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ABOUT THE USE OF HAAR WAVELETS MOTHER RESPONSES FOR PERFORMANCE ANALYSIS OF MOROCCAN ENGINEERING CASE STUDY PROCESS

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ABSTRACT

This paper aims at reviewing some Moroccan industrial applications of wavelet especially in the dynamic identification of a process model using Haar wavelet mother response. Two recent Moroccan study cases are described using dynamic data originated by a distillation column and an industrial polyethylene process plant. The purpose of the wavelet scheme is to build on-line dynamic models. In both case studies, a comparison is carried out between the Haar wavelet mother response model and a linear difference equation model. Finally it concludes, on the base of the comparison of the process performances and the best responses, which may be usefull to create an estimated on-line internal model control and its application towards model-predictive controllers (MPC). All calculations were implemented using AutoSignal Software.

KEY WORDS:

process performance, model, wavelets, Haar, Moroccan.

1. INTRODUCTION

Multiphase systems and chemical reacting flows are the basis of several important applications in chemical engineering. They often lead to field quantities that develop sharp gradients and high localized phenomena. These include strong non linearity's, coupling effects, and mass transfer resistance and fluid dynamic dispersion phenomena. Traditionally, the Fourier transform has been used to diagnose faults in the process operations and to identify processes in the frequency domain. Its effectiveness has been of great importance when solving problems in process operation and control, process modelling and simulation, identification and sensor data interpretation. Fourier transform is historically based on mathematical form that will facilitate heat transfer study. Nowadays, and instead of using a Fourier transform, a mathematical tool for the time series analysis, called wavelets [1] is being used to compress signals. Identification a process is a followed strategy to obtain an originating mathematical model of experimental data. Series of tasks must be carried out to identify a process model starting with the design of the experimental data collect, data processing that includes denoising and normalization, pattern determination and analysis [2]. Wavelet analysis is an emerging field of applied mathematics that has provided new tools and algorithms for solving such problems as are encountered in fault diagnosis, modelling, identification, and control and optimization of chemical systems, where raw sensor data have to be processed into meaningful information [3]. The theory has acquired the status of a unifying theory underlying many of the methods used in physics and signal processing. Wavelet may find many possible applications of time-frequency decomposition to the discipline of chemical engineering. In engineering process industry, signal processing and control is widely used and wavelet transform appears naturally as a useful tool [4]. We choose the Haar wavelet basis for its smoothness and compact support [5].

2. SIGNAL ANALYSIS AND WAVELET TRANSFORM

2.1. THEORICAL APPROACH

Traditionally, Fourier transform has been used to process stationary signals acquired by computers. In this way, the representative spectrum of frequencies is obtained from the time series produced during acquisition of the signal by the computer. For non stationary signals, typical of engineering processes, the existing methodologies have not been fully developed. Windowed Fourier Transform, also called short-tine Fourier Transform, was first applied using a Gaussian type window [6].

For a given signal $f(t)$, a conventionally defined signal $g(t-t_0)$ is applied to a window of time that moves along with the original signal, forming a new family of functions: $f_g(t_0,t)= f(t)g(t-t_0)$Functions formed this way are centred on t_0 and have a duration defined by the characteristic time window of the function $g(t)$. Windowed Fourier transform is thus defined as:

$$F_g[w,t_0]=\int_{-\infty}^{+\infty} f(t)g(t-t_0)e^{-jwt}dt \qquad (1)$$

This transform is calculated for all t_0 values and it gives a representation of the signal $f(t)$ in the time frequency domain. If a space function $f(x)$ instead of a time signal is considered, a representation is given in the space- frequency domain. However as a windowed Fourier transform represents a signal by the sum of it sine and cosine functions, it restricts the flexibility of the function $g(t-t_0)$ or $g(x-x_0)$ making a characterization of a signal and simultaneous location of its high frequency and low frequency components difficult in the time-frequency domain or the space-frequency domain. Wavelets transform was developed to overcome this deficiency of windowed Fourier transform in representing non-stationary signals [7]. Wavelets transform is obtained from a signal by dilatation-contraction and by the translation of a special wavelet within the time or space domain. The expansion of this signal into wavelets thus permits the signal's local transient behaviour to be captured, while the sine and the cosines can only capture the overall behaviour of the signal as they always oscillate indefinitely [8].

2.2. SIGNAL ANALYSIS AND THE HAAR WAVELET

In the Fourier analysis, every periodic function having a period of 2π and an integrable square is generated by an overlay of exponential complexes,

$$W_n(x)=e^{jnx}, n=0,\pm1,\pm2,...$$

obtained by dilations of the function

$$W(x)=e^{jx} : W_n(x)=W(nx).$$

Extending the idea to space for Ψ integrable square functions, the following is defined:

$$\Psi_{a,b} = \frac{1}{\sqrt{a}}\Psi\left(\frac{t-b}{a}\right), a,b \in R, a \neq 0 \tag{2}$$

The function Ψ is called a mother wavelet, where a the scale is factor and b is the translation parameter. The family of simpler wavelets, which will be adopted in the present work, is that the Haar wavelet [9]:

$$\Psi(x) = 1\, if\ 0 \le x \le \frac{1}{2}, \Psi(x) = -1\, if\ \frac{1}{2} \le x \le 1, \Psi(x) = 0\, if\ x \notin [0,1]$$

For the one-dimensional no stationary function $f(x)$ that decrease to zero when $x \to \infty$, the following assumption is normally adopted:

$$\Psi_{p,q}(x) = 2^{\frac{-p}{2}}\Psi\left(2^p x - q\right) \tag{3}$$

The scale factor of $2^{-p}q$ is called the localization or dyatic translation and k is the translation index associated with the localization, where p and $q \in Z$. Wavelet thus defined are orthogonal, i.e., $\left\langle \Psi_{p,q}.\Psi_{l,m} \right\rangle = \delta_{p,l}\delta_{q,m}$ for $p,q,l,m \in Z$ where $\langle . \rangle$ is equal to the scalar product and δ refer to the delta function of *Dirac* [10]. Thus the function $f(x)$ can be rewritten as follows:

$$f(x) = \sum_{p=-\infty}^{+\infty}\sum_{q=-\infty}^{+\infty} c_{p,q}\Psi_{p,q}(x) \tag{4}$$

The values of the constant $c_{p,q}$ are obtained by wavelet transform in its discrete form. Then is expanded into a series of wavelets with their coefficients obtained from

$$c_{p,q} = \left\langle f.\Psi_{p,q} \right\rangle = \int_{-\infty}^{+\infty} f(x)\Psi_{p,q}(x)dx \tag{5}$$

The wavelet transform can also be calculated using special filters called quadrature mirror filters [11]. They are defined as a low-pass filter, associated with the coarser scale, and a high-pass filter to characterize the details of the signal. The signal $f(x)$ then is described as:

$$f(x) = \sum_q c_{p_0,q}\Psi_{p_0,q}(x) + \sum_{p>p_0}\sum_q d_{p,q}\Psi_{p,q}(x) \tag{6}$$

where

$$c_{p_0,q} = \int_{-\infty}^{+\infty} f(x)\Phi_{p_0,q}(x)dx \tag{7}$$

$$d_{p,q} = \int_{-\infty}^{+\infty} f(x)\Psi_{p,q}(x)dx \tag{8}$$

In the expansion of $f(x)$ by equation 6, the first term represents the approximation of the signal and the second the signal details, filtered by the approximation. The function $\Phi_{p_0,q}(x)$ is denominated a scale function or *father wavelet*, and it is responsible for obtaining the approximation of the signal, while the mother wavelets, $\Psi_{p,q}$ are responsible for the generation of the details filtered by the approximation [12]. For the family of Haar wavelets, the scale function is $\Phi_{p_0,q}(x) = 1\, if\ x \in [0,1]$ and $\Phi_{p_0,q}(x) = 0\, if\ x \notin [0,1]$. The mother wavelets, responsible for the details in the Haar family, are expressed as:

$$\Psi_{p,q}(x) = 2^{\frac{p}{2}}, if\ 2^{-p}q \le x \le 2^{-p}\left(q + \frac{1}{2}\right) \tag{9}$$

$$\Psi_{p,q}(x) = -2^{\frac{p}{2}}, if\ 2^{-p}\left(q+\frac{1}{2}\right) \le x \le 2^{-p}(q+1) \qquad (10)$$

and $\Psi_{p,q}(x) = 0$, otherwise.

3. CASES OF THE STUDY

3.1. THE DISTILLATIN COLUMN RESPONSE

Distillation columns are one of the most commonly used thermal separation units and have been used for many centuries. Their operation is based on the difference in boiling temperatures of the liquid mixture components, and on recycling counter-current gas-liquid flow. The properly organized temperature distribution up the column results in different mixture compositions at different heights [13].

Temperature values collected at the bottom of the third plate of a distillation column (as shown in figure 1) are illustrated in figures 2 and 3 . All calculations were performed in AutoSignal environment windows software [14].

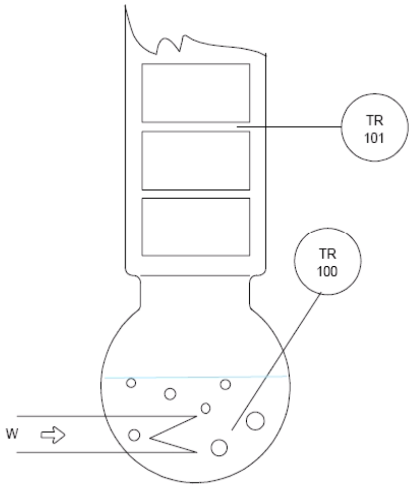

Figure 1. Temperature data in bottom and third plate of distillation column

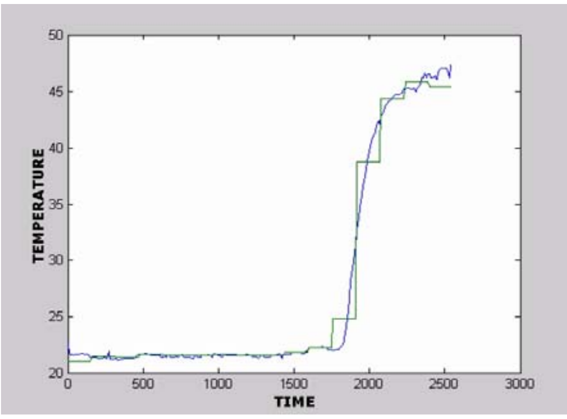

Figure 2. Identification of the third plate model using Wavelet Haar order 3 (16 coefficients)

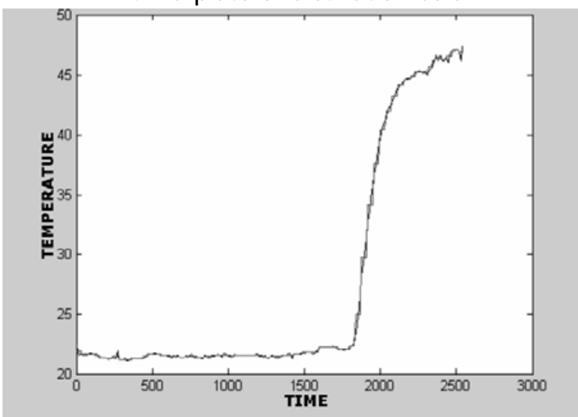

Figure 3. Wavelet Haar order 5 using 64 coefficients

When increasing the number of wavelets coefficients, the obtained signal is similar to the original one but it is less compressed (it takes less memory space). Nevertheless, the third plate response is characteristic of a linear model with dead time. If the dead time is eliminated, the signal can be approached with a linear third order differential equation written as:

$$100T(t) = 75.42\,T(t-40) + 14.81T(t-20) + 1.88\,T(t-30) + 247.5\,P(t) \qquad (11)$$

where $P(t)$ is vapour pression applied in the calderin of the column. Equation 11 was used to obtain the approximation illustrated in figure 4.

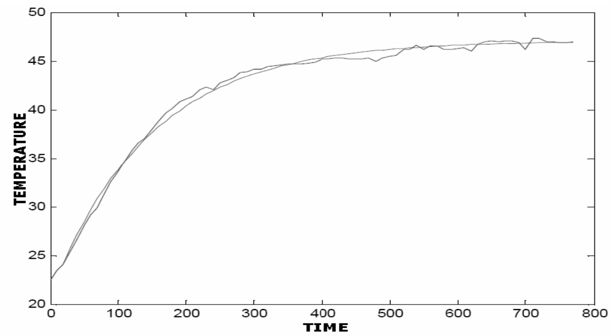

Figure 4. Third order linear model approximation

Not all processes have linear behaviour because the dead time and the saturation are two non linearity's that can be modelled perfectly using wavelets. Temperature in the bottom of the distillation column presents also dead time and saturation as two non linearity's [15]. Figure 5 shows the comparison of the bottom temperature using removal of dead time and approximating to a linear model, and simultaneously, comparing them to a Haar wavelet approach of several compression relations. It must be noticed that if other kind of wavelet is used, the approach is better since that the Haar wavelet reconstructs in a stepped way [16].

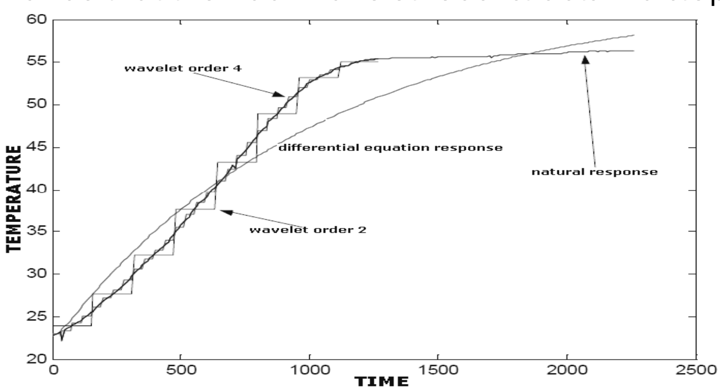

Figure 5. Comparing the natural response curve to the linear model and the wavelets model

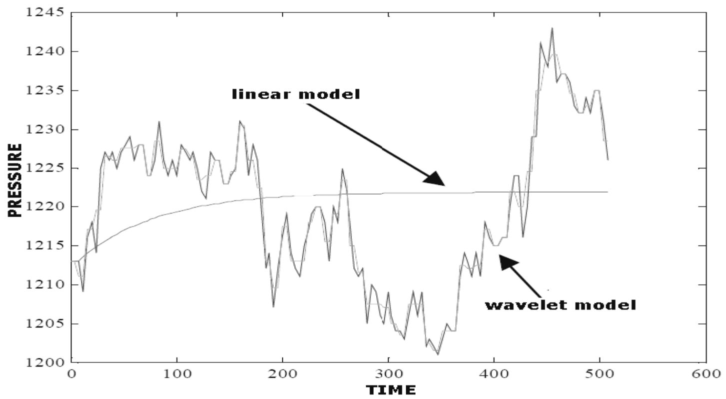

Figure 6. Comparatives responses of the original and the model. The pressure and its wavelet model follow the same tendency.

3.2 THE POLYETHYLENE ELABORATION PROCESS

Finally, figure 6 illustrates experimental values of pressure variation in a polyethylene elaboration process and it comparison to the linear and wavelets model. Dynamic identification with Haar wavelets is better than linear type identification.

4. CONCLUSIONS

In spite of its simplicity, the Haar wavelet had a better identification response than the linear model. Once the system identified, it is possible to eliminate components that represent noise in the system. This way allows better fidelity of the model. Eliminating the dead time does not guarantee a better approach to the linear model since there will be other non linerities that deviate the results. Nevertheless, in the case of on line identification, it is better to use wavelets than linear models for the following reasons:

- Not all processes are linear.
- It will be not possible to carry out step type disturbances during the process operations in all manufacturing plants.
- Wavelets give more precise results.
- It does not matter that the model calculates sixteen wavelet coefficients instead of three in the case of first order approach with dead time. This latter is much more appropriated to carry out simulated control studies. But it works poorly for on line identification.

The results of wavelets identification, even in the simplest way, make in the model predictive control (MPC) type appropriate and advantageous.

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