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DYNAMIC ENVIRONMENTS AS CONTEXTS FOR CONJECTURING AND PROVING

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ABSTRACT:
Euclidean geometry and geometric proof have occupied a central place in mathematics education from classical Greek society through to twentieth century Western culture. It is proof which sets mathematics apart from the empirical sciences, and forms the foundation of our mathematical knowledge. The latter part of the twentieth century, however, witnessed the demise of both Euclidean geometry and proof in school mathematics curricula in many countries. Debate about proof in school mathematics curricula has also been driven by the development and introduction into schools of dynamic geometry software, such as Cabri Geometry II and The Geometer's Sketchpad. With in-built Euclidean geometry tools and a drag facility, these dynamic geometry environments have the potential to transform the teaching and learning of geometry.
KEYWORDS:
Dynamic environment, geometric proof, e-learning

1. INTRODUCTION

The lack of success of traditional methods of teaching proof in mathematics has prompted researchers to seek new approaches. There is some evidence that students are better able to understand and construct proofs if they have been involved in a process of conjecturing and argumentation. Mariotti, Bartolini Bussi, Boero, Ferri, and Garuti (1) assert that successful proof construction is dependent on continuity of reasoning, or "cognitive unity", between producing a conjecture and constructing a proof of the conjecture, with the process of argumentation creating a bridge between statements made during conjecturing and statements used in the proof construction. Bartolini Bussi (2), for example, as part of a teaching project linking history and mathematics, has introduced students to conjecturing, argumentation, and proving through a study of the geometry of historical drawing instruments. Other recent research studies on the teaching of geometric proof have focused on dynamic geometry software.

A common feature of these two contrasting dynamic environments—the old technology of mechanical linkages and historic drawing instruments, and the new technology of dynamic geometry software—is the potential for dynamic visualization, which may play a key role in students' production and testing of conjectures, as well as in their proof construction. This paper will focus on the role of dynamic environments in geometric reasoning, and includes an overview of dynamic geometry software and a review of research relating to visualization and reasoning with dynamic geometry software.

2. DYNAMIC GEOMETRY SOFTWARE

"Dynamic geometry software" has become a generic term for a class of geometry software environments, for example, Cabri Geometry II, The Geometer's Sketchpad and Geometry Expert, where geometric objects can be continuously transformed on the screen by dragging, with only those features based on geometric properties remaining invariant. Accurate measurements can be performed, and the software incorporates Euclidean geometry tools, such as angle bisector and perpendicular line. The "drag mode" which distinguishes these programs from other geometry software, allows a particular drawing to be replaced by a continuum of drawings.

The dragging of an element of a drawing generates an infinity of different drawings on the screen while a geometrical figure is the set of geometrical properties and relations attached to a drawing that are invariant through the drag mode. Mariotti (1) regards Cabri screen images as representing the direct external counterpart of has called a figural concept. Screen images allow the student to take into account both the figural and conceptual components, and therefore play an important role in geometrical reasoning. Laborde (3) asserts that dynamic drawings offer stronger visual evidence than a single static drawing: "A spatial property may emerge as an invariant in the movement whereas this might not be noticeable in one static drawing". She notes that when students are engaged in problem-solving tasks in dynamic geometry computer environments, "a critical point of the solving process is the visual recognition of a geometrical invariant by the students, which allows them to move to geometry". On the other hand, questions the impact of readily produced computer images on the learner's ability to generate his/her own mental images, noting that "it is easy to become seduced by the visualizations to the extent of thinking that consideration of them is the purpose of using them in geometry".

Despite the strong feeling that the dynamic imagery associated with use of the software has the potential to play a significant part in geometric reasoning, concern has been expressed that dynamic geometry software is contributing to an empirical approach to school geometry. Instead, traditional geometry exercises have been adapted for the computer, and, of greater concern, geometry is being reduced to pattern-spotting in data generated by dragging and measurement of screen drawings, with little or no emphasis on theoretical geometry: "school mathematics is poised to incorporate powerful dynamic geometry tools in order merely to spot patterns and generate cases".

3. VISUALISATION AND REASONING WITH DYNAMIC GEOMETRY SOFTWARE

Although it appears that there are many instances of dynamic geometry software being used merely to collect empirical data, it is also possible to use the software in ways that encourage geometric reasoning. The construction of geometric shapes which retain their properties and relationships when dragged, focuses students' attention on the relationships between properties. Other activities may require students to explore, make conjectures, and prove properties for a given geometric figure, or to model and investigate a dynamic physical situation in order to understand the effect of changing parameters.

3.1. Construction tasks

In a study by Vincent (4) thirteen Year 8 students were given the task of constructing drag-resistant geometric figures in Cabri. In the first task, to draw a drag-resistant rectangle, the majority of students commenced with four carefully placed segments which they aligned with the edges of the screen, then measured the sides and/or angles to confirm that they had drawn a rectangle. As soon as they dragged their rectangles, however, they realized that their drawings did not meet the requirements, a realization that challenged them to focus on relationships between the properties of the rectangle, and to choose appropriate Cabri construction tools.

Even when a successful geometric construction of the rectangle had been completed, most students continued to use measurement of angles or sides, as well as dragging, to check the validity of their constructions. As the students progressed to their next construction task, most of them still commenced with by-eye constructions before choosing appropriate Cabri construction tools, such as Perpendicular bisector and Parallel line.

In the study reported by Vincent, only one student went straight to a correct geometric construction for each of the shapes. Significantly, although she dragged each construction briefly to check it before progressing to the next shape, she made no measurements to check her constructions.

In a further task in the same study, pairs of students were asked to replicate a given `house' shape as a Cabri figure that would retain its essential properties when dragged. As in the case of their earlier constructions, the students commenced with by-eye drawings or a combination of by-eye and geometric constructions. The feedback from dragging then either confirmed their construction or assisted them in understanding the geometry.

3.2. Proof tasks in a dynamic geometry environment

Hadas, Hershkowitz and Schwarz (5) suggest that the problem of students being too readily convinced in a dynamic geometry environment may be overcome by the use of problems that lead to unexpected or surprising situations. They designed an activity that was intended to create a contradiction between year 8 students' conjectures about the sum of the exterior angles of a polygon as the number of sides of the polygon increases, and the results obtained when the students measured the angles using dynamic geometry software. The students were first required to measure the internal angles of a number of polygons and explain their conclusion. The second task asked them to determine by measurement the sum of the exterior angles of a quadrilateral, to conjecture what would happen to the sum as the number of sides increased, and then to check their conjecture by measurement. In 37 of the 49 responses (41 written responses from 82 students working in pairs and eight students interviewed individually), students conjectured that the sum of the exterior angles would increase as the number of sides increased.

Despite their incorrect conjecture, many of the students were able to explain the actual result. Hadas report that even though the students had only just commenced their study of Euclidean geometry, nine of the 50 explanations (one interview student gave two different explanations), were "complete deductive explanations and eight more used partial deductive or inductive explanations". Seventeen of the remaining responses were categorized as "no explanation", two as "inductive", and sixteen as "visual-variations". Hadas note that the students ceased to be recipients of formal proofs, but were engaged in an activity of construction and evaluation of arguments in which certainty and understanding were at stake, and they had to use their geometrical knowledge to explain contradictions and overcome uncertainty.

3.3. Different roles of dragging in a dynamic geometry environment

Different modes of dragging of dynamic geometry figures may be used depending on the information which the user hopes to gain. If, for example, a student has constructed an isosceles triangle, dragging may be used as a check that the triangle will remain isosceles, confirming that the triangle has been appropriately constructed. This is the mode in which dragging was used by students A and G during the construction of their "house" shape. Dragging may also be used in exploratory tasks, where a figure is dragged in order to satisfy a particular visual constraint. This mode of dragging is often used in association with tracing the path of a point. Laborde (6), for example, use the example of dragging a triangle *ABC* until angle *A* is a right angle (see Figure 1), then continuing to drag point *A* to produce its trace while trying to retain the right angle intact. Laborde refer to the path of *A* as a "soft locus", as it is a visual path obtained by deliberate dragging, rather than a locus in the sense of a point being constrained to move along a certain path due to a particular property being incorporated into the construction. Laborde suggest that this exploration provides a starting point for the conjecture that the path of point *A* is a circle that in turn may lead students to the construction of a circle with the midpoint of *BC* as centre. Dragging may then be used as a check to confirm the conjecture. Laborde note that "the final step is the question "Why?" where students must use geometry to explain their observation.

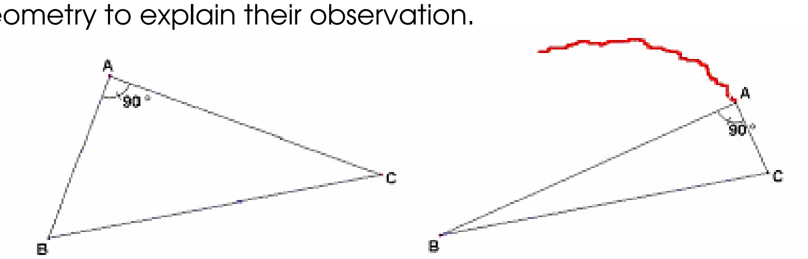

Figure 1. Tracing the path of A as it is dragged to retain the right angle at A

Arzarello (7) assert that students' use of dragging when investigating a problem in a dynamic geometry environment changes as they develop a greater understanding of the problem, and that the different modes of dragging play a part in the progression to deductive reasoning. Arzarello describe a study undertaken with a class of 27 students who used Cabri to investigate the following problem (Figure 2).

Let *ABCD* be a quadrangle. Consider the bisectors of its internal angles and their intersection points *H, K, L, M* of pairwise consecutive bisectors. Drag *ABCD,* considering all its different configurations: what happens to the quadrangle *HKLM*? What kind of figure does it become?

Arzarello report that the more able students tended to drag the quadrilateral *ABCD* systematically (Figure 3): they dragged *ABCD* until it was a parallelogram and noticed that *HKLM* was a rectangle; when *ABCD* was a rectangle, *HKLM* was a square; and finally, when *ABCD* was a square, *H*, *K*, *L*, and *M* were concurrent.

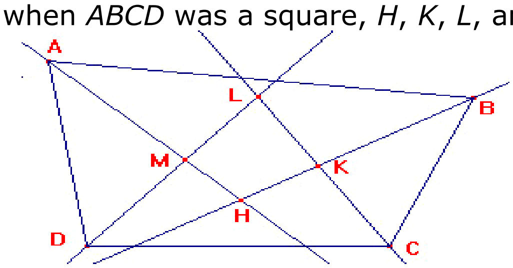

Figure 2. Cabri construction of the angle bisectors of a quadrilater ABCD

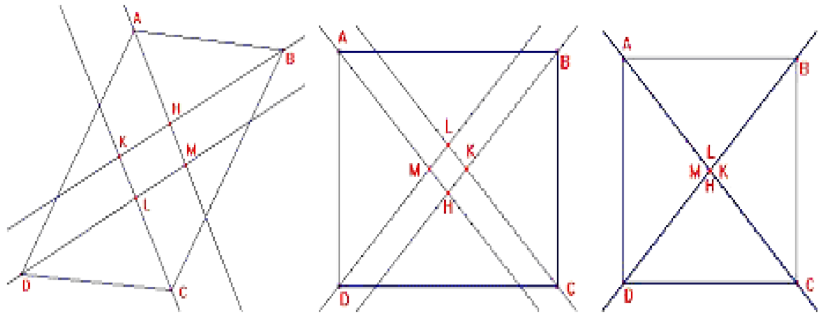

Figure 3. Systematic dragging of quadrilateral ABCD

The students then dragged quadrilateral *ABCD* so that the four points *H*, *K*, *L*, and *M* remained concurrent, and noticed that the sum of the lengths of each pair of opposite sides of *ABCD* were equal (Figure 4). Based on their previous geometry knowledge, they conjectured that this would occur if the quadrilateral were circumscribed to a circle. Arzarello note that the students then commenced with a circle, constructed a circumscribed quadrilateral, and showed that its angle bisectors were concurrent (Figure 5). Arzarello refer to this reversal of reasoning as abduction.

Construction of the angle bisectors and a dragging test confirmed that the angle bisectors were concurrent. Arzarello use the terms ascending and descending control of meaning to describe the two phases of the students' exploration of the problem. The ascending phase involves the use of wandering and lieu muet dragging, and represents the empirical and abductive stages, while the descending phase is associated with deductive reasoning and use of the dragging test. In the discussion which followed the exploration, one student commented: "We proved the same thing but starting from a circle too; we drew the tangent lines and we came to the same conclusion. Arzarello note that the students now had "all the elements they need to prove the statement". It is not clear from the report, however, whether the students did actually construct a formal deductive proof.

The middle ability level students were observed to be less systematic in their dragging. Arzarello did not report the number of students in the class who were able to arrive at the circumscribed quadrilateral conjecture, but they do suggest that learning situations could be designed to encourage this purposeful lieu muet dragging rather than it depending "only on the ingenuity of some pupils". The task described by Arzarello could, for example, be modified to a more directed activity in which students were asked to explore the conditions under which the angle bisectors were concurrent.

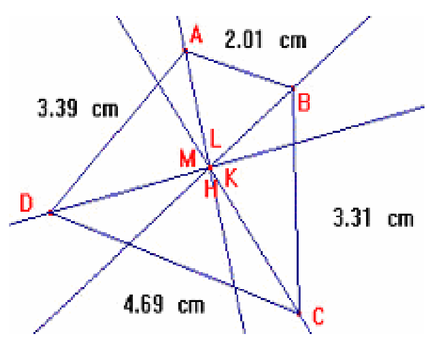

Figure 4. Concurrent angle bisectors and the circumscribed quadrilateral property

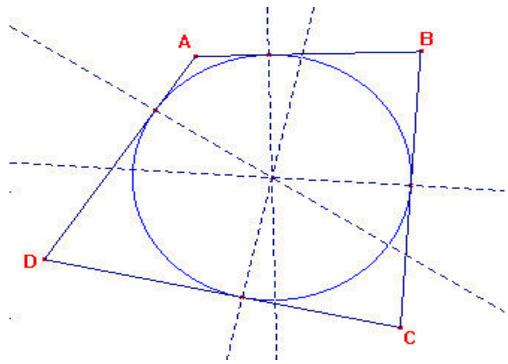

Figure 5. Inscribed circle in quadrilateral ABCD

3.4. CONNECTING EMPIRICAL AND DEDUCTIVE REASONING IN A DYNAMIC GEOMETRY ENVIRONMENT

Laborde (6) assert that the changes in the solving process brought by the dynamic possibilities of Cabri come from an active and reasoning visualization, from what we call an interactive process between inductive and deductive reasoning.

Similarly, Laborde (3) notes that learning geometry involves "not only learning how to use theoretical statements in deductive reasoning but also learning to recognise visually relevant spatial-graphical invariants attached to geometrical invariants". She reports that observations of students working on a geometry problem in pencil-and-paper and dynamic geometry environments showed that the problem made sense for the students only after they were able to visually manipulate their screen construction. Laborde argues that in a pencil-and-paper environment, students' movement between the spatial-graphical and theoretical domains is restricted, whereas the software environment promotes links between the two domains.

Scher (8) asserts that dynamic geometry software can influence the style of experimentation and reasoning so that "the boundary between deductive reasoning and dynamic geometry becomes blurred: the software finds its way into the proof process". De Villiers (9) notes that in cases of his own personal discoveries using dynamic geometry software, actual conviction based on continual experimental confirmation preceded the eventual proof, and that manipulation of the dynamic geometry construction provided him with the necessary insight to develop the proof.

Although it might be expected that in a dynamic geometry environment students would make connections between empirical data and deductive reasoning, studies have shown that this is not necessarily the case. Scher describe aspects of a classroom research project in which fifteen 15-year-old students of above average mathematical attainment were introduced to a culture of conjecture and proof in a Cabri environment. Despite a deliberate attempt in the research design to facilitate links between inductive and deductive reasoning, it was found that many of the students failed to make connections between their empirical Cabri work and proofs.

The teaching experiment consisted of three classroom sessions with three follow-up homework activities. During the first session the students used Cabri to explore the conditions for triangle congruency. The second session, which introduced the students to formal proof writing, drew upon their "actions, conjectures and explanations" from the first session and the homework activity.

4. CONCLUSIONS

There is a strong belief amongst many mathematics educators that proof should be a part of school mathematics, and should be seen by students as fundamental to the way mathematics is constructed. The difficulties which students experience with proof indicate, however, that new ways of approaching the teaching and learning of proof are necessary. There appear to be two main issues associated with these difficulties—firstly, students' motivation and cognitive need for proof, and secondly, their understanding of how to construct proofs particularly deductive proofs. The first of these implies an acceptance of the need for proof as fundamental to how our mathematical knowledge has been built up, and recognition of the purposes of proof. Where students are presented with a statement to prove, as was the case for many generations of students, construction of the proof frequently becomes an end in itself, with the majority of students failing to understand either the need for, or the purposes of, proof. Proof-related activities need to be designed so that students experience a genuine cognitive need for conviction beyond the conviction traditionally engendered by a textbook or the authority of the teacher. Equally important, the proving process should offer students the satisfaction of being able to explain why their conjectures are true.

Research suggests that if students are to be successful in constructing proofs, they must be allowed to engage in tasks where they can produce their own conjectures, and develop their own reasoned arguments through a process of classroom argumentation. During the argumentation process, where justifications are being continually modified and refined, the continuity that exists between conjecturing and proving appears to facilitate the students' logical ordering of statements in their proofs. Classroom argumentation has been criticized, however, on the grounds that the natural language of arguments, where the aim is to convince at all costs, conflicts with mathematical reasoning. If students are left to themselves to argue, this criticism may indeed apply. It is essential, then, that the teacher fulfils the role of 'qualified judge' with regard to the validity of students' arguments, and to the appropriateness and acceptability of these arguments for the particular level.

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