expressions of the density, can better approximate the experimental data. Further on, we introduced the χ^2 truncated repartition, accompanied by numerical examples.

KEYWORDS:
The truncate χ^2 distribution, coefficient of correlation.

1. INTRODUCTION

The classical χ^2 distribution, have the density distribution

$$\text{fclas}(x) = \frac{1}{2^{\frac{n}{2}} \cdot \sigma^n \cdot \Gamma\left(\frac{n}{2}\right)} \cdot x^{\frac{n}{2}-1} \cdot \exp\left(\frac{-x}{2 \cdot \sigma^2}\right) \tag{1}$$

This distribution wills by approximate white the function

$$\text{ftrunc}(x, n, \sigma, T) = \begin{cases} K(T) \dfrac{1}{2^{\frac{n}{2}} \cdot \sigma^n \cdot \Gamma\left(\frac{n}{2}\right)} \cdot x^{\frac{n}{2}-1} \cdot \exp\left(\dfrac{-x}{2 \cdot \sigma^2}\right), & x \in [0, T] \\ 0, \quad x \notin [0T] \end{cases} \tag{2}$$

For the function ftrunc (x,n,σ,T) to be a distribution density we impose the conditions

$$K(T) > 0 \ , \ T>0 \tag{3}$$

and

$$\int_{-\infty}^{\infty} \text{ftrunc}(t, n, \delta, K(T))\, dt = 1 . \tag{4}$$

Thus we determine expression of the truncate distribution density

$$\text{ftrunc2}(x,n,\sigma,T) := \text{if}\left(x < T, \frac{1}{\displaystyle\int_0^T \frac{1}{2^{\frac{n}{2}}\cdot\sigma^n\cdot\Gamma\left(\frac{n}{2}\right)}\cdot x^{\frac{n}{2}-1}\cdot\exp\left(\frac{-x}{2\cdot\sigma^2}\right)dx}\cdot\frac{1}{2^{\frac{n}{2}}\cdot\sigma^n\cdot\Gamma\left(\frac{n}{2}\right)}\cdot x^{\frac{n}{2}-1}\cdot\exp\left(\frac{-x}{2\cdot\sigma^2}\right), 0\right) \tag{5}$$

We remark the fact that for $T \to \infty$, the function ftrunc (x,n,σ,T) coincides with the function fclas (x).

2. PRACTICAL CASE

In the practical case result the next value of experimental date

$$x := \begin{pmatrix} 56.515 \\ 61.645 \\ 62.388 \\ 64.565 \\ 66.214 \\ 66.488 \\ 69.035 \\ 70.348 \\ 74.451 \\ 75.384 \\ 76.47 \\ 76.898 \\ 77.325 \\ 77.881 \\ 87.534 \end{pmatrix} \qquad y := \begin{pmatrix} 0.011 \\ 8.563\times 10^{-3} \\ 8.186\times 10^{-3} \\ 8.583\times 10^{-3} \\ 4.507\times 10^{-3} \\ 8.223\times 10^{-3} \\ 7.486\times 10^{-3} \\ 3.425\times 10^{-3} \\ 2.573\times 10^{-3} \\ 2.406\times 10^{-3} \\ 4.449\times 10^{-3} \\ 3.081\times 10^{-3} \\ 2.986\times 10^{-3} \\ 2.866\times 10^{-3} \\ 0 \end{pmatrix}$$

where x represent the value of independent variable, and y represent the value of dependent variable.

The parameters n, σ and T will be determined by imposing the condition of minimizing the sum of the difference squares between of the value of theoretic function ftrunc(x,n,σ,T) and experimental value of dependent variable y, that is

$$\sum_{i=1}^{nv}\left[\left[\text{if}\left[x_i < Tp_{kt}, \text{inte}\cdot\frac{1}{2^{\frac{np_{it}}{2}}\cdot(\sigma p_{jt})^{np_{it}}\cdot\Gamma\left(\frac{np_{it}}{2}\right)}\cdot(x_i)^{\frac{np_{it}}{2}-1}\cdot\exp\left[\frac{-x_i}{2\cdot(\sigma p_{jt})^2}\right], 0\right] - y_i\right]\right]^2 \tag{5}$$

where

$$\text{inte} := \frac{1}{\displaystyle\int_0^{Tp_{kt}} \frac{1}{2^{\frac{np_{it}}{2}}\cdot(\sigma p_{jt})^{np_{it}}\cdot\Gamma\left(\frac{np_{it}}{2}\right)}\cdot x^{\frac{np_{it}}{2}-1}\cdot\exp\left[\frac{-x}{2\cdot(\sigma p_{jt})^2}\right]dx} \tag{6}$$

Thus we use the next program MathCAD for minimize the function (6)

$\text{ORIGIN} \equiv 1$ $\quad nv := \text{length}(x)$ $\quad i := 1..nv$

$mx := \text{mean}(x)$ $\quad mx = 70.876$ $\quad sx := \text{stdev}(x)$ $\quad sx = 7.818$

$na := 2$ $\quad nb := 11$ $\quad \sigma a := 1$ $\quad \sigma b := 4$

$Ta := 10$ $\quad Tb := 100$ $\quad nt := nrdi$ $\quad it := 1..nt + 1$

$$np_{it} := na + (it-1)\cdot\frac{(nb-na)}{nt}$$

$$
\begin{array}{l}
prg := \\
\quad nt \leftarrow nrdi \\
\quad fmin \leftarrow 10^{10} \\
\quad \text{for } it \in 1..nt+1 \\
\qquad np_{it} \leftarrow na + (it-1)\cdot\frac{(nb-na)}{nt} \\
\qquad \text{for } jt \in 1..nt+1 \\
\qquad\quad \sigma p_{jt} \leftarrow \sigma a + (jt-1)\cdot\frac{(\sigma b-\sigma a)}{nt} \\
\qquad\quad \text{for } kt \in 1..nt \\
\qquad\qquad Tp_{kt} \leftarrow Ta + (kt-1)\cdot\frac{(Tb-Ta)}{nt} \\
\qquad\qquad \text{inte} \leftarrow \dfrac{1}{\displaystyle\int_0^{Tp_{kt}} \frac{1}{2^{\frac{np_{it}}{2}}\cdot(\sigma p_{jt})^{np_{it}}\cdot\Gamma\left(\frac{np_{it}}{2}\right)}\cdot x^{\frac{np_{it}}{2}-1}\cdot\exp\left[\frac{-x}{2\cdot(\sigma p_{jt})^2}\right]dx} \\
\qquad\qquad f \leftarrow \displaystyle\sum_{i=1}^{nv}\left[\text{if}\left[x_i < Tp_{kt},\ \text{inte}\cdot\frac{1}{2^{\frac{np_{it}}{2}}\cdot(\sigma p_{jt})^{np_{it}}\cdot\Gamma\left(\frac{np_{it}}{2}\right)}\cdot(x_i)^{\frac{np_{it}}{2}-1}\cdot\exp\left[\frac{-x_i}{2\cdot(\sigma p_{jt})^2}\right],0\right]-y_i\right]^2 \\
\qquad\qquad \text{if } (f \le fmin) \\
\qquad\qquad\quad nf \leftarrow np_{it} \\
\qquad\qquad\quad \sigma f \leftarrow \sigma p_{jt} \\
\qquad\qquad\quad Tf \leftarrow Tp_{kt} \\
\qquad\qquad\quad fmin \leftarrow f \\
\quad \begin{pmatrix} nf \\ \sigma f \\ Tf \\ fmin \end{pmatrix}
\end{array}
$$

$$prg = \begin{pmatrix} 10 \\ 2 \\ 80 \\ 2.03\times 10^{-5} \end{pmatrix}$$

$n := prg_1 \quad n = 10 \quad T := prg_3 \quad T = 80$

$\sigma := prg_2 \quad \sigma = 2$

Thus, from this program result the value of parameters n, σ and T.

These values substituted in relation (4) lead to expression of truncate density distribution.

For this modeling result the value of the correlation coefficient

$$rtrunc2 := \sqrt{1 - \frac{\sum_i (y_i - ftrunc2(x_i, n, \sigma, T))^2}{\sum_i (y_i - mean(y))^2}}, \qquad rtrunc2 = 0.925 \tag{7}$$

And the standard deviation is

$$\sigma trunc2 := \sqrt{\frac{\left|\sum_i [y_i - (ftrunc2(x_i, n, \sigma, T))]^2\right|}{nv}}, \qquad \sigma trunc2 = 1.163 \times 10^{-3} \tag{8}$$

The graphic of distribution density is representing in Figure 1.

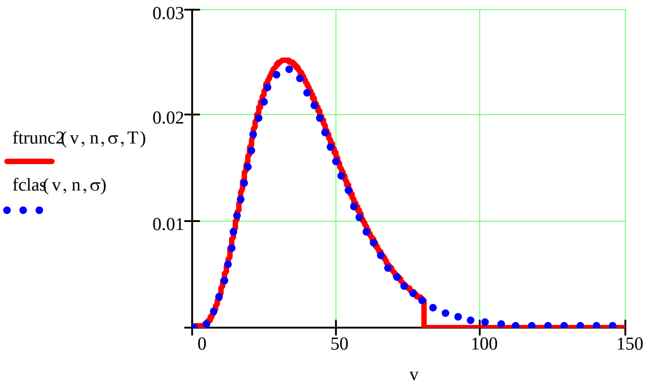

Figure 1. The truncate distribution density

The graphic of truncate repartition function is representing in Figure 2.

$var1 := 1.3 \quad var2 := 7 \quad kf := 20 \quad k := 1..kf$

$$xv_k := (0) + \frac{(max(x) + var2) - (0)}{kf - 1} \cdot (k - 1)$$

$$Ftrunc2(x, n, \sigma, T) := \int_{-.1}^{x} ftrunc2(u, n, \sigma, T)\, du \quad wFtrunc2_k := Ftrunc2(xv_k, n, \sigma, T)$$

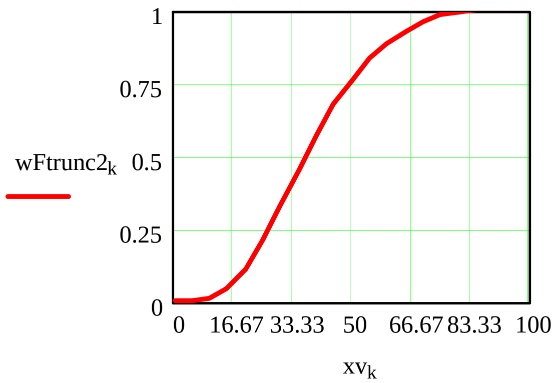

Figure 2. The truncate repartition function

And the graphic of the truncated characteristic function is represented in Figure 3.

$$ex(t) := \int_{-1}^{6} e^{i\cdot t\cdot x}\cdot if\left(x<T, \frac{1}{\int_0^T \frac{1}{2^{\frac{n}{2}}\cdot\sigma^n\cdot\Gamma\left(\frac{n}{2}\right)}\cdot x^{\frac{n}{2}-1}\cdot\exp\left(\frac{-x}{2\cdot\sigma^2}\right)dx}\cdot\frac{1}{2^{\frac{n}{2}}\cdot\sigma^n\cdot\Gamma\left(\frac{n}{2}\right)}\cdot x^{\frac{n}{2}-1}\cdot\exp\left(\frac{-x}{2\cdot\sigma^2}\right), 0\right) dx$$

$$t := -20, -15 .. 200$$

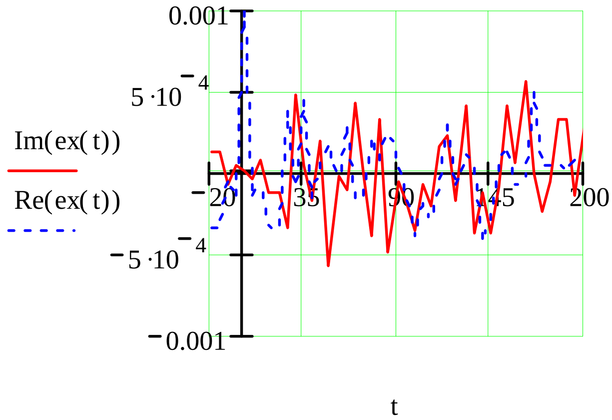

Figure 3. The truncated characteristic function

We notice that the, for classical distribution density, for the same experimental date, per minimizing the sum of the difference squares between of the value of theoretic function fclas(x,n,σ) and experimental value of dependent variable y, we obtain the value

$$n := 8 \qquad \sigma := 2.2 .$$

Thus, for this value substituted in relation (1), we obtain the value of correlation coefficient

$$rcl := \sqrt{1 - \frac{\sum_i \left(y_i - fclas(x_i)\right)^2}{\sum_i \left(y_i - mean(y)\right)^2}}, \qquad rcl = 0.893 \tag{9}$$

In the case of truncate distribution density, white this value of n and σ, we obtain for T the value $T := 79.5$

Thus, if we modeling the experimental date with the function

$$\text{ftrunc1}(x,T) := \text{if}\left(x < T, \frac{1}{\displaystyle\int_0^T \frac{1}{2^{\frac{n}{2}}\cdot\sigma^n\cdot\Gamma\left(\frac{n}{2}\right)}\cdot x^{\frac{n}{2}-1}\cdot\exp\left(\frac{-x}{2\cdot\sigma^2}\right)dx}\cdot\frac{1}{2^{\frac{n}{2}}\cdot\sigma^n\cdot\Gamma\left(\frac{n}{2}\right)}\cdot x^{\frac{n}{2}-1}\cdot\exp\left(\frac{-x}{2\cdot\sigma^2}\right), 0\right) \quad (10)$$

obtain the value of correlation coefficient

$$\text{rtrunc1} := \sqrt{1-\frac{\sum_i \left(y_i - \text{ftrunc1}\left(x_i, T\right)\right)^2}{\sum_i \left(y_i - \text{mean}(y)\right)^2}}, \qquad \text{rtrunc1} = 0.914 \,. \quad (11)$$

3. CONCLUSIONS

If we compare the value obtains in tree modeling, results that the best modeling is on tree parameters, that is ftrunc (x, n, σ, T)

In conclusion this type of modeling is recommendation in any practical cases.

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