ANNALS OF THE FACULTY OF ENGINEERING HUNEDOARA
2006, Tome IV, Fascicole 2, (ISSN 1584 – 2665)
FACULTY OF ENGINEERING HUNEDOARA,
5, REVOLUTIEI, 331128, HUNEDOARA

STUDY OF THE NON-DIMENSIONAL SOLUTION OF DYNAMIC EQUATION OF MOVEMENT ON THE PLANE PLAQUE WITH CONSIDERATION OF TWO-ORDER SLIDING PHENOMENON

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Abstract:
Considering a sliding boundary layer of two orders in the neighborhood of an unlimited plane plaque we emphasize a new approach for the numerical solution of the involved viscous fluid flow.

Key words: Sliding boundary layer, polynomial velocity profile
AMS: 76D10

1. INTRODUCTION

Let us envisage a fluid stream past a semi-infinite plane plaque with the "attack" angle zero. Suppose that the fluid is viscous incompressible while the flow is plane (in Oxy). The plane plaque is considered located on the real axis Ox, its "attack" edge being at O.

According to the well known boundary layer approximation (Prandtl), the Navier -Stokes equations

$$\rho\vec{v}.\nabla u = -\frac{\partial p}{\partial x} + \mu\Delta u, \qquad \rho\vec{v}.\nabla v = -\frac{\partial p}{\partial y} + \mu\Delta v, \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

lead to

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\,\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

where ρ, p, and $\vec{v}(u,v)$, are the mass density, the pressure and the plane velocity respectively while μ is the viscosity coefficient.

To these equations one attaches the boundary conditions

$$\begin{aligned}&u(x,0) = L_1 \frac{\partial u}{\partial y}(x,0) - M_1 \frac{\partial^2 u}{\partial y^2}(x,0),\\ &v(x,0) = 0, \qquad u(x,\infty) = u_{\infty},\end{aligned} \tag{4}$$

the first one signifying the fact that the fluid slides on the plaque surface (in stead of the adherence on the plaque, i.e. of the classical non-slip condition $u(x,0) = 0$).

In a previous attempt we have given a numerical method for approaching this problem which is based on a particular polynomial development of the velocity profile inside the boundary layer.

Such sliding boundary conditions have been considered before [1].

In the papers [2], [3], [4] and [5] is study the dynamic problem in hypotheses

$$u(x,0) = L \frac{\partial u}{\partial y}(x,0)\,. \tag{5}$$

2. Results

By using the expression of *v*, which comes from (3), the above relation (2) leads to

$$\rho\left[u\frac{\partial u}{\partial x} - \left(\int_0^y \frac{\partial u}{\partial x} dy\right)\frac{\partial u}{\partial y}\right] = \mu \frac{\partial^2 u}{\partial y^2}\,. \tag{6}$$

Integrating this integro-differential equation across the boundary layer, namely, from $y = 0$ to $y = \delta(x)$ - the upper border of the boundary layer, i.e.,

$$\rho\int_0^\delta u\frac{\partial u}{\partial x}dy - \rho\left[u\int_0^y \frac{\partial u}{\partial x}dy\right]_0^\delta + \rho\int_0^\delta u\frac{\partial u}{\partial x}dy = -\tau_w\,, \tag{7}$$

we get the integral relation,

$$\rho u_\infty^2 \frac{d}{dx}\int_0^\delta \frac{u}{u_\infty}\left(\frac{u}{u_\infty} - 1\right)dy = -\tau_w\,, \tag{8}$$

where

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_w\,. \tag{9}$$

In this paper, we solve this equation by considering a velocity profile of a polynomial type (within the boundary layer) but this time of higher (6^{th}) degree which seems to be a more accurate approach.

Precisely we suppose that

$$\frac{u}{u_\infty} \equiv \bar{u} = a_0 + a_1\eta + a_2\eta^2 + a_3\eta^3 + a_4\eta^4 + a_5\eta^5 + a_6\eta^6, \quad 0 \le \eta \le 1,$$

and $\dfrac{u}{u_\infty} \equiv \bar{u} = 1, \quad \eta \ge 1$,

where $\eta \equiv \dfrac{y}{\delta(x)}$. (11)

The coefficients a_i can be determined by using the appropriate conditions

$$\overline{u} = L\frac{\partial \overline{u}}{\partial \eta} - M\frac{\partial^2 \overline{u}}{\partial \eta^2}, \quad \frac{\partial^3 \overline{u}}{\partial \eta^3} = 0, \quad \frac{\partial^4 \overline{u}}{\partial \eta^4} = 0, \quad \text{for } \eta = 0, \tag{12}$$

$$\overline{u} = 1, \quad \frac{\partial \overline{u}}{\partial \eta} = 0, \quad \frac{\partial^2 \overline{u}}{\partial \eta^2} = 0, \quad \frac{\partial^3 \overline{u}}{\partial \eta^3} = 0, \quad \text{for } \eta = 1, \tag{13}$$

where

$$L = \frac{L_1}{\delta(x)} \quad , \quad M = \frac{M_1}{\delta^2(x)}. \tag{14}$$

Following the calculations, it results for the non-dimensional profile of the horizontal component of the velocity the expression

$$\overline{u} = \frac{1}{2 + 6L + 10M}\left(6L + 10M + 6\eta - 5\eta^2 + 2\eta^5 - \eta^6\right). \tag{15}$$

In figure (1) and (2) is present the profile of the velocity for L = 0, respectively L = 0,5.

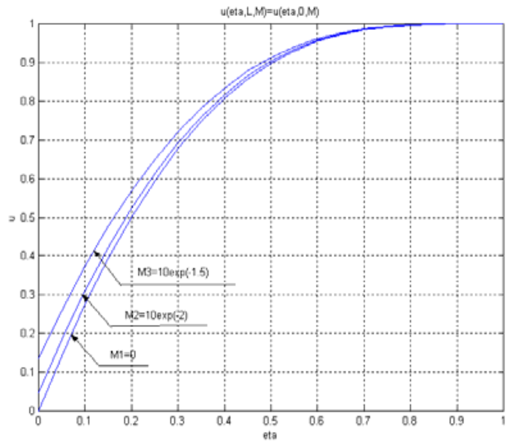

Figure 1. Profile of the velocity u for L=0

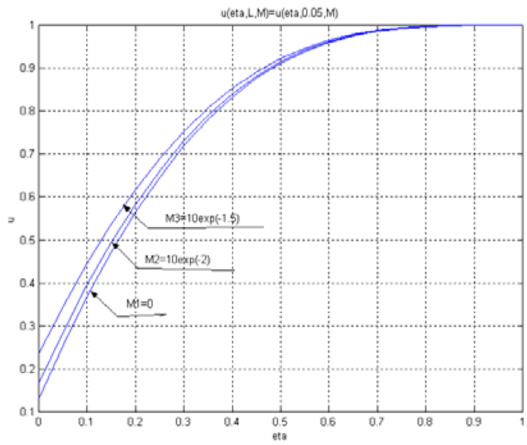

Figure 2. Profile of the velocity u for L=0,05

In figure (3) and (4) the influence of the M parameter on the velocity's profile is presented.

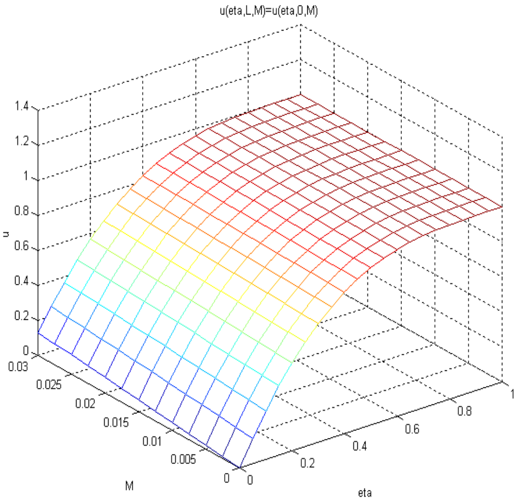

Figure 3. The influence of the y and M parameters on the velocity's profile (L=0).

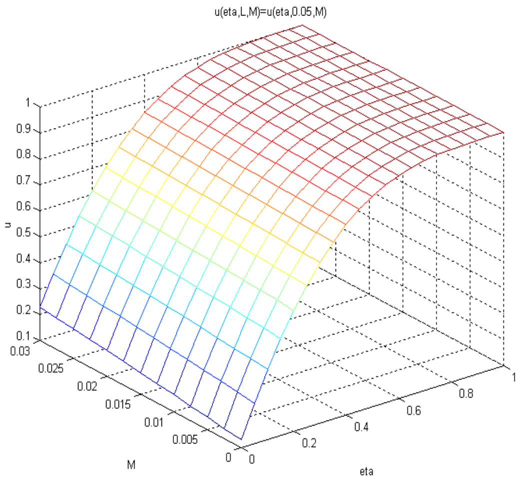

Figure 4. The influence of the y and M parameter on the velocity's profile (L=0,05).

In figure (5) and (6) is present the contour lines for u(eta,0,M) respectively u(eta,0.05,M) .

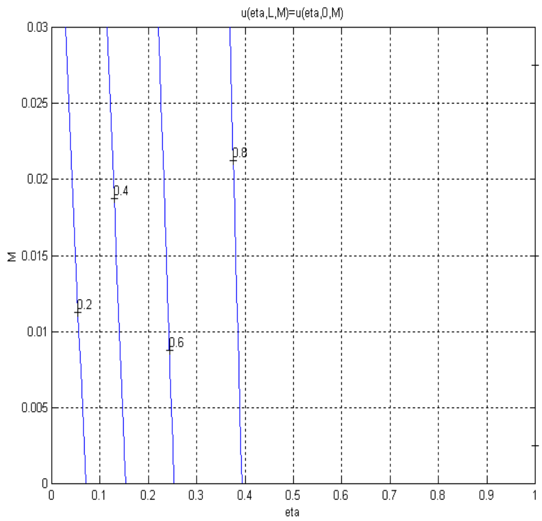

Figure 5. The contour lines for u(eta,0,M)

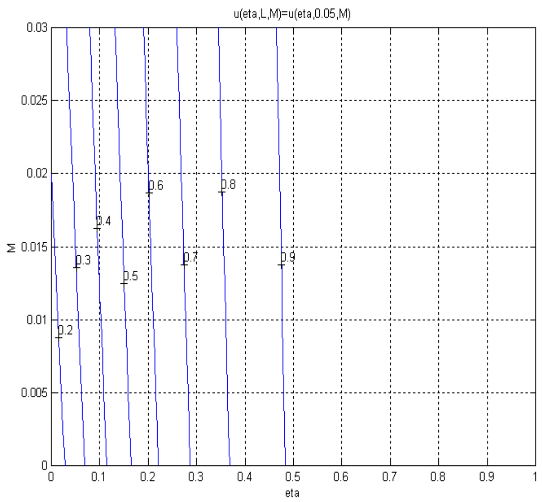

Figure 6. The contour lines for u(eta,0.05,M)

The thickness of the impulse losses

$$\theta = \delta\int_0^1 \frac{u}{u_\infty}\left(1-\frac{u}{u_\infty}\right)d\eta, \tag{16}$$

has in this case the following form,

$$\theta = \frac{\delta}{N^2}\left(\frac{20}{7}L + \frac{100}{21}M + 0.4\right) \tag{17}$$

where $N = 2 + 6L + 10M$.

In figure (7) and (8), in the section x = ct., is present the influence of the L and M parameters on the velocity in y = 0 respectively y = 0,1, and in figure 9 and 10 is present the contours lines corresponding.

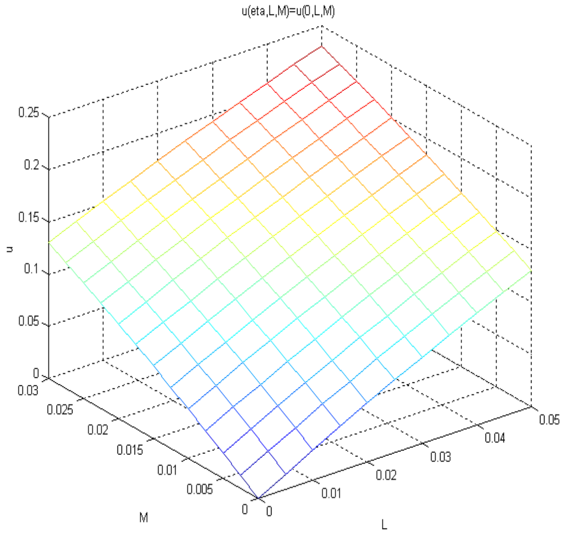

Figure 7. The influence of the L and M parameters on the velocity in y=0

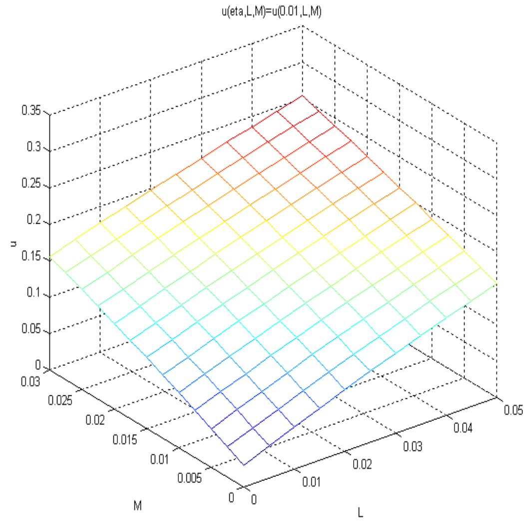

Figure 8. The influence of the L and M parameters on the velocity in y=0,01

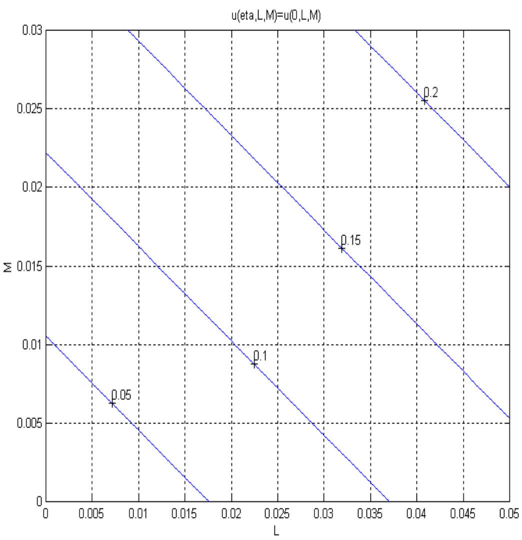

Figure 9. The contour lines for Figure 7

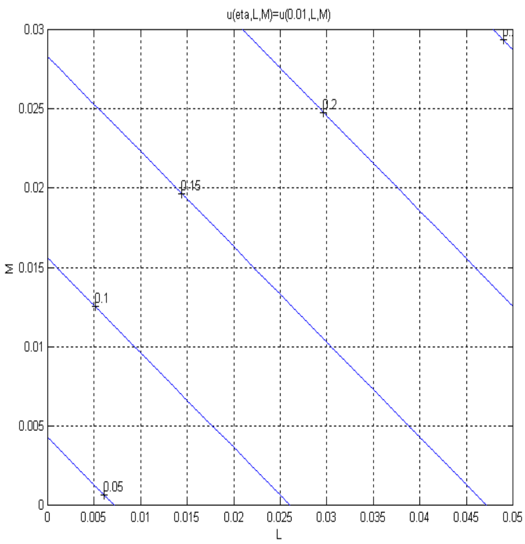

Figure 10. The contour lines for Figure 8

3. Conclusions

This paper study the influence of two order term from boundary condition corresponding to motion of viscous incompressible fluid on the plane plaque.

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