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DEVELOPMENT OF A MATHEMATICAL MODEL FOR ELASTIC FIELD SIMULATION DURING UNLOADING CONDITION IN SURFACE BURNISHING PROCESSES

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ABSTRACT:
In manufacturing engineering it is imperative to improve the surface quality of the machine parts, which ensures their durability and reliability. To achieve this, the residual stresses which are developed in them during the machining processes are required to be estimated for which one has to know the elastic and plastic stress components. To improve the surface quality and hardness of machine parts, Surface Rolling is an effective cold forming process. The stress fields during this process are constituted of elastic and plastic fields which must be known to evaluate the effectiveness of this process. In this work, a proposed simulation model was implemented to determine and to analyze the elastic stress field during unloading in Surface Rolling process. This work is frame-worked for the analytical determination of elastic field for applying in a residual stress-forecasting module of surface burnishing processes.
KEYWORDS:
Elastic field, Surface burnishing, Residual stresses

1. INTRODUCTION

Surface Rolling is a process in which a smooth hard tool having a planetary rotation is rubbed (using sufficient pressure) on the metal surface. It is a Super-finishing process as shown in Figure-1. Surface Rolling is authentic from burnishing as because in burnishing, the ball or roller does not rotate whereas in Surface rolling the tool has a planetary rotation. By causing plastic flow of metal, the irregular spots are smoothened. To illustrate this point further, all machined surfaces can be described as a series of peaks and valleys of irregular height and spacing where the plastic deformation resulting from Surface Rolling causes displacement of the material in the peaks which cold flows under pressure into the valleys. *Surface Rolling* improves metallurgical properties that being better the surface finish, increased surface hardness, wear-resistance, and fatigue and corrosion resistance.

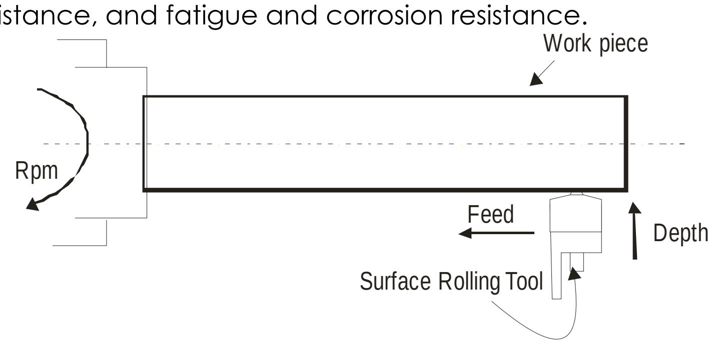


Figure 1: Surface Rolling

Deformation is the change in shape or configuration of a material due to exertion of a sufficient amount of load to the metal or the structural material under consideration. Deformation may occur due to tensile (pulling) forces, compressive (pushing) forces, shear, bending or torsion (twisting). The resulting deformation may eventuate depending upon the type of material, size and geometry of the object, and the forces applied. In Surface Rolling, the work-piece goes under two kinds of deformation, which are Elastic Deformation proceeds the Plastic Deformation. Many research works [1-6] were carried out to calculate the residual stresses in fracture mechanics. But in this paper a mathematical model was developed for surface burnishing process and validated the model with the experimental [7] one.

Elastic Deformation: Elastic deformation is an impermanent and self-reversing alteration in shape, which regains its original shape when the applied force is withdrawn. In other words, elastic deformation is a change in shape of a material at low stress that is recoverable after the stress is removed. During this type of deformation, only stretching of the bonds takes place rather than the sliding between the atoms.

Plastic Deformation: In physics and materials science, plasticity is a property of a material due to which, when a force is implied the material undergoes a non-reversible alteration of shape. The term "Yield" which is used in engineering, in fact, refers to the same property. When a permanent misshape of the metal results due to the applied stress, it is called plastic deformation.

Deformation Process Modeling during Surface Rolling: In Surface Rolling, when the roller is indented into the cylindrical work-piece, the deformation occurring in the vicinity of the roller is divided into four zones as shown in Figure 2. They are:

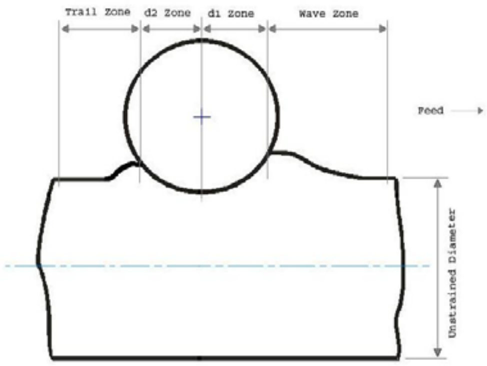


Figure 2: Deformation during Surface Rolling

1. Wave Zone
2. d_1 Zone
3. d_2 Zone
4. Trail Zone

- Trail Zone: It is the portion of the work-piece coinciding with the trailing contact point of the roller. In this zone, a wave is formed in the work-piece. This wave is diminished as soon as the roller is withdrawn, but it continues to move along with the roller in the direction of feed. It can be considered as a wave in the work-piece material, which always has a contact with the trailing contact point.
- d_2 Zone: This zone starts from the trailing contact point and ends at the normal to the work-piece axis which goes through the roller center. The length of this zone is represented by d_2.
- d_1 Zone: This zone starts from end point of the d_2 zone and ends at the leading contact point of the roller. The length of this zone is represented by d_1.
- Wave Zone: The zone starting from the leading contact point, going forth in the direction of feed is called the wave zone. In this zone the indentation of the roller causes some material of the work-piece to be pushed up, resulting into formation of a wave. This wave has the highest wake at the leading contact point and diminishes gradually. Like the wave formed in the trail zone, it also moves along with the roller.

Simplified Model of the Process:

In order to simulate the elastic stress field, it is necessary to develop a simplified model from plastic field as shown in Figure 3. To define the different geometrical parameters of the process a coordinate system must be established. In this coordinate system, the lowest point of the contact arc is defined as the origin. X-axis is defined as parallel to the work-piece axis going through this origin. Y axis is essentially perpendicular to the work-piece axis and goes through the roller center. Other geometrical parameters are defined as follows:

r= Radius of the roller

R= Radius of the Work-piece

d_2= Distance between trailing contact point of the roller and Y-axis.

d_1= Distance between leading contact point of the roller and Y-axis.

L= Distance between Y-axis and the front wave end point where unstrained radius of the work-piece commences.

h_f= Height of the leading contact point from X-axis

h_d= Height of the unstrained surface boundary of the work-piece from X-axis

h_b= Height of the leading contact point from the unstrained surface boundary of the work-piece

Δ=Height of the hardened surface boundary of the work-piece from X-axis

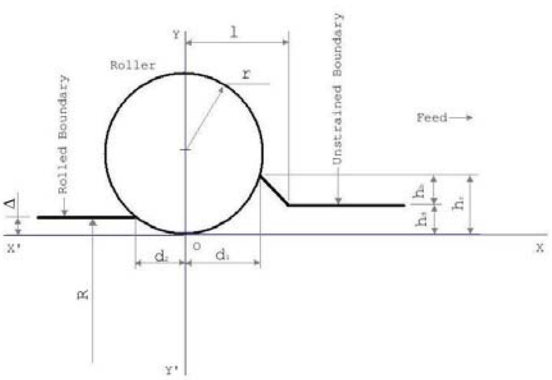


Figure 3: Geometrical Model of Surface Rolling

For simplicity, in this model the trail zone wave is assumed as a straight line coinciding with the rolled boundary of the work-piece. Again, the front wave is approximated to an inclined straight line connecting the leading contact point and the end point of the wave. These assumptions ease the development of the simulation model to a great extent with sufficient accuracy for practical application.

2. SIMULATION MODEL

During hardening process by surface rolling under the line of contact between the instrument and the work-piece there forms the zone of plastic deformation. Elements of the surface layer undergo deformation while passing through this zone. During this period it undergoes several cycles of loading and unloading. As a result the effects of deformations are added each time. Final condition of particle will be permanent while it goes out of the deformation. So the final components of residual stresses will depend upon the conditions in the border of the deformation zone. Above way of determination of residual stress can be realized by using theorem of unloading. According to this theory components of the residual stress of a particle after unloading may be written as

$$\sigma_{ij}^{R}=\sigma_{ij}^{L}+\sigma_{ij}^{UL}$$

σ_{ij}^{R} = Stress components during loading, σ_{ij}^{L} = Stress components during unloading

σ_{ij}^{UL} = Residual stress components

So for determination of residual stress component it is required to determine the elastic components in the deformation zone. For explaining the character of formation_of the residual stress field the stress field is simulated. According to *Theorem of Unloading* while calculating the residual stress, the force causing elastic field during unloading is equal to the force that causes plastic deformation with the direction opposite to it. For such type of problem it is required to calculate the elastic stress field in the lateral sectional plane of the work-piece with the distributed force along the contact arc of the instrument and the work piece. Stress for this type of distributed force can be calculated with sufficient accuracy for the practical problem by using *principle of Superposition* of stresses, for application of each of the force or parts of the force. But such type of problem is difficult to solve due to the curvature of the contact line.

Calculation of Contact Thickness

When a sphere is intended into a cylinder without any deformation of the sphere, the contact is called circular contact and it is elliptical in shape (Figure 4).

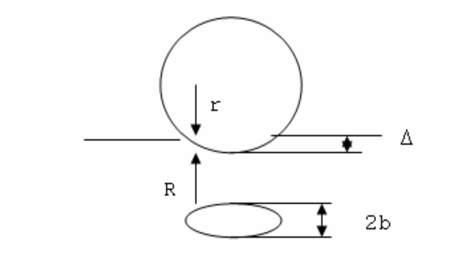


Figure 4: Circular contact, Parameter of the plan view of contact ellipse

Thickness of the contact area is calculated by using formula for indenting a sphere into the cylinder.

$$b=r\sqrt{1-\frac{r^2+(r-\Delta+R)^2-R^2}{2r\ (r-\Delta+R)}}$$

b= half thickness of the contact, mm
r= local value of the radius of the sphere, mm
R= local value of the radius of the deformed cylinder, mm
Δ= indentation depth, mm

For the contact zone of the surface rolling above parameters vary with the location of the contact point along the length. These values can be calculated by using the figure 5.

For the front side of the contact following formula are used.

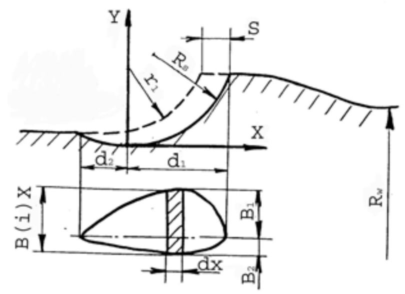


Figure 5: Calculation of Contact Thickness

$$r_1 = \sqrt{R_S^2 - x^2}$$

$$R_1 = R_W - h_d + h_x + h_{cd}$$

$\Delta_1 = h_{cd}$

R_S - Radius of the ball, mm

R_W - Radius of the work piece, mm

x - Co-ordinate of the point of calculation, mm

h_d - Real value of depth of indentation, mm

h_{cd} - Current value of indentation, mm

h_x - Current height of tool profile, mm

For the back side, $r_2 = r_1$, $R_2 = R_W - h_d + h_x + h_{res}$ $\Delta_2 = h_{res}$, where, h_{res} = Height of the restored profile, mm.

For $x \geq d_1 - S$, $h_d = h_r - h_x$ where, $h_r = R_S - \sqrt{R_S^2 - d_1^2}$

For $d_1 - S > x \geq 0$, $h_d = h_{xres} - h_x$ where, h_{xres} = Height of the resorted profile.

For $x < 0$, $h_d = h_z - h_x$ where, $h_z = \sqrt{R_S^2 - d_2^2}$

h_x and h_{xres} was calculated by the formula, $h_x = R_S - \sqrt{R_S^2 - x^2}$, $h_{xres} = R_{res} - \sqrt{R_{res}^2 - x^2} + h_z$ h_{res} was calculated.

$$h_{xres} = 0.12 \times h_d$$

Radius of the restored profile was calculated considering the wave direction,

$$R_{res} = \frac{(d_1 - S)^2 + (h_r - h_z)^2}{2(h_r - h_z)}$$

Area of the contact can be calculated by the formula $A = r\sum_{i=1}^{n} dx_i \frac{b_i(x) + b_{i-1}(x)}{2}$

Method of Calculation

It is required to calculate the components of elastic field in a point m(x, y) as shown in Figure-6. Let the thickness perpendicular to xy plane is equal to 1 and force P is equally distributed along the thickness. In the first step, let the border ABCD is far from the point O. This assumption gives opportunity to consider simple radial distribution of elastic stresses. Then components of stresses can be expressed by the following formula

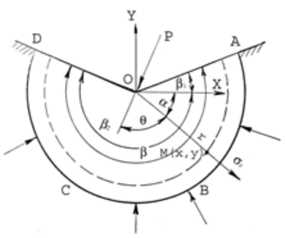


Figure 6: Method of Calculation

$$\sigma_r = -\frac{A_p P \cos\theta}{r}, \ \sigma_\theta = 0, \tau_{r\theta} = 0 \ \ldots \qquad (1)$$

Constant A in the equation (1) is calculated considering the equilibrium condition of point 0.

$$-\int_{\beta_1}^{\beta_2} \frac{A_p P \cos^2\theta}{r} r d\theta = -P \Rightarrow \int_{\beta_1}^{\beta_2} A_p \cos^2\theta \, d\theta = 1 \Rightarrow A_p = \frac{2}{\beta + \frac{1}{2}\sin\beta} \qquad (2)$$

Putting the value in the equation (1) we get

$$\left.\begin{aligned} &\sigma_r = -\frac{2P\cos\theta}{r(\beta + \frac{1}{2}\sin\beta)}, \ \sigma_x = \sigma_r \cos^2\alpha = \frac{2P\cos\theta\cos^2\alpha}{r(\beta + \frac{1}{2}\sin\beta)} \\ &\sigma_x = \sigma_r \sin^2\alpha = \frac{2P\cos\theta\sin^2\alpha}{r(\beta + \frac{1}{2}\sin\beta)}, \ \sigma_z = \mu\ (\sigma_x + \sigma_y) \\ &\tau_{xy} = \sigma_r \sin a \cos a = \frac{2P\ \cos\theta\ \sin a \cos\ \alpha}{r(\beta + \frac{1}{2}\sin\beta)} \end{aligned}\right] \qquad (3)$$

Above method can be used for approximate calculation of the elastic unloading stress field by simplifying the distribution of the unloading force on the arc of contact. For this purpose the area of the contact were divided into r element portion. Total force as shown in Figure 7 working on this portion can be calculated by the following formula:

$$\Delta P_n = \sigma_n dA \cdot N_i ; \qquad \Delta P_\tau = \Delta P_n f \cdot N_i$$

Here,

$\Delta P_n =$ Normal force of the portion ; $\Delta P_\tau =$ Friction force of the portion

$\sigma_n =$ Normal stress; dA = Area of the elemental portion; f= friction factor

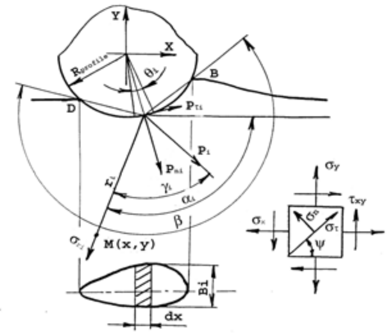


Figure 7: Calculation of Force Components

Then forces equally distributed to the unit thickness can be calculated by the formulae,

$$P_{ni} = \sigma_n r \ dx \ b_i(x)$$

$P_{Ti} = P_{ni} f$, Therefore, $P_i = \sqrt{P_{ni}^2 + P_{Ti}^2}$ $\quad \gamma = \tan^{-1}\frac{P_{Ti}}{P_{ni}}$

P_{ni}, P_{Ti}, P_i - normal, tangential and resultant forces respectively.

For using equation (3) for calculating the components of elastic stress, it is required to replace the arc of contact by two lines connecting the point of application of the force P_i and the point B and D.

Then stress components as per equations (3) will be,

$$\sigma_{xi} = \frac{2P_i \cos\theta_i \cos^2 \alpha_i}{r_i(\beta + \frac{1}{2}\sin\beta_i)}$$

Meaning of r_i, α_i, θ_i and β is shown in fig. 7. Total stress can be calculated by using principle of superposition i.e. by adding the components of stresses from elemental forces.

$$\sigma_x = \sum_{i=1}^{n} \sigma_{xi} \quad \sigma_y = \sum_{i=1}^{n} \sigma_{yi} \quad \sigma_z = \mu \ (\sigma_x + \sigma_y) \quad \tau_{xy} = \sum_{i=1}^{n} \tau_{xyi}$$

Calculation of Normal Stress along Contact Arc: Normal stresses are calculated form the calculated maximum shear stress of the hardened material for this purpose. Following formula are used (4).

$$\left.\begin{array}{l} k_o = \frac{\sigma_o}{2}; \; k_{max} = \frac{\sigma_{max}}{2}; \quad k_{max} = k_o + k_{stren} \ \varepsilon_{max}^n \\ \varepsilon_{max} = \frac{A_m d_1}{R_{profile}} \end{array}\right] \tag{4}$$

Here,

$k_o =$ Ultimate Shear Strength, MPa, $\sigma_o =$ Ultimate Tensile Strength,MPa

$k_{max} =$ Maximum Shear Stress, MPa, $\sigma_{max} =$ Maximum Tensile Stress, MPa

$\varepsilon_{max} =$ Maximum Strain, $n =$ Hardening Index, $A_m =$ Constant for Calculating ε_{max}

$R_{profile} =$ Roller Profile Radius, mm, $k_{stren} =$ Strength Coefficient, MPa

Where x<=0, the material particles there has gone through more number of cycles of loading and unloading. Hence, it may be assumed that, normal stress in those particles has reached maximum value. Therefore, this phenomenon is taken into consideration by using the following formula (4). $k_n = k_0$, Where x>0, the material particles there has gone through less number of cycles of loading and unloading. Hence, the local normal stress developed there has a lower magnitude than the left side particles of the material (where X<=0). This phenomenon is taken into consideration by using the following formulae (2).

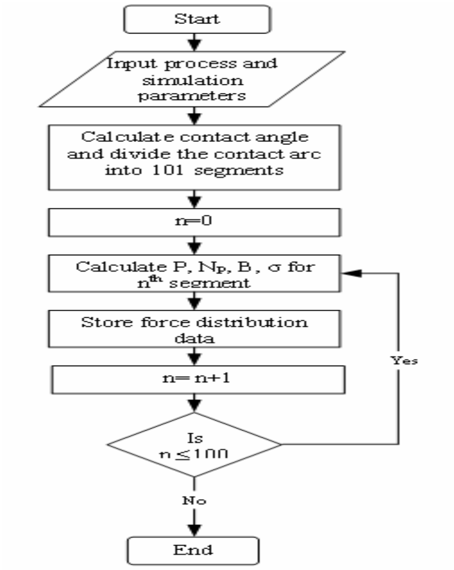
$$k_n = k_0 + [k_{max} - k_0] e^{-c(x/l)^2}; \; c = \frac{2\pi\sqrt{ab}}{h_g}, \; h_g = 1.47 \times d_1^{0.5}, \; b = \frac{2A}{\pi(d_1 + d_2)}, \; a = \frac{d_1 + d_2}{2}$$

Here,
k_n = Local Shear Stress, MPa, c = Constant, a = Half Width of the Contact Ellipse, mm,
b = Half Thickness of the Contact Ellipse, mm, A = Total Area of Contact

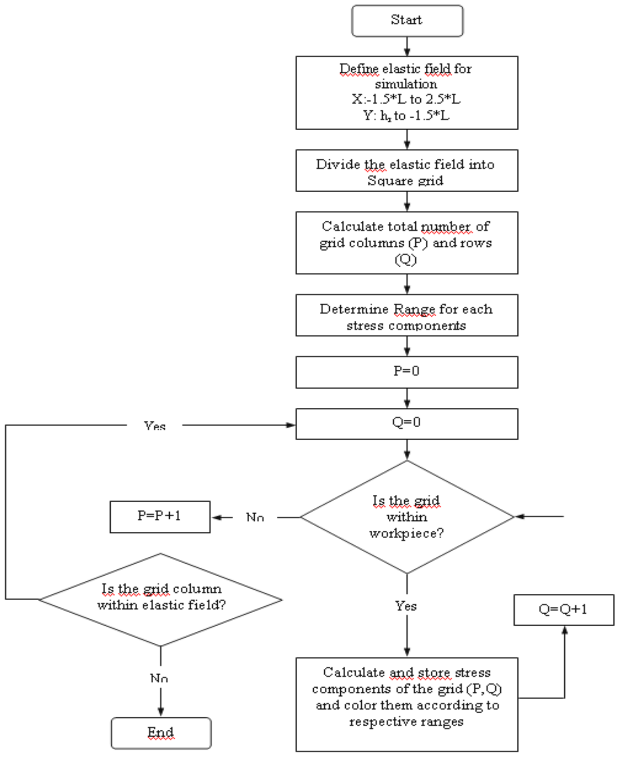
Simulation of Elastic Field: For simulation a square net with dx and dy with a number of dimensions which are equal to 50 microns, 25 microns, 5 microns and 1 micron were taken. Stresses are calculated. Stresses are grouped with a small $d\sigma$ and a square portion around. The points are colored with different intensities. Tests were done to find out the location of the point. If the point lies within the material, it is calculated. Otherwise calculation for that point is aborted.

Simulation Flowchart:

The simulation process was done according to the following flowcharts.



Flowchart 1: Force Distribution Calculation



Flowchart 2: Stress Components Calculation

Description of the Software: Computer software was developed to simulate the elastic field. This program was developed in visual basic 6.0. Several types of controls and data containers, which were grouped in five different forms, were required for designing a comprehensive interface of the application. These forms let the user to input data, view the simulation results and save the results in storage devices. Images of two important forms are shown in figure 8 and 9.

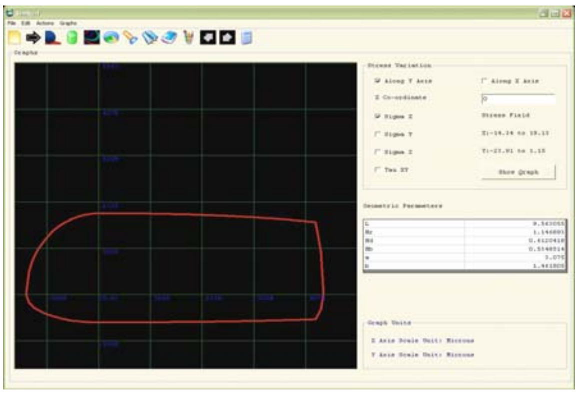


Figure 8: Root interface of the software

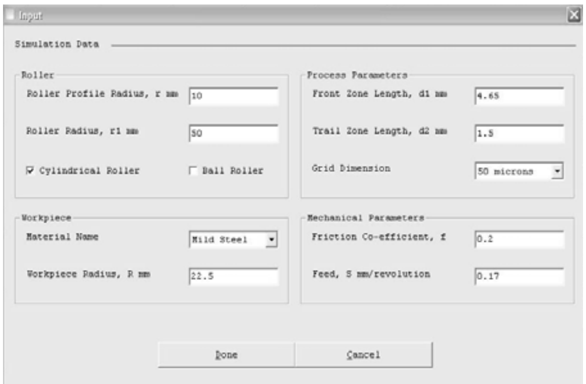


Figure 9: Input form of the software

Simulated Data & Analysis: Stress Components Isobars

In the simulated elastic field, isobar lines represent those grids, which have equal stress acting upon them with different colors. As there are four stress components ($\sigma_x, \sigma_y, \sigma_z, T_{xy}$), four isobar images are obtained from the elastic field. It may be mentioned worthy that, the stress field is continuous and the black region has continuous value of stress. But to find out the pattern of the stress distribution, some values between maximum and minimum stress was taken and only those points with ±5% of those definite values of stresses were painted with different colors. The first set of isobars given below, figure 10-12 is simulated from the following data, presented in Table 1.

Table 1: Input Data, Set 1

Parameter	Value
d_1, mm	4.65
d_2, mm	1.5
Roller Profile Radius, mm	10
Roller Radius, mm	50
Workpiece Diameter, mm	45
Feed, mm/revolution	0.17
Friction Factor	0.2

Work-piece material was Mild Steel and the roller was cylindrical.

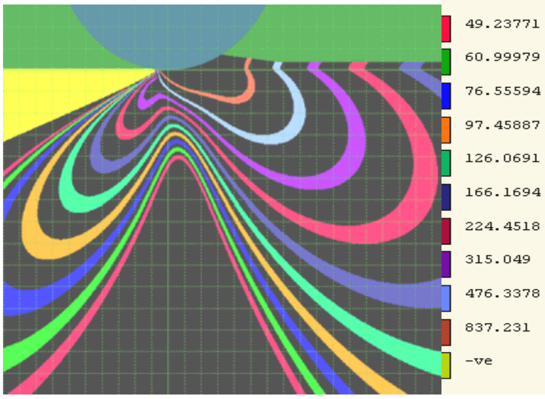


Figure 10: Isobar σ_x

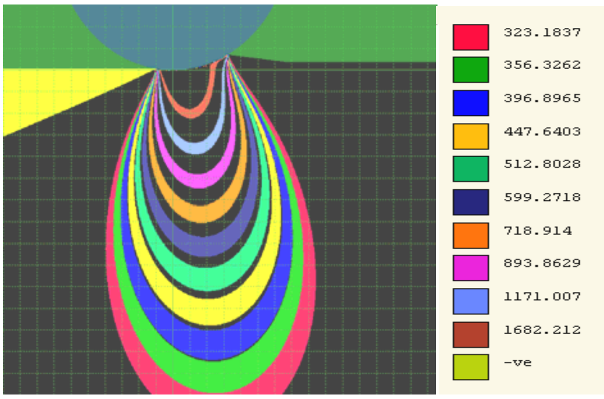


Figure 11: Isobar σ_y

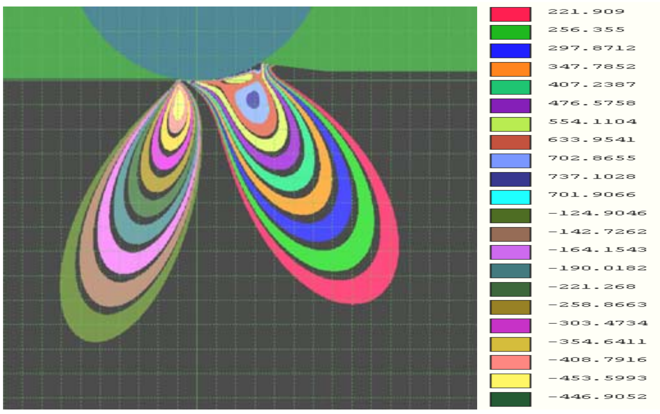


Figure 12: Isobar τ_{xy}

3. CORRECTION FACTOR

For calculation of contact stress set of formula (2) were used. These formulae are deduced assuming plain stress condition in the deformation zone. But in fact there will be a narrow zone along the contact line where three-dimensional deformation will take place. To consider this fact a correction factor is used. To find out the correction factor radial forces are calculated and compared with the experimental radial forces. The correction factor was determined by dividing the experimental value by the calculated value. Variation of the correction factor with d_1 is due to linearization of the relationship of accumulated strain with d_1. Experiments with different materials and size of balls shows that average correction factor for optimum rolling condition is 3. Sample of calculation for mild steel with ball diameter, 5mm are shown in the table 2 and figure 13. As this correction factor will affect accuracy of calculation of the stress field detail experimental work is going on to study the effect of different technological parameters on this correction factor.

Table 2: Experimental and simulated data with correction factor for Ball Roller with 5mm Diameter

d_1 (mm)	Simulated P_y (N)	Experimental P_y (N)	Correction factor	Average correction factor	Corrected simulated force P_y (N)
0.325	100.352	250	2.491231	3.01495489	302.556753
0.45	174.472	500	2.865789		526.025209
0.625	318.3709	1000	3.140991		959.873901
0.8	486.6216	1600	3.287976		1467.14217
0.85	547.3141	1800	3.288788		1650.12732

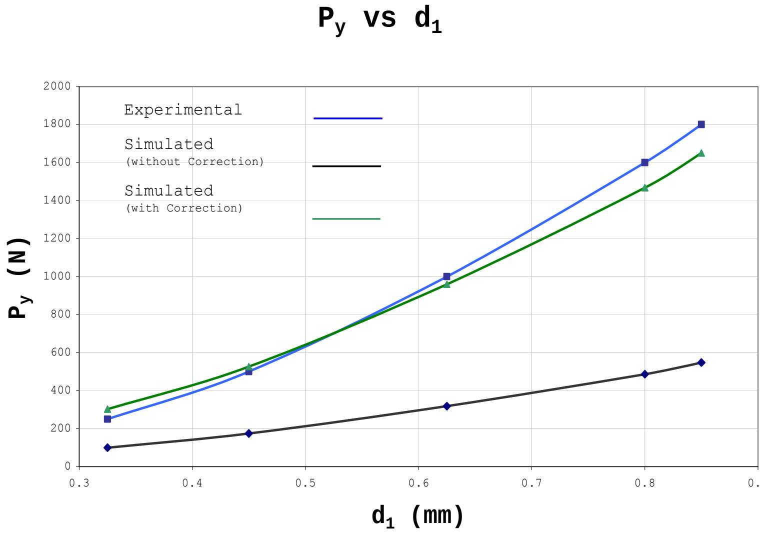


Figure 13: Correction Factor

Simulated Contact Area & Stress Variations

The subsequent contact area was simulated (Figure 14) using following data, (Table 3). X and y axis units are micron. The curves (Figure 15-16) show the change in normal stress along the axis's and the contact arc for the material mild steel (d_1=0.325 mm, d_2=0.2 mm).

Table 3: Input Data, Set 2

Parameter	Value
d_1, mm	0.95
d_2, mm	0.40
Roller Profile Radius, mm	10
Roller Radius, mm	50
Work-piece Diameter, mm	45
Feed, mm/revolution	0.15
Friction Factor	0.2

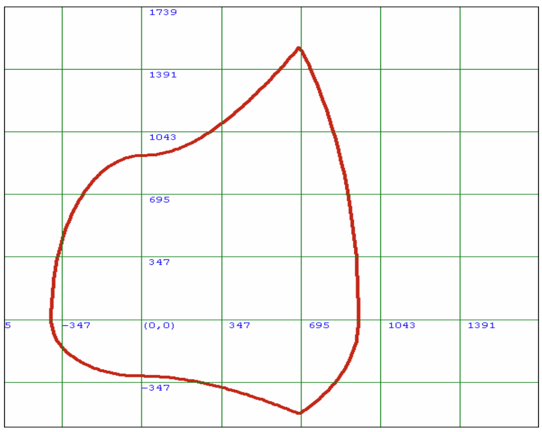


Figure14: Contact Area, Cylindrical Roller

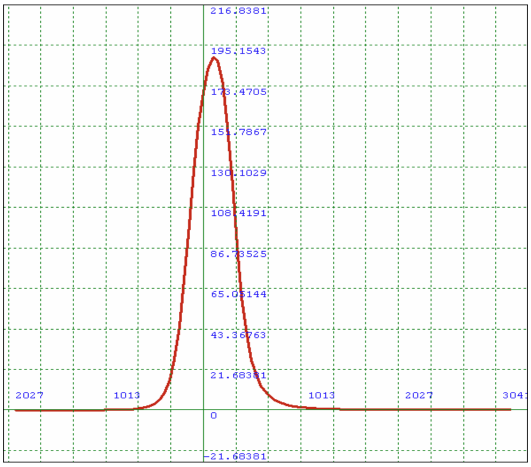


Figure 15: σ_y along x axis

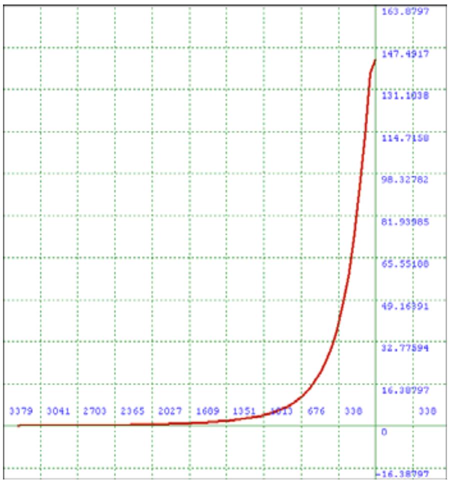


Figure 16: σ_x along y axis

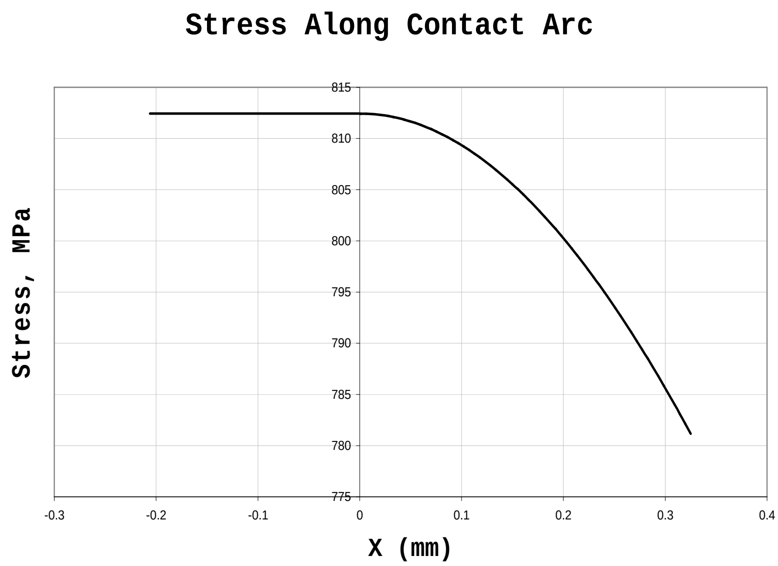


Figure 17: Stress along Contact Arc

4. CONCLUSION

A surface rolling simulation software applicable for different materials and rolling tools was developed in this project. Using that software elastic force and stress data was generated and analyzed for different sets of parameters with a view to conduct a comprehensive study about experimental and simulated elastic field.

It has been observed that stress variations along the cross-section of the work-piece going through the contact points (front and trail) represent the stress variation in the whole elastic field. It was also observed that d_1 is the governing parameter in this process and others parameters could be expressed in terms of d_1. This kind of simulation testing is more economic both in the context of time and cost.

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