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A WAVELET BASED METHOD APPLIED TO ANALYZE MOROCCAN PHOSPHATE DEPOSIT "DISTURBANCES"

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Abstract
For geophysical processes, in particular, tools that offer the ability to examine the variability of a process at different scales are especially important. The wavelet transform offers such a tool and has already proven useful in the study of many processes in diverse areas of science and engineering including resistivity survey. In this paper we have focused on presenting the most important properties of wavelet transforms that make them attractive for Moroccan geoelectrical application. We review the basic results of wavelet analysis applied as an attractive and powerful tool for filtering resistivity data in Moroccan phosphate deposit "disturbances".
Key words:
resistivity, Schlumberger, phosphate, wavelet, Morocco.

Introduction

Wavelet transforms are relatively recent developments that have fascinated the scientific, engineering, and mathematics community with their versatile applicability.

Wavelets have become pervasive in various scientific and engineering researches. Wavelet transform has provided not only a wealth of new mathematical results, but also a common language and rallying call for researchers in a remarkably wide variety of fields: mathematicians working in harmonic analysis because of the special properties of wavelet bases; mathematical physicists because of the implications for time-frequency or phase-space analysis and relationships to concepts of renormalization; digital signal processors because of connections with multirate filtering [1]. Within geophysics, there have been numerous applications already, such as geological or geophysical variables for data compression, noise reduction, and feature extraction. Wavelets are essentially used in two ways when studying geophysical processes or signals: as an integration kernel for analysis to extract information about the process and also as a basis for representation or characterization of the process [2][3][4][5]. Evidently, in any analysis or representation the choice of the kernel or basis function determines the nature of information that can be extracted or represented about the process comparatively to the Fourier process.

Fourier transforms provide a powerful tool for the analysis of stationary processes because for such processes the Fourier components are uncorrelated. The decision as to which representation (expansion) to use for a signal, for example, wavelet expansion versus Fourier or spline expansion, depends on the purpose of the analysis [6]. Wavelets have become increasingly popular for analyzing data in the geosciences. Wavelets re-express data collected over a time span or spatial region such that variations over temporal/spatial scales are summarized in wavelet coefficients. Individual coefficients depend upon both a scale and a temporal/spatial location, so wavelets are ideal for analysing geo-systems with interacting scales [7]. We applied this powerful tool to resistivity data map, which represents in fact Moroccan phosphate deposit "disturbances" map (figure 1).

* "DERANGEMENT" OU "DISTURBANCE"

Figure.1. Example of "disturbances" in the Moroccan phosphatic series

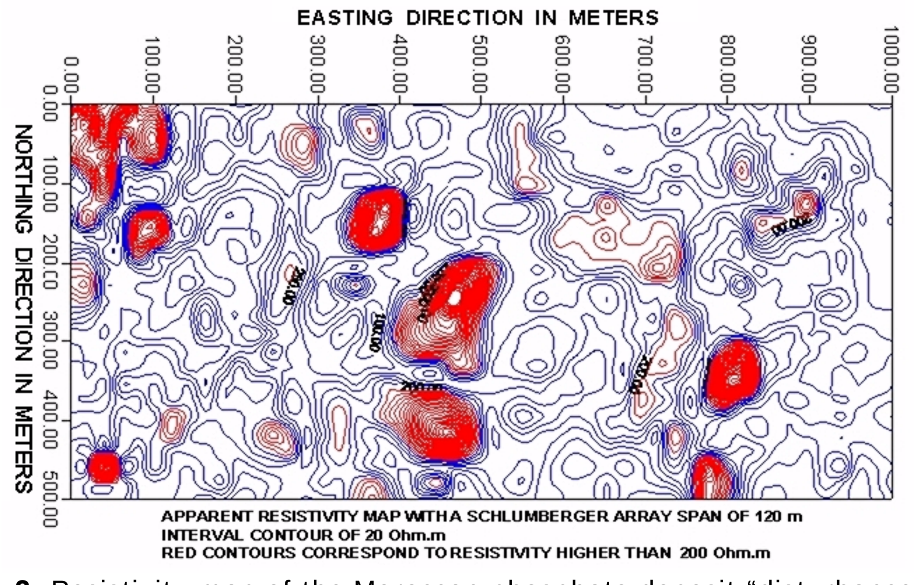

Figure.2. Resistivity map of the Moroccan phosphate deposit "disturbances" map

The resistivity map (figure 2) obtained from a further survey was considered in fact a map of discrete potentials on the free surface, and any major singularity in the apparent resistivities due to the presence of a perturbation will be due to the crossing from a "normal" into a "perturbed" area or vice versa. In other words, the apparent resistivity map have been considered a map of scalar potential differences assumed to be harmonic everywhere except over the perturbed areas. Interpretation of resistivity anomalies, in this context, was the process of extracting information on the position and composition of a target mineral body in the ground. In the present case the targets were essentially the inclusions called *perturbations.* The amplitude of an anomaly may be assumed to be proportional to the volume of a target body and to the resistivity contrast with the mother lode. If the body has the same resistivity as the mother lode no anomaly will be detected. Thus assumed in fact and in first approach that the resistivity anomalies would be representative of the local density contrast between the disturbances and the mother lode. Level disturbance of the anomalous zones is proportionnal to resistivity intensity (figure 3) [8][9].

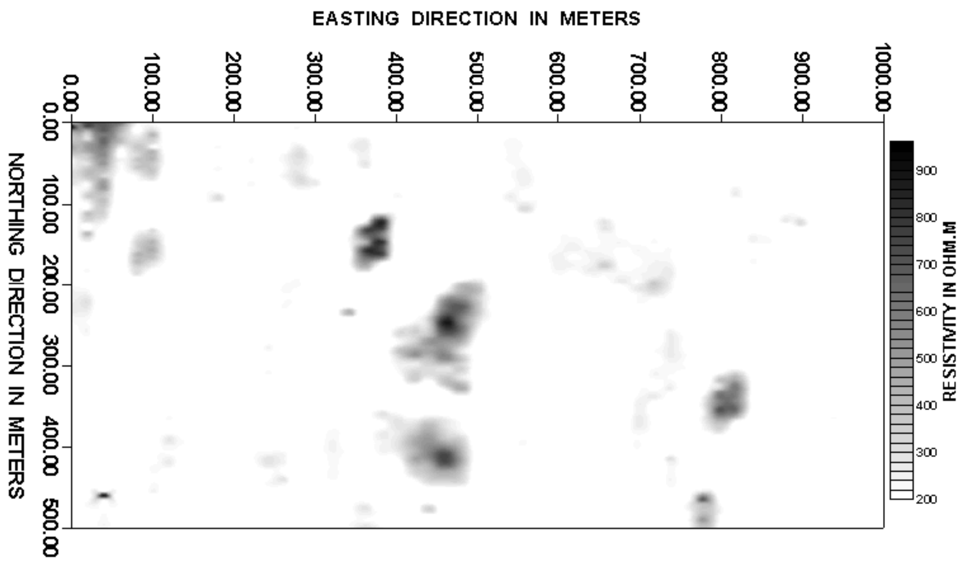

Figure.3. A map of disturbed phosphate zones corresponding to figure 2

2D wavelet transform & processing data

The wavelet transform is a time-frequency decomposition, which links a time (or space) domain function to its time-scale wavelet domain representation. The concept of scale is broadly related to frequency. Small scales relate to short duration, high frequency features and correspondingly, large scales relate to long duration, low frequency features. Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other function. In the signal analysis framework, the wavelet transform of the time (or space) varying signal depends on the scale that is related to frequency and time (or space) [10].

The 2D wavelet method provides information on many more resolution than the former method. It is a powerful tool particulary suitable in denoising, filtering and analyzing problems and potential singularities [11]. Moreover this property is crucial for performing an efficient linear denoising resistivity anomaly map of the moroccan phosphate deposit “disturbances”.

The wavelet transform of a 2D signal $f(x,y)$, where x and y represent respectively the easting and the northing directions, and f the apparent resistivity data ρ, is defined as:

$$\omega(X,Y,a,b)=\int\int \frac{1}{\sqrt{|XY|}} f(x,y)\psi\left(\frac{x-a}{X},\frac{y-b}{Y}\right)dxdy \qquad (1)$$

where $\frac{1}{\sqrt{|XY|}}\psi\left(\frac{x-a}{X},\frac{y-b}{Y}\right)$ is the wavelet coefficient associated to the scales X and Y at the point with coordinates a and b [12]. The limits in the double integral are $-\infty$ and $+\infty$ for the two variables. ψ is the wavelet "mother" function that satisfies the constraints

$$\iint dxdy\,\psi = 0 \quad (2)$$

and

$$\iint dxdy\,\psi^2 = 0 \quad (3)$$

The 'admissibility' condition that allows to reconstruct the function f, i.e there exists the following integral:

$$C_\psi = (2\pi)^2 \iint dk_1 dk_2 \frac{\left|\hat{\psi}(k_1,k_2)\right|^2}{|k_1 k_2|} \quad (4)$$

where $\hat{\psi}(k_1,k_2)$ represents the 2D Fourier transform of ψ and denotes the modulus of the complex number C_ψ. A reconstruction of the original geophysical signal corresponding to resitivity data can be achieved using the inversion formula expressed by:

$$f(x,y) = \frac{1}{C_\psi}\iint \frac{dXdY}{|XY|^2} dadb\,\omega(X,Y,a,b)\frac{1}{\sqrt{|XY|}}\psi\left(\frac{x-a}{X},\frac{y-b}{Y}\right) \quad (5)$$

We have chosen in our study the Morlet wavelet "mother" [13] (figure 4) defined in 1D by the following equation:

$$\psi_{Morlet}(x) = \frac{1}{\pi^4} e^{(j\alpha_0 x)} e^{(-x^2/2)} \quad (6)$$

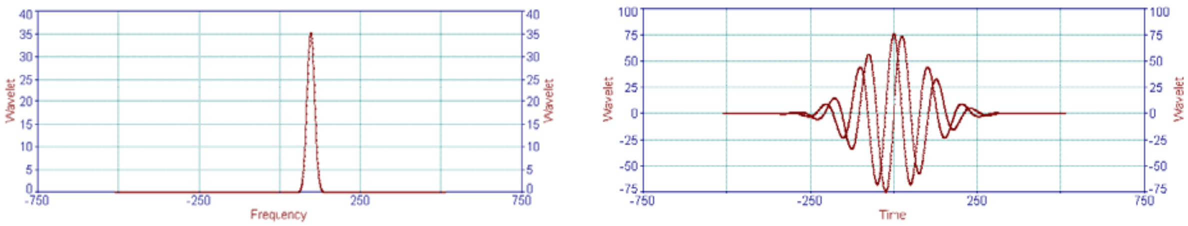

Figure.4. Time and frequency Morlet wavelet "mother" responses

where α_0 is an adjustable parameter (wavenumber). The adjustable parameter has been put to 6. This is the smallest wavenumber that allows for an accurate resistivity signal reconstruction.

The resistivity database is a compilation of 51 traverses at a spacing of 20 m. There were 101 stations at 5 m distance for every traverse, which makes 5151 stations all together in the resistivity survey. We calculated the magnitude square of the wavelet transform coefficients using AutoSignal routine [14] for each resistivity traverse (figure 5). Then we deferred all the results to built a 2D wavelet spectrum regular maps which represent in fact filtering and denoising map of the phosphate deposit "disturbances". Since a major potential application of wavelets is in image processing, the 2D wavelet transform is a necessity to be applied as a detector and analyser of singularities like edges, contours or corners [15].

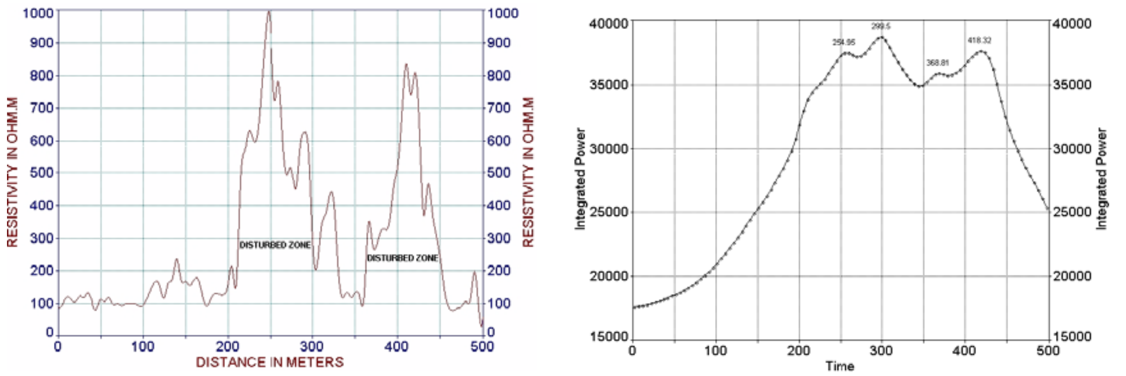

Figure.5. Example of real resistivity traverse data of the survey and the corresponding continuous wavelet transform using Morlet "mother".

Results & conclusions

Figure 6 represents an indicator of the level of variation of the contrast of density between the disturbances and the normal phosphate-bearing rock. The surface modeling of resistivity anomalies is obtained by AutoSignal routine from our apparent resistivity survey. This procedure enables us to define the surface phosphate disturbed zones. The continous wavelet analysis surface of phosphate deposit disturbance zones modelling as obtained by the above procedure in the study area provided a direct image for an interpretation of the resistivity survey.

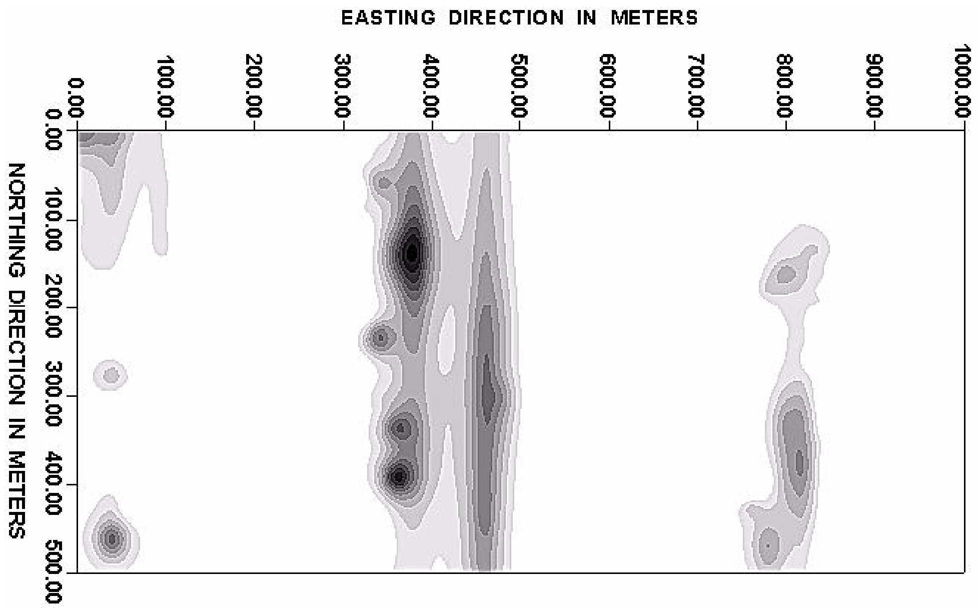

Figure.6. Wavelet output of the phosphate deposit "disturbances" map given in figure 3

This method enables us to identify the anomalies area, which turned out to be strongly correlated with the disturbances. The use of magnitude square of the continous wavelet transform represent an effective filtering method which makes it possible to attenuate considerably the noise represented by the minor dispersed and random disturbances. The overall effect is that of scanning and denoising the anomalous bodies. Comparatively to classical approaches used in filtering and denoising geophysical data maps, the advantage of the 2D continous wavelet transform method is doesn't introduce significant distorsion to the shape of the original resistivity signal. The wavelet output of the apparent resistivity which correspond to the wavelet output of the anomalous phoshate deposit map obtained from such a technical tool represent the crossing dominate area from a "normal" into a "perturbed" area or vice versa.

Moreover the level of disturbance is very clearly shown. The proposed filtering and denoising method using 2D continous wavelet transform tends to give a real estimation of the surface of the phosphate deposit "disturbances" zones with a significant suppression of the noise. The level disturbance resulting from such method is also more defined in all the disturbed zones. Comparatively to others methods like non parametric regression method [16] or Savitzky-Golay filtering method [17] also used to denoise the same resistivity data map, we have described an analytical procedure to analyze the anomalies of a specific problem in the phosphate mining industry. The results proved satisfying. Data processing procedures as 2D wavelet transform of resistivity data map was found to be consistently useful and the corresponding map may be used as auxiliary tools for decision making under field conditions.

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