



## RESPONSE SURFACE METHODOLOGICAL APPROACH FOR THE PREDICTION OF TANGENTIAL CUTTING FORCES IN END MILLING OF STAINLESS STEEL

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### Abstract:

The present paper discusses a response surface methodological approach for the prediction of tangential cutting force produced in end-milling operation of Stainless steel (SS304). It is difficult to predict accurately the cutting forces encountered in end milling operations due to large number of independent variables involved. In this work, an approach was undertaken to develop mathematical model based on RSM design for predicting the average tangential cutting force in end milling of SS304 in terms of cutting parameters cutting speed, feed rate, and axial depth of cut. All the individual cutting parameters affect on cutting force as well as their interaction are also investigated in this study. The experimental results indicate that the proposed mathematical models suggested could adequately describe the performance indicators within the limits of the factors that are being investigated. The adequacy of the predictive model was verified using ANOVA at 95% confidence level. This paper presents an approach to predict cutting force model in end milling of stainless steel using coated TiN insert under dry conditions and full immersion cutting.

### Keywords:

Cutting Forces, RSM, coated TiN, model

## 1. INTRODUCTION

Details of the metal cutting forces are required not only for the design of machine tools and cutting tools, but also for the determination of the cutting conditions for the various machining operations, especially for the programming on the CNC machining. It is necessary to select appropriate cutting conditions but avoid conditions that lead to excessive cutter deflection and cutter breakage, which are becoming increasingly important in finish machining. The traditional mechanistic approach is often used in predicting the cutting forces, where the cutting coefficients are identified through empirical curve fit to measured average milling forces. Peripheral milling is a widely used metal removal process in automobile, aerospace, textile machinery and other manufacturing industries for the roughing and finish cutting of profiled components, such as aircraft structural parts, dies and molds. The contour accuracy of the milled profiles is influenced by a number of factors including tool deflection, work-piece deflection, machine tool geometric errors, thermal effect, tool wear, etc. Among these factors, tool deflection caused by the cutting forces is the most dominant factor [1–3]. So a proper model of cutting forces provides a basis for surface accuracy prediction and improvement [4–6]. Presently developed cutting force models generally resort to the integrations of local cutting forces along the cutting flutes over cutter rotational cycles by numerical calculation [7], convolution analysis [8] and analytical formulation [9]. These models are obtained on condition of steady state cutting, in which the

machining parameters are fixed. But in reality, variability in cutting parameters such as radial depth of cut is frequently encountered in applications such as die sinking or pocketing operations, where smaller-diameter end mills have to remove material, especially at corners, not removed by larger-diameter roughing end mills. The radial depth of cut generally varies due to the rough corner radius as the end mill enters the corner, which results in the changes of cutting forces. Many researchers focus on the cutting forces for the cases of varying machining conditions. Fussell and Srinivasan [10] investigate experimentally the cutting forces for the cases of changing axial and radial depths of cut and feed rate, as well as the startup transients in the force as the cutter engages with the workpiece. Li and Liang [11] predict the milling forces in the axial, feed and cross-feed directions during transient-state cutting as the cutter engages with and disengages from a workpiece. Budak *et al.* presented a unified mechanics of cutting approach in predicting the milling force coefficients for cylindrical helical end mills [12]. Many other researches have followed purely experimental approaches to study the relationship between cutting forces and independent cutting conditions. This has reflected on the increased total cost of the study as a large number of cutting experiments is required. Furthermore, with this purely experimental approach, researchers have investigated the effect of cutting parameters on cutting forces using machining experiments based on a one-factor-at-a-time design, without having any idea about the behavior of cutting forces when two or more cutting factors are varied at the same time. The present study considers the effect of simultaneous variations of three cutting parameters (cutting speed, feed rate and axial depth of cut) on the behavior of cutting forces. For this purpose, the response surface methodology RSM is utilized. RSM is a group of mathematical and statistical techniques that are useful for modeling the relationship between the input parameters (cutting conditions) and the output variables (cutting force) [13]. RSM saves cost and time on conducting metal cutting experiments by reducing the overall number of required tests. In addition, RSM helps describe and identify, with a great accuracy, the effect of the interactions of different independent variables on the response when they are varied simultaneously [14–16]. In this paper, the technique is used to develop a mathematical model that utilizes the response surface methodology and method of experiments to predict the Cutting Force when milling stainless steel SS 304 using TiN coated Tungsten carbide inserts. The predicted Cutting Force results are presented in terms of mean values with 95% confidence interval.

## 2. Mathematical Model

Cutting force model for end milling in terms of the parameters can be expressed in general terms as:

$$F_t = C V^k a^m f_z^l \quad (1)$$

Where  $F_t$  is the predicted Cutting Force (N),  $V$  is the cutting speed (m/min),  $f_z$  is the feed per tooth (mm/tooth), and  $a$  is the axial depth of cut (mm).  $C$ ,  $k$ ,  $l$ , and  $m$  are model parameters to be estimated using the experimental results. To determine the constants and exponents, this mathematical model can be linearized by employing a logarithmic transformation, and Eq. (1) can be re-expressed as:

$$\ln F_t = \ln C + k \ln V + m \ln a + l \ln f \quad (2)$$

The linear model of Eq. 2 is :

$$F_t = \beta_0 x_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \quad (3)$$

where  $F_t$  is the true response of Cutting Force on a logarithmic scale,  $x_0 = 1$  (dummy variable),  $x_1$ ,  $x_2$ ,  $x_3$  are logarithmic transformations of speed, depth of cut, and feed, respectively, while  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  are the parameters to be estimated. Eq (3) can be expressed as:

$$\hat{F}_t = F_t - \varepsilon = b_0 x_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 \quad (4)$$

where  $\hat{F}_t$  is the estimated response and  $F_t$  the measured Cutting Force on a logarithmic scale,  $\varepsilon$  the experimental error and the  $b_i$  values are estimates of the  $\beta_i$  parameters.

The second-order model can be extended from the first-order model equation as:

$$\hat{F}_{2t} = F_t - \varepsilon = b_0x_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3 \quad (5)$$

where,  $\hat{F}_{2t}$  is the estimated response based on the second order model. Analysis of variance is used to verify and validate the model.

### 3. EXPERIMENTAL DETAILS

#### 3.1 Experimental setup

Cutting tests was conducted mainly on Vertical Machining Center (VMC ZPS, Model: 1060) powered by a 30 KW motor with a maximum spindle speed of 8000 rpm. Fig.1 shows the experimental set up cutting test conditions on end milling for machining of stainless steel with TiN inserts. Kistler Rotating Cutting Force Dynamometer was used for measuring cutting forces. The complete measuring system consists of rotor (type 9125A) stator (type 5235), connecting cable (type 1500A37), signal conditioner (5267A1/A2) (Fig.1) and computer interface (RS-232C).

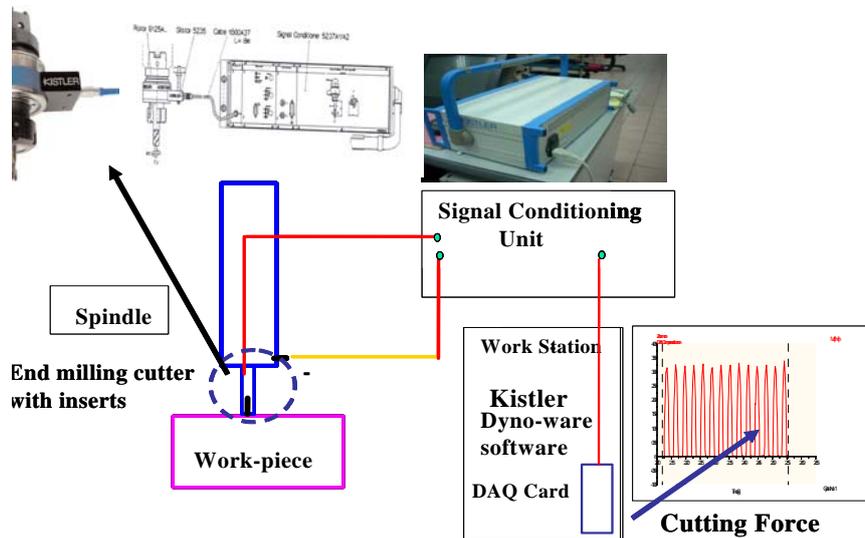


Fig.1: Rotating High-Speed Cutting Force Measuring System

The computer software used was Kistler Dyno-Ware (type: 2825D1-2, version 2.31) which is universal and operator's friendly software. The instrument can measure two components of cutting force i.e. Thrust for along z-axis ( $F_z$ ) in Newton and Moment along z-axis ( $M_z$ ) in Newton-meter. The Torque in z-axis  $M_z$  was then converted to Tangential force ( $F_t$ ) by dividing it with the radius of the tool holder diameter as follows:

$$F_t = \frac{M_z}{\text{Radius\_of\_tool\_Holder} \times 0.001(\text{m})} [\text{N}]$$

#### 3.2. Coding of the independent variables

The independent variables were coded taking into consideration the limitation and capacity of the cutting tools. Levels of independent and coding identification are presented in Table 1, for experiment using Coated TiN inserts, respectively. Table1 Coding Identification for end milling using Coated TiN insert.

Tabel 1.

Level of coding	Lowest -√2	Low -1	Centre 0	High +1	Highest +√2
x <sub>1</sub> cutting speed, v m/min	59.5	71	109.2	168	200.78
x <sub>2</sub> axial depth of cut, mm	1.005	1.15	1.59	2.2	2.516
x <sub>3</sub> Feed, mm/tooth	0.039	0.05	0.089	0.16	0.204

The transforming equations for each of the independent variables are:

$$\begin{aligned}
 x_1 &= \frac{\ln V - \ln 109.2}{\ln 168 - \ln 109.2}; \\
 x_2 &= \frac{\ln a - \ln 1.59}{\ln 2.2 - \ln 1.59}; \\
 x_3 &= \frac{\ln f_z - \ln 0.089}{\ln 0.16 - \ln 0.089}
 \end{aligned}
 \tag{6}$$

The above relationships were obtained from the following transforming equation:

$$x_1 = \frac{\ln x_n - \ln x_{n0}}{\ln x_{n1} - \ln x_{n0}}$$

where, x is the coded value of any factor corresponding to its natural value  $x_n$ .  $x_{n1}$  is the +1 level and  $x_{n0}$  is the natural value of the factor corresponding to the base of zero level.

### 3.3. Experimental Design

The design of the experiments has an effect on the number of experiments required. Therefore, it is important to have a well-designed experiment to minimize the number of experiments which often are carried out randomly. Cutting conditions in coded factors and the Cutting Force values obtained using TiN coated cemented carbide insert are presented in Table 2. In the experiment, small central composite design was used to develop the Cutting Force model. The analysis of mathematical models was carried out using Design-expert 6.0 package [17].

Table 2. Cutting Force and cutting conditions in coded factors

Std Ord.	Type	Coding of Level			Cutting Forces (N)
		$x_1$	$x_2$	$x_3$	
		Cutting speed, /min	Axial Depth of cut, mm	Feed, mm/tooth	
1	Factorial	1	1	-1	330
2	Factorial	1	-1	1	410
3	Factorial	-1	1	1	120
4	Factorial	-1	-1	-1	220
5	Centre	0	0	0	386
6	Centre	0	0	0	388
7	Centre	0	0	0	394
8	Centre	0	0	0	390
9	Centre	0	0	0	392
10	Axial	-1.414	0	0	420
11	Axial	1.414	0	0	380
12	Axial	0	-1.414	0	265
13	Axial	0	1.414	0	580
14	Axial	0	0	-1.414	238
15	Axial	0	0	1.414	750

## 4. RESULTS AND DISCUSSIONS

### 4.1 Development of second order model using CCD design

The Fit and summary test which are shown in Table 3, indicate that the quadratic model CCD models was more significant than linear model and it also proved that linear model has a significant lack of fit (LOF). Therefore, the quadratic model was chosen in order to develop the CCD model.

Table 3 Fit and Summary test of the second order CCD model

Sequential Model Sum of Squares						
Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Mean	514.4203	1	514.4203			Suggested
Block	0.214392	1	0.214392			
Linear	0.478996	3	0.159665	0.859622	0.4932	
2FI	1.366371	3	0.455457	6.493049	0.0197	
<b>Quadratic</b>	<b>0.490754</b>	<b>3</b>	<b>0.163585</b>	<b>2487.952</b>	<b>&lt; 0.0001</b>	<b>Suggested</b>
Cubic	0	0				Aliased
Residual	0.000263	4	6.58E-05			
Total	516.9711	15	34.46474			

The second order Cutting Force model is given as:

$$\hat{y}_2 = 6.14 - 0.035 x_1 + 0.28 x_2 + 0.41 x_3 - 0.16 x_1^2 - 0.17 x_2^2 - 0.13 x_3^2 + 0.50 x_1 x_2 + 0.48 x_1 x_3 - 0.44 x_2 x_3$$

To verify the adequacy of the proposed second order CCD model, ANOVA was used and the results are shown in the Table 4.

Table 4. ANOVA for second order CCD model

Sequential Model Sum of Squares						
Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Block	0.214392	1	0.214392			
Model	2.33612	9	0.259569	3947.772	< 0.0001	Significant
x <sub>1</sub>	0.005008	1	0.005008	76.17175	0.0009	
x <sub>2</sub>	0.306778	1	0.306778	4665.774	< 0.0001	
x <sub>3</sub>	0.658725	1	0.658725	10018.52	< 0.0001	
x <sub>1</sub> <sup>2</sup>	0.196499	1	0.196499	2988.537	< 0.0001	
x <sub>2</sub> <sup>2</sup>	0.220028	1	0.220028	3346.391	< 0.0001	
x <sub>3</sub> <sup>2</sup>	0.134437	1	0.134437	2044.647	< 0.0001	
x <sub>1</sub> x <sub>2</sub>	0.506174	1	0.506174	7698.37	< 0.0001	
x <sub>2</sub> x <sub>3</sub>	0.466072	1	0.466072	7088.462	< 0.0001	
Residual	0.394125	1	0.394125	5994.236	< 0.0001	
Lack of Fit	0.000263	4	6.58E-05			
Pure Error	2.550775	14				
Cor Total	0.214392	1	0.214392			

The quadratic CCD model shows that feed has the most significant effect on Cutting Force, followed by axial depth of cut and cutting speed.

**Cutting Force comparison of Different model with actual**

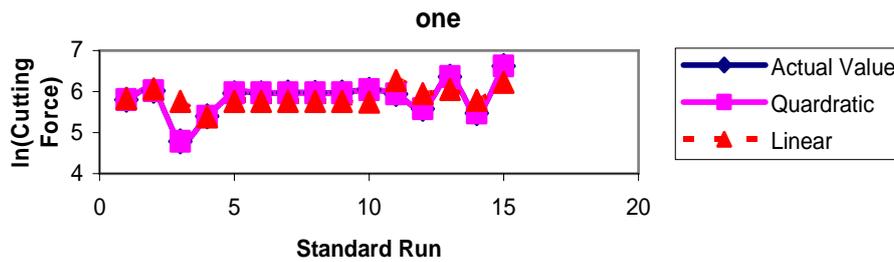


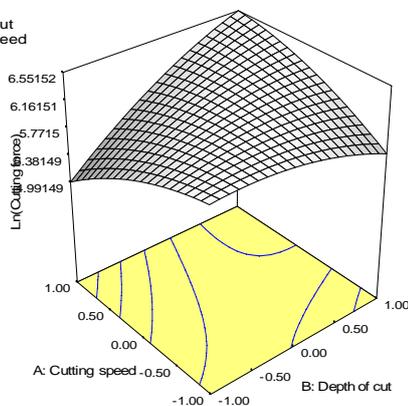
Fig 2 Cutting Force contours of experimental and quadratic CCD predicted values

Fig. 2 indicates the contours of actual Cutting Force values and the corresponding predicted values of quadratic CCD models. The graphs indicated that the quadratic model could predict cutting force values very close to the actual values. It can be also observed from Fig.2 that quadratic model leads to the values more close to the actual values compared to linear model.

DESIGN-EXPERT Plot

Ln(Cutting force)  
X = B: Depth of cut  
Y = A: Cutting speed

Actual Factor  
C: Feed = 0.00



DESIGN-EXPERT Plot

Ln(Cutting force)  
X = B: Depth of cut  
Y = C: Feed

Actual Factor  
A: Cutting speed = 0.00

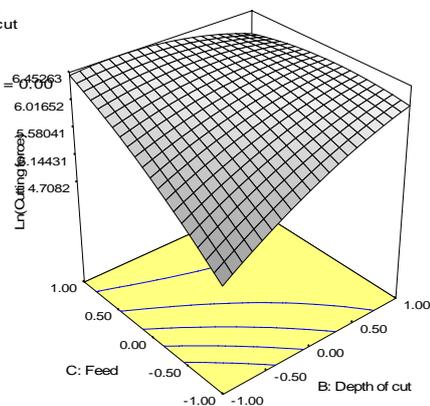


Fig 3 The response surface of the quadratic CCD model for end milling using coated TiN insert

Figure 3 shows the 3D-response surface of quadratic CCD model based on the effect of speed and depth of cut on Cutting Force and depth of cut and feed on Cutting Force. The contours affirm that Cutting Force can be affected by the feed followed by axial depth of cut and cutting speed. The normal probability plots of the residuals and the plots of the residuals versus the predicted response for Cutting Force are shown in Fig.4, Fig 5 respectively. A check on the plots in Fig.4 revealed that the residuals generally fall on a straight line implying that the errors are distributed normally.

Also Fig 5 revealed that they have no obvious pattern and unusual structure. This implies that the models proposed are adequate and there is no reason to suspect any violation of the independence or constant variance assumption.

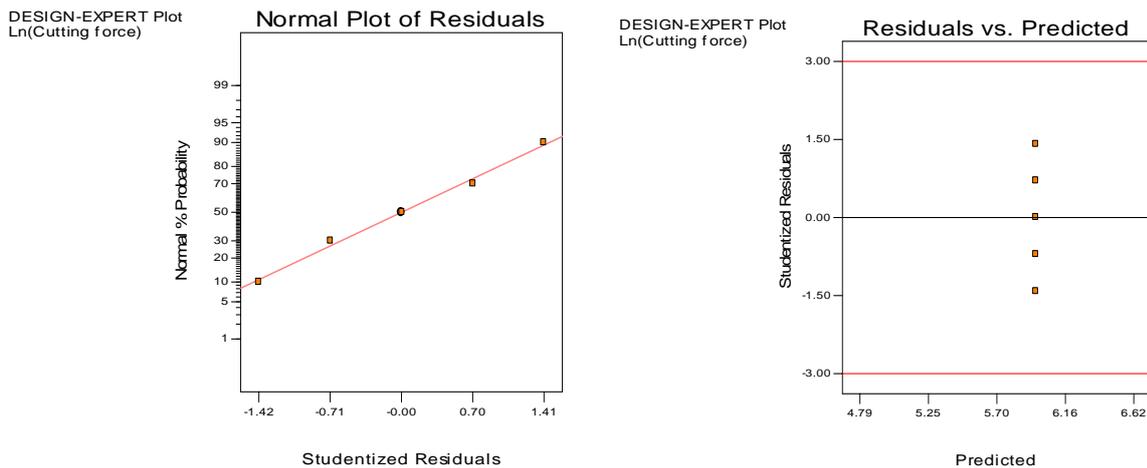


Fig 4 :Normal probability plot of residuals for  $F_{\alpha}$  data. Fig 5: Plot of residuals vs. predicted response for  $F_{\alpha}$  data

**Effect of Feed on Cutting Force:** By analysis the developed quadratic model, it has been observed that with the increase of feed the tangential cutting force increases as shown in Fig 6.

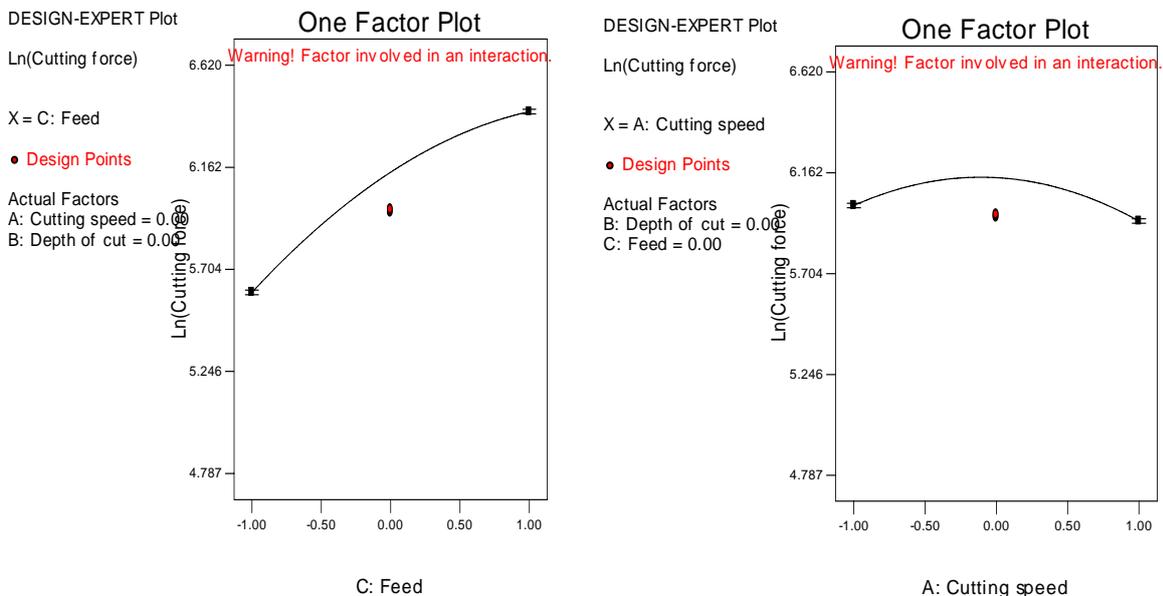


Fig. 6: Effect of feed on cutting force Fig. 7 Effect of Cutting speed on cutting force

**Effect of Cutting speed on Cutting Force:** By analysis the developed quadratic model, it has been also observed that with the increase of Depth of cutting speed the tangential cutting force decreases as shown in Fig 7.

**Effect of Depth of cut on Cutting Force:** By analysis the developed quadratic model, it has been also observed that with the increase of depth of cut the tangential cutting force increases as shown in Fig 8.

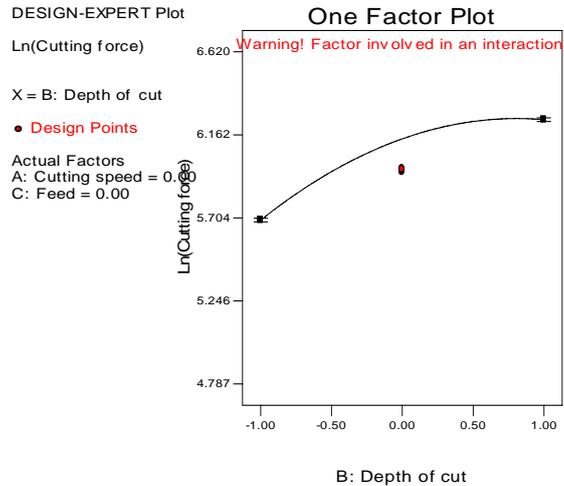


Fig.8: Effect of Depth of on cutting force

## 5. CONCLUSIONS

This research paper discussed the development of a theoretical and experimental model for improving the efficiency of face milling of stainless steel (SS304) using coated TiN inserts. The general conclusions can be summarized as follows:

- ✚ The quadratic CCD indicate that the feed was the most significant influence on Cutting force, followed by depth of cut and cutting speed
- ✚ An increase in either the feed or the axial depth of cut increases the Cutting force, whilst an increase in the cutting speed decreases the Cutting force.
- ✚ The CCD model developed by RSM using Design Expert package is able to provide accurately the predicted values of Cutting force close to actual values found in the experiments. The equations are checked for their adequacy with a confidence level of 95%.
- ✚ Contours of surface outputs are constructed in planes containing two of the independent variables. These contours were further developed to enable the selection on the proper combination of cutting parameters to increase the metal removal rate without sacrificing the quality of the surface finish produced.

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