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CONSIDERATIONS ON DISTRIBUTION OF STRESS IN NON-DIMENSIONAL BOUNDARY LAYER IN THE CASE OF MOUVMENT WITH PHENOMENON OF TWO ORDERS SLIDING

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Abstract:
Considering a sliding boundary layer of two orders in the neighborhood of an unlimited plane plaque, we study the distribution of stress in boundary layer.

Key words: Sliding boundary layer, polynomial velocity profile
AMS: 76D10

1. Introduction

Let us envisage a fluid stream past a semi-infinite plane plaque with the "attack" angle zero. Suppose that the fluid is viscous incompressible while the flow is plane (in Oxy). The plane plaque is considered located on the real axis Ox, its "attack" edge being at O.

According to the well known boundary layer approximation (Prandtl), the Navier -Stokes equations

$$\rho\vec{v}.\nabla u = -\frac{\partial p}{\partial x} + \mu\Delta u, \qquad \rho\vec{v}.\nabla v = -\frac{\partial p}{\partial y} + \mu\Delta v, \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

lead to

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\,\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

where ρ, p, and $\vec{v}(u,v)$, are the mass density, the pressure and the plane velocity respectively while μ is the viscosity coefficient.

To these equations one attaches the boundary conditions

$$u(x,0) = L_1 \frac{\partial u}{\partial y}(x,0) - M_1 \frac{\partial^2 u}{\partial y^2}(x,0), \quad v(x,0) = 0, \quad u(x,\infty) = u_{\infty}, \tag{4}$$

the first one signifying the fact that the fluid slides on the plaque surface (in stead of the adherence on the plaque, i.e. of the classical non-slip condition $u(x,0) = 0$).

In a previous attempt we have given a numerical method for approaching this problem which is based on a particular polynomial development of the velocity profile inside the boundary layer.

Such sliding boundary conditions have been considered before [1].
In the papers [2], [3], [4] and [5] is study the dynamic problem in hypotheses

$$u(x,0) = L \frac{\partial u}{\partial y}(x,0) \tag{5}$$

2. Results

By using the expression of v which comes from (3), the above relation (2) leads to

$$\rho\left[u\frac{\partial u}{\partial x} - \left(\int_0^y \frac{\partial u}{\partial x} dy\right)\frac{\partial u}{\partial y}\right] = \mu \frac{\partial^2 u}{\partial y^2}. \tag{6}$$

Integrating this integro-differential equation across the boundary layer, namely, from $y = 0$ to $y = \delta(x)$ - the upper border of the boundary layer, we get the integral relation,

$$\rho u_{\infty}^2 \frac{d}{dx}\int_0^{\delta} \frac{u}{u_{\infty}}\left(\frac{u}{u_{\infty}} - 1\right) dy = -\tau_w, \tag{7}$$

where

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_w. \tag{8}$$

Distribution of non-dimensional velocity in boundary layer is given of the expression [2]

$$\bar{u} = \frac{1}{2 + 6L + 10M}\left(6L + 10M + 6\eta - 5\eta^2 + 2\eta^5 - \eta^6\right). \tag{9}$$

Where

$$\bar{u} = \frac{u}{u_{\infty}}, \quad \eta \equiv \frac{y}{\delta(x)}, \quad 0 \le \eta \le 1, \tag{10}$$

This expression satisfy the conditions of boundary conditions

$$\bar{u} = L\frac{\partial \bar{u}}{\partial \eta} - M\frac{\partial^2 \bar{u}}{\partial \eta^2}, \quad \frac{\partial^3 \bar{u}}{\partial \eta^3} = 0, \quad \frac{\partial^4 \bar{u}}{\partial \eta^4} = 0, \quad \text{for } \eta = 0, \tag{12}$$

$$\bar{u} = 1, \quad \frac{\partial \bar{u}}{\partial \eta} = 0, \quad \frac{\partial^2 \bar{u}}{\partial \eta^2} = 0, \quad \frac{\partial^3 \bar{u}}{\partial \eta^3} = 0, \quad \text{for } \eta = 1, \tag{13}$$

where

$$L = \frac{L_1}{\delta(x)}, \quad M = \frac{M_1}{\delta^2(x)}. \tag{14}$$

In this paper, we study the distribution of stress in boundary layer considering a velocity profile of a polynomial type (within the boundary layer) but this time of higher (6^{th}) degree which seems to be a more accurate approach.

The thickness of the impulse losses

$$\theta = \delta\int_0^1 \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right)d\eta, \tag{16}$$

has in this case the following form,

$$\theta = \frac{\delta}{N^2}\left(\frac{20}{7}L + \frac{100}{21}M + 0.4\right) \tag{17}$$

where

$N = 2 + 6L + 10M$.

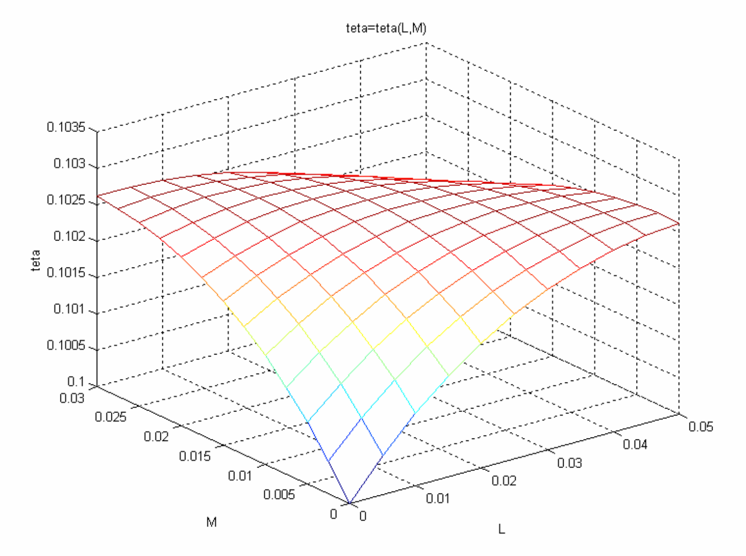

Figure 1. The distribution of thickness of the impulse losses in a section of boundary layer

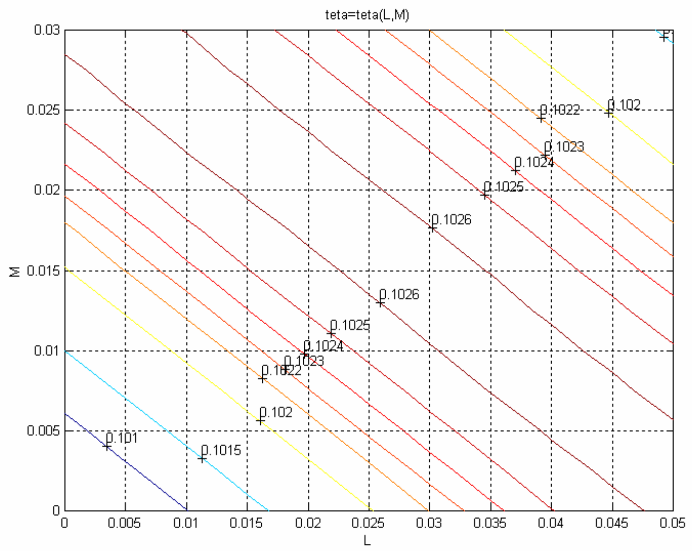

Figure 2. The level lines corresponding

The local tension between two neighboring layers

$$\tau = \mu\left(\frac{\partial u}{\partial y}\right), \tag{18}$$

has the expression

$$\tau = \frac{\mu\ u_\infty}{\delta\left(2+6L+10M\right)}\left(6-10\eta+10\eta^4-6\eta^5\right) \tag{19}$$

The local stress on the plaque has the structure

$$\tau_w = \frac{3\mu\ u_\infty}{\delta(1+3L+5M)}. \tag{20}$$

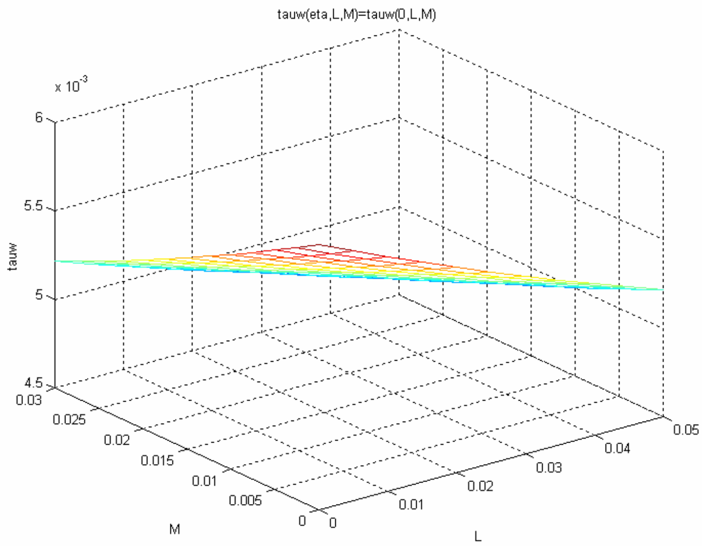

Figure 3. Distribution of the stress of friction on plaque

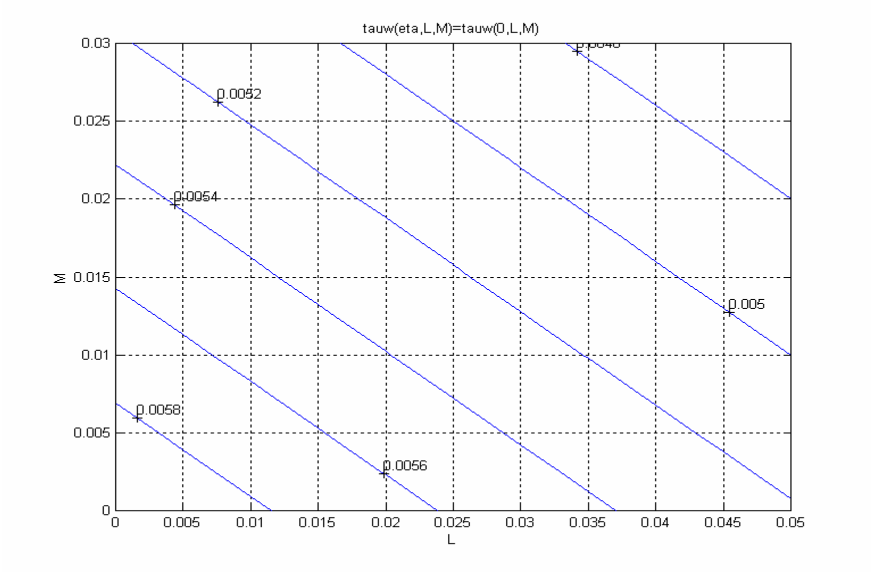

Figure 4. The level lines corresponding

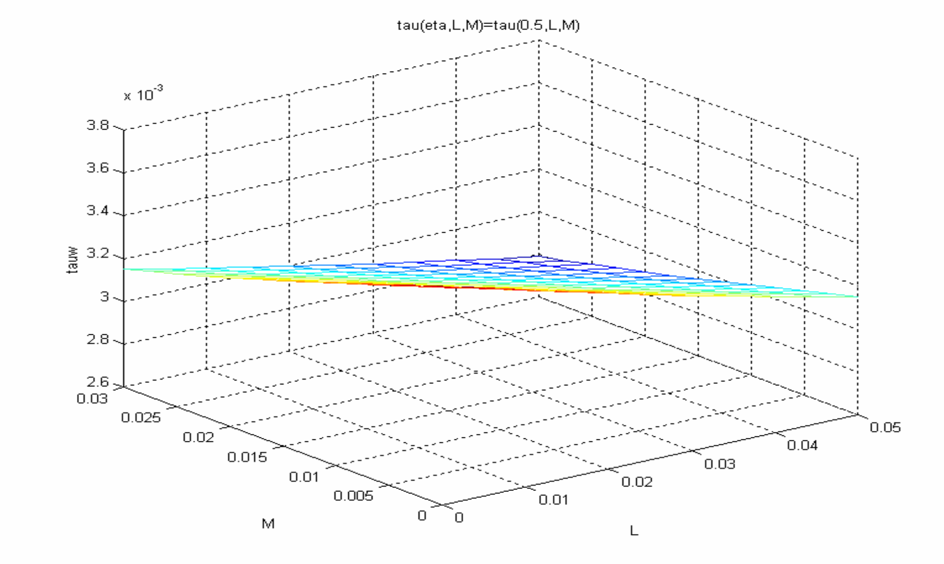

Figure 5. Distribution of the stress of friction in layer

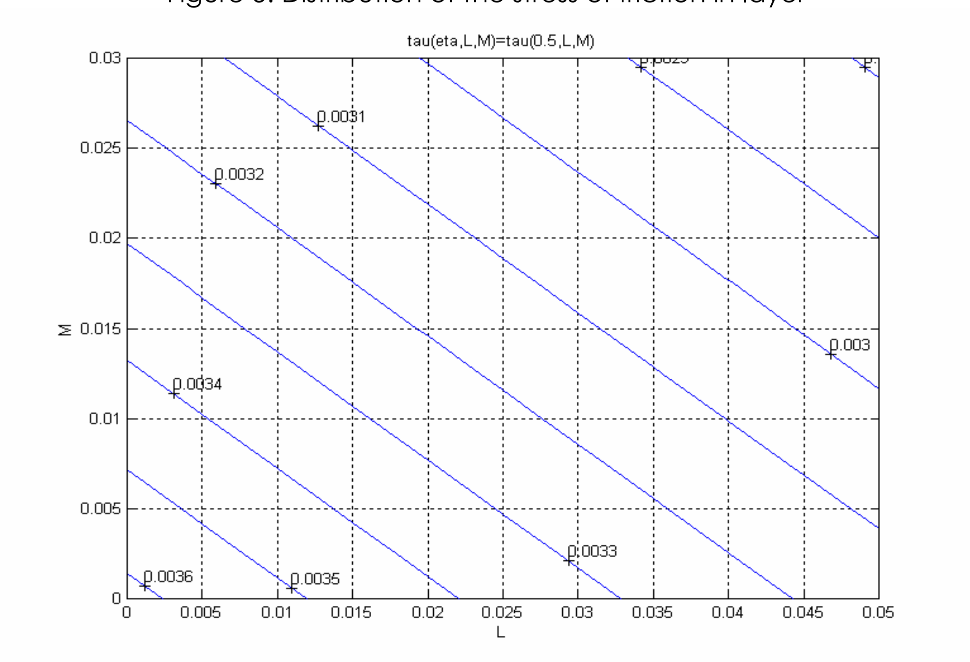

Figure 6. The level lines corresponding

3. Conclusions

This paper study the influence of two order term on stress of friction from boundary condition corresponding to motion of viscous incompressible fluid on the plane plaque.

REFERENCES

[1.] Schaaf, S.A., Shermann, F.S., *Mechanika*, 1 (29), 1955, pp. 130-140;

[2.] Maksay, St., *Dynamic boundary layer with sliding on a plane plaque*, Politehnica University of Timisoara, Anal. Fac. Ing. of Hunedoara, Tom III, Fasc.5, 2001, pp.39;

[3.] Maksay, St., *Solving the dynamical problem of the fluids movement on a plane plaque considering the sliding phenomenon*, Politehnica University of Timisoara, Anal. Fac. Ing. of Hunedoara, Tom I, Fasc. 4, 1999, pp. 39 – 46;

[4.] Petrila, T., Maksay, St., *New numerical approach for the dynamic boundary layer sliding on a plate*, Bul. St. – Univ. Pitesti, Ser. Matem & Inform., 9, 2003, pp. 249;

[5.] Petrila, T., Maksay, S., A *new numerical approach of a sliding boundary layer*, Computers and Mathematics with applications, 50 (2005), 113-121;

[6.] Maksay, S., Stoica, D., *Study of dynamic equation of movement on the plane plaque with consideration of two order sliding phenomenon*, Proceedings of the eleventh Symposium of Mathematics and its Applications, 2006 (in press).

[7.] Hasimoto, H., *Boundary layer slip solutions for a flat plate*, Journal of the Aeronautical Sciences 25, No.1, 68-69(1958).