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GENERALIZATION OF THE SINGLE RULE REASONING METHOD SURE-LS FOR THE CASE OF ARBITRARY POLYGONAL SHAPED FUZZY SETS

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ABSTRACT:
Fuzzy rule interpolation (FRI) based methods present advantageous solutions for cases when the rule base does not ensure a full coverage of the input space. Several of them apply the two-step approach using in their second step a single rule reasoning (SRR) method for the determination of the conclusion from the observation and the interpolated rule.
This paper presents a generalized version of the SRR method SURE-LS that was originally developed for the triangle shaped fuzzy sets. The new version extends the application area of the method to the arbitrary polygonal shaped valid membership functions. The dissimilarity of the antecedent and observation linguistic terms is measured α-cut wise and the conservation of the consequent partition's characteristic shape type is ensured by the method of least squares.
KEYWORDS:
single rule reasoning, SURE-LS, LESFRI, fuzzy rule interpolation

1. INTRODUCTION

Single rule reasoning as part of fuzzy rule interpolation based inference has been intensively studied since the early 1990s. The research in this area was motivated mainly by the need for complexity reduction in case of fuzzy systems with multidimensional input spaces [8]. Besides, the results were also applicable in systems where the knowledge base is incomplete (sparse) owing to the lack of information. Figure 1. illustrates the antecedent space of a sparse rule base. Each rule is represented by a brick defined by the supports of the antecedent sets.

The mainstream of the proposed solutions reduced the complexity of the fuzzy systems by creating a rule base containing only the relevant rules. However, this approach implied the need for development of new reasoning methods, because the classical compositional inference techniques (e.g. Mamdani [9], Takagi-Sugeno [12], etc.) were not able to produce a proper conclusion when none of the antecedents of the known rules were "hit" by the system input (observation).

The fuzzy inference methods developed for sparse rule bases use two or more rules in the neighbourhood of the observation and apply interpolation for the calculation of the conclusion (system output). They can be categorized either as one-step or as two-step methods.

The techniques that belong to the first category obtain the conclusion directly from the observation and the rules taken into consideration. Here belongs among others the linear fuzzy rule interpolation (KH method) [8] proposed by Kóczy and Hirota, the MACI [13] (Tikk and Baranyi) that solved the problem of abnormal conclusion using a co-ordinate transformation, and the FIVE [7] (Kovács and Kóczy) that applied the concept of vague environment for rule interpolation.

The members of the second category determine first a new rule in the position of the observation (first step) and next fire the new rule (second step) applying a single rule reasoning method. This approach follows the concepts of generalized methodology of fuzzy rule interpolation (GM) [2] introduced by Baranyi, Kóczy and Gedeon. For example here belong the techniques suggested by Branyi et al in [1][2][3], the alpha-cut based LESFRI [5] (Johanyák and Kovács), the polar-cut based FRIPOC [4] (Johanyák and Kovács), and the vague environment based VEIN [6] (Johanyák and Kovács). The members of both groups can be applied to fuzzy logic control [10].

This paper focuses on the second step of LESFRI, where the task of single rule reasoning is solved by the SURE-LS [5] method. Although its key idea indicated a wider application area, the original version of SURE-LS covered only the case of triangle shaped fuzzy sets. Recognizing this deficiency we elaborated the generalization of the method for the case of arbitrary polygonal shaped fuzzy sets.

The rest of this paper is organized as follows. Section 2 recalls the basic ideas of single rule reasoning and SURE-LS. Section 3 introduces the proposed generalized version. The conclusions are presented in section 4.

2. SINGLE RULE REASONING BASED ON THE METHOD OF LEAST SQUARES

In the general case a fuzzy system has some input and some output state variables, i.e. it is a MIMO (Multiple Input Multiple Output) system. However, conform to the literature we will only consider MISO (Multiple Input Single Output) systems, because MIMO systems are always decomposable to and recomposable from the respective MISO ones. This admittance simplifies the creation, analysis and discussion of fuzzy systems. SISO (Single Input Single Output) systems are treated as special cases of the MISO ones.

In case of single rule reasoning (also called revision) methods one has one or more fuzzy sets describing the input (observation), and a rule whose antecedent linguistic terms are situated in the same position as the corresponding observation sets. The position of a fuzzy set is defined by its reference point (RP). RPs are widely used in fuzzy reasoning in order to facilitate the set-distance evaluation, and ordering of the linguistic terms. An RP is an element of a set considered as the most representative one. Usual options are the centre of the core, centre of the support, centre of gravity, etc. SURE-LS uses the first one, because it simplifies the division of the set shape into two flanks, which is necessary in course of the calculations.

Each revision method implements a generalized modus ponens by stating that the more similar the observation to the rule antecedent is the more similar should be the final conclusion to the consequent of that rule. Although the methods start from the same principle they diverge by applying different similarity/distance measures. There are α-cut (ST [15], FVL [11], SURE-LS [5]), polar-cut (SURE-p [4]) or vague environment (REVE [6]) based similarity evaluation solutions. The advantage of SURE-LS lies in its low computational complexity, easy applicability and implementability as well as its ability to preserve the characteristic shape type of the consequent partition.

SURE-LS (Single rUle REasoning based on the method of Least Squares) was developed originally especially for the case when all linguistic terms of a consequent partition belong to the same shape type (e.g. triangle) and all characteristic (break) points are situated at same α-levels. It also means that the height of all sets is the same.

In such circumstances it seems to be a natural demand that the conclusion should also adhere to this regularity. Thus one does not

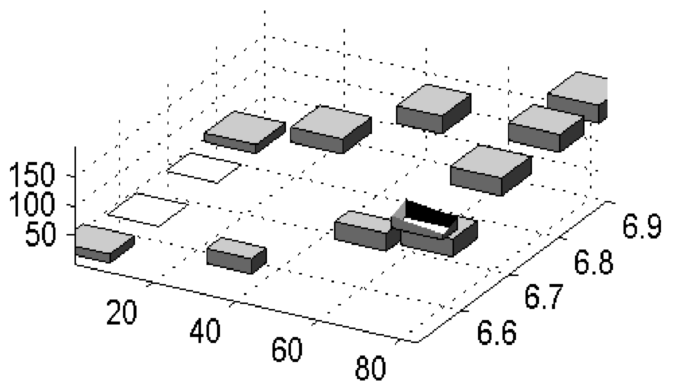

FIGURE 1. ANTECEDENT SPACE OF A SPARSE RULE BASE WITH THREE INPUT DIMENSIONS

seek an arbitrary shaped conclusion but a special form with the characteristic points having predetermined ordinate values. Therefore only the abscissas of the characteristic points have to be calculated.

SURE-LS uses a set of α-levels compiled together by taking into consideration the break-point levels of all antecedent dimensions and the current consequent partition. The calculations are done separately for the left and right flanks. The method calculates on each side and for each level the weighted average of the distances between the α-cuts' endpoints of the rule antecedent and observation sets. The weighting makes possible to take into consideration the input state variables with different influences.

The basic idea of the method is the conservation of the weighted average differences measured on the antecedent side. Conservation means that the raw characteristic point set of the conclusion is determined by modifying the rule consequent with these values. The conclusion with the desired shape type is calculated next applying the method of Least Squares

Due to the last step of the calculation the resulting set is always a valid one and it conserves the piece-wise linearity. Furthermore if the rule antecedent fits the observation perfectly the conclusion will be identical with the consequent of the rule. The method has two shortcomings. Firstly due to the α-cut based approach the heights of all sets have to be the same. However, usually this condition is fulfilled. Secondly the original version of SURE-LS covered only the popular case of triangle shaped fuzzy sets.

3. GENERALIZED VERSION OF SURE-LS

This section presents the generalized version of SURE-LS capable to handle any polygonal shaped valid fuzzy set. The method separates the set shapes at their reference points into two flanks. Thus the calculations can be done independently for the left and right flanks. First one compiles together two sets of α-levels (Λ_L and Λ_R) corresponding to the break-points of the two flanks of the fuzzy sets. Here both the antecedent and consequent linguistic terms are taken into consideration

$$\Lambda_L = \left(\bigcup_{i=1}^{N} \Lambda_{Li}^{a} \right) \bigcup \Lambda_L^c \ , \tag{1}$$

$$\Lambda_R = \left(\bigcup_{i=1}^{N} \Lambda_{Ri}^{a} \right) \bigcup \Lambda_R^c \ , \tag{2}$$

where *N* is the number of antecedent dimensions, *a* and *c* denote the antecedent respective consequent parts, *L* and *R* denote the left respective right sides.

In each antecedent dimension one determines the distances ($d_{i\alpha}^{aL}$ and $d_{i\alpha}^{aR}$) between the endpoints of the corresponding α-cuts of the rule antecedent (A_i) and observation (A_i^*) sets (see figure 2.)

$$d_{i\alpha}^{aL} = inf\left\{ \left[A_i \right]_\alpha \right\} - inf\left\{ \left[A_i^* \right]_\alpha \right\} \ , \tag{3}$$

$$d_{i\alpha}^{aR} = sup\left\{ \left[A_i \right]_\alpha \right\} - sup\left\{ \left[A_i^* \right]_\alpha \right\} \ . \tag{4}$$

The average distances are defined by

$$ad_\alpha^{aL} = \frac{\sum_{i=1}^{N} d_{i\alpha}^{aL}}{N} \tag{5}$$

$$ad_\alpha^{aR} = \frac{\sum_{i=1}^{N} d_{i\alpha}^{aR}}{N} \tag{6}$$

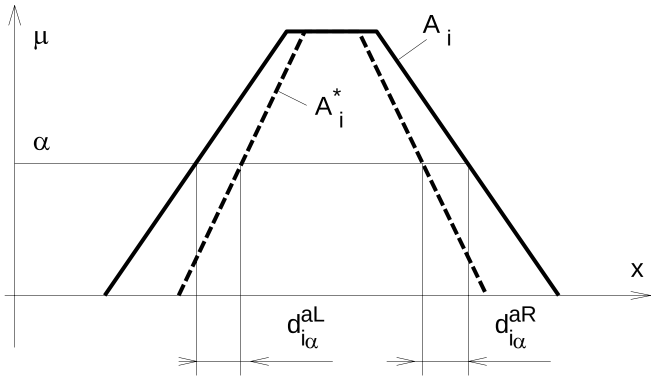

FIGURE 2. DISTANCES ($d_{i\alpha}^{aL}$ AND $d_{i\alpha}^{aR}$) BETWEEN THE ENDPOINTS OF THE CORRESPONDING α-CUTS OF THE RULE ANTECEDENT (A_i) AND OBSERVATION (A_i^*) SETS IN THE ITH ANTECEDENT DIMENSION

The basic idea of the method is the principle of conservation of the average differences measured on the antecedent side. Therefore the corresponding α-cut endpoints on the consequent side will be modified by $d_{i\alpha}^{aL}$ and $d_{i\alpha}^{aR}$

$$d_\alpha^{cL} = inf\left\{[B\ ]_\alpha\right\} - inf\left\{[B^*]_\alpha\right\} = ad_\alpha^{aL} \quad , \tag{7}$$

$$d_\alpha^{cR} = sup\left\{[B\ ]_\alpha\right\} - sup\left\{[B^*]_\alpha\right\} = ad_\alpha^{aR} \quad , \tag{8}$$

where *B* is the rule consequent and *B** is the conclusion. The raw characteristic point set of the conclusion is determined by

$$inf\left\{[B^*]_\alpha\right\} = Min\left\{ inf\left\{[B]_\alpha\right\} - ad_\alpha^{aL}, RP(B^*)\right\} \quad , \tag{9}$$

$$sup\left\{[B^*]_\alpha\right\} = Max\left\{ sup\left\{[B]_\alpha\right\} - ad_\alpha^{aR}, RP(B^*)\right\} \quad , \tag{10}$$

where $RP(B^*)$ is the reference point of the conclusion. Thus the raw characteristic points for the two flanks are

$$P_L = \left\{(x,\mu) \mid x = inf\left\{[B^*]_\alpha\right\}, \mu = \alpha, \alpha \in \Lambda_L\right\} , \tag{11}$$

$$P_R = \left\{(x,\mu) \mid x = sup\left\{[B^*]_\alpha\right\}, \mu = \alpha, \alpha \in \Lambda_R\right\} , \tag{12}$$

and the joint point set is

$$P = P_L \bigcup P_R \ . \tag{13}$$

Further on for the sake of simplicity we will denote the reference point of the conclusion (*B**) by *RP*. The characteristic break-point level set of the consequent partition for the left flanks is

$$\Lambda_L^c = \left\{\alpha_{l_0}, \alpha_{l_1}, \ldots, \alpha_{l_{m_l}}\right\}, \tag{14}$$

where α_{l_0} is the ordinate of *RP*, $\alpha_{l_{m_l}} = 0$ and $\alpha_{l_i} > \alpha_{l_{i+1}}$ $(0 \le i < m_l)$. The characteristic break-point level set of the consequent partition for the right flanks is

$$\Lambda_R^c = \left\{ \alpha_{r_0}, \alpha_{r_1}, \ldots, \alpha_{r_{m_r}} \right\}, \tag{15}$$

where α_{r_0} is the ordinate of RP, $\alpha_{r_{m_r}} = 0$ and $\alpha_{r_i} > \alpha_{r_{i+1}}$ $(0 \le i < m_r)$.

The next task is to find the abscissas $x_{l_0}, x_{l_1}, \ldots, x_{l_{m_l}}$ and $x_{r_0}, x_{r_1}, \ldots, x_{r_{m_r}}$ of the break-points of the polygonal shaped membership function that approximates the given set of points. Owing to the symmetry of the problem it is enough to deal with the abscissas of the left flank, therefore only that is going to be presented below.

One can distinguish basically two polygon shape types (see figures 3. and 4.):

1. The set has only one point with the maximal membership value (peek point, vertex), i.e. the reference point is the only one point, where the ordinate value is $\alpha_{l_0} = \alpha_{r_0}$.
2. The top of the shape is a plateau (edge), i.e. there are $x_l, x_r \left(x_l < RP < x_r, RP = \dfrac{x_l + x_r}{2}\right)$, such that (x_l, α_{l_0}) and (x_r, α_{r_0}) are vertices of the polygon.

Be:

$$P_{l_1} = \{(x_j, \mu_j) | x_j \le RP \, and \, \alpha_{l_0} > \mu_j \ge \alpha_{l_1}\}, \tag{16}$$

$$P_{l_i} = \{(x_j, \mu_j) | x_j \le RP \, and \, \alpha_{l_{i-1}} \ge \mu_j \ge \alpha_{l_i}\} \; (2 \le i \le m_l), \tag{17}$$

where n_{l_i} is the cardinality of P_{l_i}, i.e. the number of elements – points – in P_{l_i} - $(1 \le i \le m_l)$.

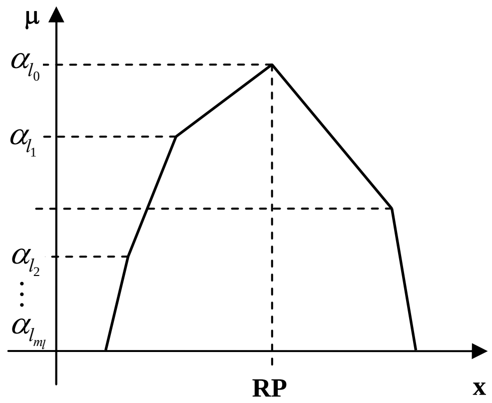

FIGURE 3. SET SHAPE WITH ONE POINT HAVING THE MAXIMAL MEMBERSHIP VALUE (VERTEX TYPE TOP)

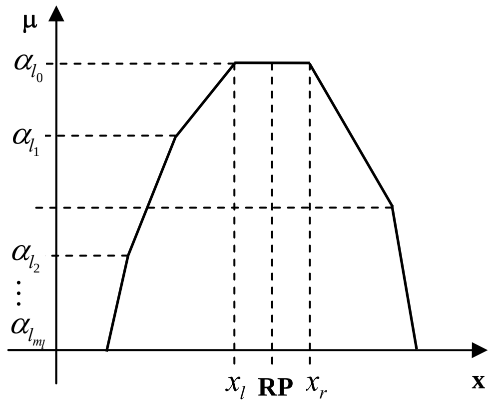

FIGURE 4. SET SHAPE WITH MANY POINTS HAVING THE MAXIMAL MEMBERSHIP VALUE (PLATEAU TYPE TOP)

Let us consider the first case, when the top of the polygonal shaped fuzzy set is a vertex of the polygon. The proposed algorithm is the following:

1. In order to obtain a first approximation of the equation of the first edge – the edge between the break-points α_{l_0} and α_{l_1} – one starts from
$$x = m_1\mu + b_1. \tag{18}$$
One endpoint of the edge is (RP, α_{l_0}), therefore
$$x = m_1(\mu - \alpha_{l_0}) + RP. \tag{19}$$
Applying now the method of least squares
$$\sum_{(x_j,\mu_j)\in P_{l_1}} (m_1(\mu_j - \alpha_{l_0}) + RP - x_j)^2 \to \min, \tag{20}$$
$$\frac{\partial}{\partial m_1} = 2\sum_{(x_j,\mu_j)\in P_{l_1}} (m_1(\mu_j - \alpha_{l_0}) + RP - x_j)(\mu_j - \alpha_{l_0}) = 0, \tag{21}$$
$$m_1 = \frac{\sum_{(x_j,\mu_j)\in P_{l_1}} (x_j - RP)(\mu_j - \alpha_{l_0})}{\sum_{(x_j,\mu_j)\in P_{l_1}} (\mu_j - \alpha_{l_0})^2}. \tag{22}$$
2. In order to obtain a first approximation of the equation of the edge i $(2 \le i \le m_l)$ – the edge between break-points $\alpha_{l_{i-1}}$ and α_{l_i} – one starts from
$$x = m_i\mu + b_i. \tag{23}$$
Applying again the method of least squares
$$\sum_{(x_j,\mu_j)\in P_{l_i}} (m_i\mu_j + b_i - x_j)^2 \to \min, \tag{24}$$
$$\frac{\partial}{\partial m_i} = 2\sum_{(x_j,\mu_j)\in P_{l_i}} (m_i\mu_j + b_i - x_j)\mu_j = 0, \tag{25}$$
$$\frac{\partial}{\partial b_i} = 2\sum_{(x_j,\mu_j)\in P_{l_i}} (m_i\mu_j + b_i - x_j) = 0. \tag{26}$$
Thus one gets the following equation system
$$m_i \sum_{(x_j,\mu_j)\in P_{l_i}} \mu_j^2 + b_i \sum_{(x_j,\mu_j)\in P_{l_i}} \mu_j = \sum_{(x_j,\mu_j)\in P_{l_i}} x_j\mu_j, \tag{27}$$
$$m_i \sum_{(x_j,\mu_j)\in P_{l_i}} \mu_j + b_i n_{l_i} = \sum_{(x_j,\mu_j)\in P_{l_i}} x_j. \tag{28}$$
Using the notation
$$M_i(v) = \frac{\sum_{(x_j,\mu_j)\in P_{l_i}} v_j(x_j,\mu_j)}{n_{l_i}} \tag{29}$$
The solution is
$$m_i = \frac{(M_i(x\mu) - M_i(x)M_i(\mu))}{(M_i(\mu^2) - M_i^2(\mu))} \tag{30}$$
$$b_i = M_i(x) - m_i M_i(\mu) \tag{31}$$
3. Be:

$x_{l_0} = RP$

For $i = 1, \ldots, m_l$ do:

i.
$$x_{l_i} = \frac{x_{l_i}^{(1)} n_{l_i} + x_{l_i}^{(2)} n_{l_{i+1}}}{n_{l_i} + n_{l_{i+1}}}, \tag{32}$$

where

$$x_{l_i}^{(1)} = m_i \alpha_{l_i} + b_i \text{ and } x_{l_i}^{(2)} = m_{i+1}\alpha_{l_i} + b_{i+1}. \tag{33}$$

Note: in order to use this general formula we apply

$$m_{l_{m+1}} = b_{l_{m+1}} = n_{l_{m+1}} = 0. \tag{34}$$

ii. $j = i$

While $x_{l_j} > x_{l_{j-1}}$ do:

$$x_{l_i} = x_{l_{i-1}} = \ldots = x_{l_j} = x_{l_{j-1}} = \frac{\sum_{k=j-1}^{i}(x_{l_k}^{(1)} n_{l_k} + x_{l_k}^{(2)} n_{l_{k+1}})}{\sum_{k=j-1}^{i}(n_{l_k} + n_{l_{k+1}})} \tag{35}$$

$j = j - 1$

Figure 5. illustrates the need for and the result of the correction done by step ii. In the figure the original edge *i* is shown with normal line width, while the correction is drawn by a bold line, which is a vertical edge.

Note that the edge *i-1* was corrected with the correction of the edge *i* as well. When the edge *i-1* is corrected, it could occur an $x_{l_{i-1}} > x_{l_{i-2}}$. In this case the correction have to be continued and the edge *i-1* and the edge *i* will become a single vertical edge.

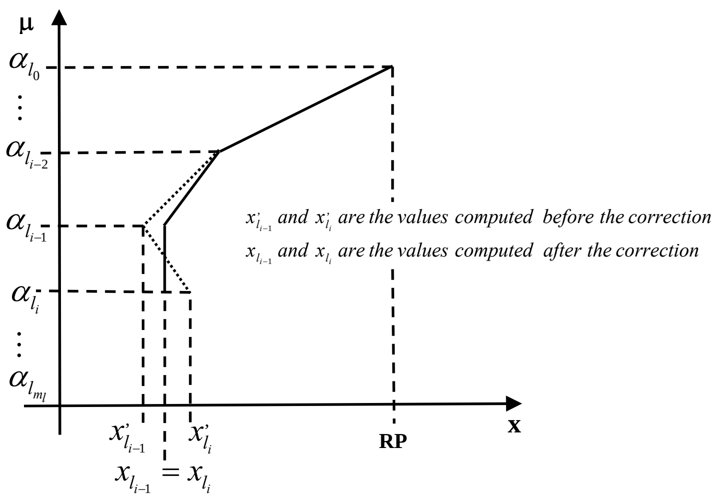

FIGURE 5. THE CORRECTION OF THE EDGE i.

The correction of the edge *i* might imply the need for the correction of the edge *i+1* too, but we do not deal with this correction immediately, it will be done in the next step, when $x_{l_{i+1}}$ is calculated and the edge *i+1* is examined.

Let us consider now the second case, when the top of the polygonal shaped fuzzy set is a plateau (an edge of the polygon). The above given algorithm has to be modified as follows:

1. Obtain a first approximation of the equation of the edge *i* $(1 \le i \le m_l)$ – the edge between break-points $\alpha_{l_{i-1}}$ and α_{l_i}. This step is the same as the previous one with only one difference: one obtains here a first approximation of the equation of the first edge as well.
2. Be:

$$x_{l_0} = x_{l_0}^{(2)} = m_1 \alpha_{l_0} + b_1,\ x_{l_0}^{(1)} = n_{l_0} = 0 \tag{36}$$

For $i = 1,\ldots,m_l$ do:

i.

$$x_{l_i} = \frac{x_{l_i}^{(1)} n_{l_i} + x_{l_i}^{(2)} n_{l_{i+1}}}{n_{l_i} + n_{l_{i+1}}}, \tag{37}$$

where $x_{l_i}^{(1)} = m_i \alpha_{l_i} + b_i$ and $x_{l_i}^{(2)} = m_{i+1}\alpha_{l_i} + b_{i+1}$

($m_{l_{m+1}} = b_{l_{m+1}} = n_{l_{m+1}} = 0$)

ii. $j = i$

While $x_{l_j} > x_{l_{j-1}}$ and $j > 0$ do:

$$x_{l_i} = x_{l_{i-1}} = \ldots = x_{l_j} = x_{l_{j-1}} = \frac{\sum_{k=j-1}^{i} x_{l_k}^{(1)} n_{l_k} + x_{l_k}^{(2)} n_{l_{k+1}}}{\sum_{k=j-1}^{i} n_{l_k} + n_{l_{k+1}}} \tag{38}$$

$j = j - 1$

We obtained the polygonal shape of the left flank and similarly we can obtain the polygonal shape of the right flank. After the determination of the two polygonal shapes we have to adjust x_{l_0} and x_{r_0} in such a form, that $RP = \frac{x_{l_0} + x_{r_0}}{2}$ holds.

Adjusting x_{l_0} and x_{r_0}:

Calculate:

$$d_1^{(1)} = RP - x_{l_0}, \tag{39}$$

$$d_1^{(2)} = x_{r_0} - RP, \tag{40}$$

$$d_1 = \frac{d_1^{(1)} + d_1^{(2)}}{2}, \tag{41}$$

$$d_2 = RP - x_{l_1}, \tag{42}$$

$$d_3 = x_{r_1} - RP, \tag{43}$$

$$d = \min(d_1, d_2, d_3), \tag{44}$$

then do:

$$x_{l_0} = RP - d, \tag{45}$$

$$x_{r_0} = RP + d, \tag{46}$$

Figures 6. and 7. illustrate the adjustment of x_{l_0} and x_{r_0}.

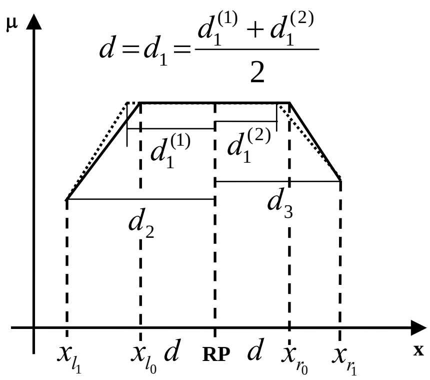

FIGURE 6. ADJUSTING x_{l_0}

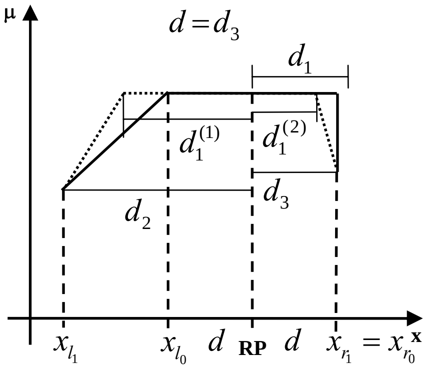

FIGURE 7. ADJUSTING x_{r_0}

4. CONCLUSIONS

The practical applicability of the SRR methods depends on their computational complexity, their capability to produce always a valid fuzzy set as well as the range of shape types they can handle.

Although SURE-LS has several advantages its original version covered only the popular case of triangle shaped fuzzy sets. This paper introduced a generalization of the method to the arbitrary shaped valid polygonal set types. The new technique preserved the advantageous features of the old one, e.g. the low computational complexity, the adherence to the characteristic shape type of the consequent partition, etc. Future research will be focused on increasing the area of applications [14].

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