JOURNAL OF ENGINEERING
annals OF
FACULTY OF ENGINEERING HUNEDOARA
TOME VI (year 2008), FASCICULE 3, (ISSN 1584 – 2673)

# ABOUT SEQUENCE IMPEDANCE COMPONENTS AT ELECTRICAL THREE-PHASES CIRCUITS IN SINUSOIDAL NON-SYMMETRICAL SITUATIONS

Marilena MATICA[1.], Radu MATICA[2.], Marius LOLEA[3.], Viorel HAS[3.]

[1.] Department of Electrical Drives and Automatics, University of Oradea, Faculty of Electrical Engineering and Information Technology
[2.] S.C.Electrica Service S.A. – A.I.S.E. Oradea, University of Oradea, Faculty of Energetic
[3.] University of Oradea, Faculty of Energetics, Romania

**Abstract:**
It is necessary to know sequence impedance components in purpose to analyze the electrical networks. For this reason the paperwork describes computed formula of sequence impedance components matrix at electrical three-phase circuits in sinusoidal non-symmetrical situations. Analyzed model regard electrical lines with three and four conductors.
**Keywords:**
the non-symmetry ratio, the minimum non-symmetry ratio, the maximum non-symmetry ratio

## 1. GENERAL CONSIDERATIONS

The three-phase electric networks were conceived in order to function in balanced symmetrical situations. In these situations the component elements: transformers, lines, condenser batteries, reactance coils etc. present circuit parameters identical on each phase, and the voltage and current system in any section are symmetric. If one of the network's elements, or the consumers that are supplied by the network with electric energy, becomes unbalanced, the situation becomes non-symmetric, the voltage and current systems from different knots lose their symmetry.

## 2. NEW CHARACTERISTIC MEASURES OF THE NON-SYMMETRIC SITUATION

To characterize the non-symmetric measures in a knot of the electro-energetic system the following measures can be used: the reverse non-symmetry coefficient (bi-symmetry), the coefficient of homo-polar non-symmetry (anti-symmetry), the total non-symmetry coefficient: $K_{ns} = K_{ns}^{-} + K_{ns}^{0}$, the non-symmetry ratio: $r_{ns} = A_{min} / A_{max}$, the differential symmetry ratio.

If we consider that for the supplying voltages the unbalance is not high, sometimes even neglected regarding the outpacing, the arithmetic mean of phase voltages represents the voltage of direct sequence. Consequently the deviation or the ratio between the maximum and minimum values of voltages and the medium value is a useful index of the non-symmetry degree. So we can define the following non-symmetry ratios:

$$r_{min} = \frac{\min(U_R, U_S, U_T)}{U_{med}} \tag{1}$$

$$r_{max} = \frac{\max(U_R, U_S, U_T)}{U_{med}} \tag{2}$$

Where $r_{min}$ stands for the minimum non-symmetry ratio, $r_{max}$ stands for the maximum non-symmetry ratio. We can easily find out $r_{min} / r_{max} = r_{ns}$.

## 3. THE METHOD OF SEQUENCE COMPONENTS. THE FORM OF THE SEQUENCE MATRIX

The analysis of the non-symmetric situation is facilitated, as it is known, by the use of sequence components method (Fortescue components). If we apply the sequence components method, the real three-phase network is decomposed in three networks of non-coupled sequence, inductive or capacitate, among them. In the case of short circuits or interruptions, the networks must be considered linked among them, in the case of some unbalanced consumers they are not linked.

To be able to form such networks it is necessary to know the sequence parameters of each element of network or system. This is the reason why it will determine the form of the sequence impedance of an element from electrical network, expressed in symmetric components, knowing the phase impedance. The network element is considered linear and of a general form, with and without neuter conductor.

a) For the three-phase element without neuter conductor from system described in fig.1, if we refer to the longitudinal aspect we can write the following relations among the voltages and phase currents:

$$\begin{aligned} \underline{U}_R &= \underline{Z}_{RR} \cdot \underline{I}_R + \underline{Z}_{RS} \cdot \underline{I}_S + \underline{Z}_{RT} \cdot \underline{I}_T \\ \underline{U}_S &= \underline{Z}_{SR} \cdot \underline{I}_R + \underline{Z}_{SS} \cdot \underline{I}_S + \underline{Z}_{ST} \cdot \underline{I}_T \\ \underline{U}_T &= \underline{Z}_{TR} \cdot \underline{I}_R + \underline{Z}_{TS} \cdot \underline{I}_S + \underline{Z}_{TT} \cdot \underline{I}_T \end{aligned} \quad (3)$$

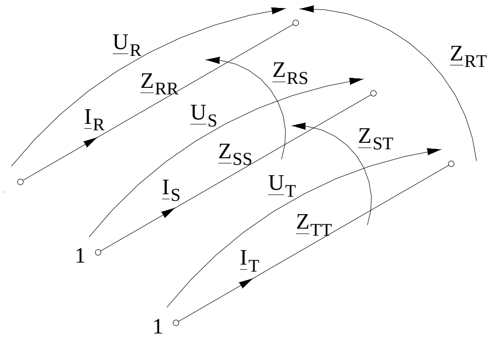


Figure 1. Element of three-phased system without neuter conductor

To make the change of measures: currents or voltages, from the system of phase measures, in the system of sequence components, it was applied the changing relations:

$$\begin{aligned} [\underline{U}_{Scv}] &= [\underline{T}_U] \cdot [\underline{U}_f] \\ [\underline{I}_{Scv}] &= [\underline{T}_I] \cdot [\underline{I}_f] \end{aligned} \quad (4)$$

$$[\underline{T}_U] = [\underline{T}_I] = [\underline{T}] = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \quad (5)$$

„a” being the complex operator.

For the sequence measures, a similar relation can be written, so $[\underline{U}_{Scv}] = [\underline{Z}_{Scv}] \cdot [\underline{I}_{Scv}]$, where $[\underline{Z}_{Scv}]$ is the matrix of sequence impedance. Generally speaking this one has the following form:

$$[\underline{Z}_{Scv}] = \begin{vmatrix} \underline{Z}^{++} & \underline{Z}^{+-} & \underline{Z}^{+0} \\ \underline{Z}^{-+} & \underline{Z}^{--} & \underline{Z}^{-0} \\ \underline{Z}^{0+} & \underline{Z}^{0-} & \underline{Z}^{00} \end{vmatrix} \quad (6)$$

For the application of sequence component method, it is desirable to have a diagonal matrix. If we consider the relations (4), (5), (6), the following expression results for the matrix of sequence impedance:

$$\begin{aligned} [\underline{Z}_{Scv}] &= [\underline{T}_U] \cdot [\underline{Z}_f] \cdot [\underline{T}_I]^{-1} \\ &= [\underline{T}] \cdot [\underline{Z}_f] \cdot [\underline{T}]^{-1} \end{aligned} \quad (7)$$

The matrix $[\underline{Z}_{Scv}]$ receives particular forms if we consider that the network's system's elements dispose of certain symmetry. Thus for the network's elements, we can admit, with a certain error, that it presents a total symmetry, that is characterized by the proper and mutual impedance equality, indifferent of the phases way.

The matrix of sequence impedance becomes:

$$[\underline{Z}_{Scv}] = \frac{1}{3^2}\begin{bmatrix}1 & a^2 & a\\ 1 & a & a^2\\ 1 & 1 & 1\end{bmatrix}\cdot\begin{bmatrix}\underline{Z} & \underline{Z}' & \underline{Z}''\\ \underline{Z}'' & \underline{Z} & \underline{Z}'\\ \underline{Z}' & \underline{Z}'' & \underline{Z}\end{bmatrix}\cdot\begin{bmatrix}1 & a & a^2\\ 1 & a^2 & a\\ 1 & 1 & 1\end{bmatrix} = \begin{bmatrix}\underline{Z}^{++} & 0 & 0\\ 0 & \underline{Z}^{--} & 0\\ 0 & 0 & \underline{Z}^{00}\end{bmatrix} \quad (8)$$

If we dispose of a cyclic symmetry, characterized by the proper impedance's equality $\underline{Z}_{RR} = \underline{Z}_{SS} = \underline{Z}_{TT} = \underline{Z}$, equality of mutual impedance considered in direct direction $\underline{Z}_{RS} = \underline{Z}_{ST} = \underline{Z}_{TR} = \underline{Z}'$ and equality of the mutual impedance in opposite direction $\underline{Z}_{SR} = \underline{Z}_{TS} = \underline{Z}_{RT} = \underline{Z}''$ the matrix of phase impedance has the form:

$$[\underline{Z}_f] = \begin{bmatrix}\underline{Z} & \underline{Z}' & \underline{Z}''\\ \underline{Z}'' & \underline{Z} & \underline{Z}'\\ \underline{Z}' & \underline{Z}'' & \underline{Z}\end{bmatrix} \quad (9)$$

and the sequence impedance is:

$$[\underline{Z}_{Scv}] = \begin{bmatrix}\underline{Z} + a^2\underline{Z}' + a\underline{Z}'' & 0 & 0\\ 0 & \underline{Z} + a\underline{Z}' + a^2\underline{Z}'' & 0\\ 0 & 0 & \underline{Z} + \underline{Z}' + \underline{Z}''\end{bmatrix} \quad (10)$$

b) For the three-phase element with neuter conductor, fig.2, the following relations can be written between the falling of voltages and currents:

$$\begin{bmatrix}\Delta\underline{U}_f\\ \Delta\underline{U}_n\end{bmatrix} = [\underline{Z}_f]\cdot\begin{bmatrix}\underline{I}_f\\ \underline{I}_n\end{bmatrix} \quad (11)$$

$$[\underline{Z}_f] = \begin{bmatrix}\underline{Z}_{RR} & \underline{Z}_{RS} & \underline{Z}_{RT} & \underline{Z}_{Rn}\\ \underline{Z}_{SR} & \underline{Z}_{SS} & \underline{Z}_{ST} & \underline{Z}_{Sn}\\ \underline{Z}_{TR} & \underline{Z}_{TS} & \underline{Z}_{TT} & \underline{Z}_{Tn}\\ \underline{Z}_{nR} & \underline{Z}_{nS} & \underline{Z}_{nT} & \underline{Z}_{nn}\end{bmatrix} \quad (12)$$

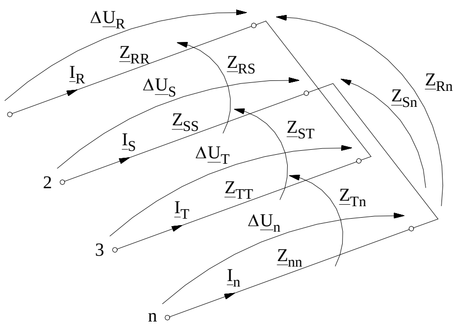


Figure 2. Three-phased element with neuter conductor

If the voltages applied to the three phases, as compared to the neuter conductor, are $U_R$, $U_S$, $U_T$, ; by decreasing the last line of the system (11) from the three firsts and considering that $\underline{I}_N = -(\underline{I}_R + \underline{I}_S + \underline{I}_T)$ we get the system $[\underline{U}_f] = [\underline{Z}_f]\cdot[\underline{I}_f]$, where $[\underline{Z}_f]$ is obtained from the relation (12), by giving up the last line and last column, so:

$$[\underline{Z}_f] = \begin{bmatrix}\underline{Z}_{RR} - \underline{Z}_{Rn} - (\underline{Z}_{Rn} - \underline{Z}_{nn}) & \underline{Z}_{RS} - \underline{Z}_{Rn} - (\underline{Z}_{Sn} - \underline{Z}_{nn}) & \underline{Z}_{RT} - \underline{Z}_{Rn} - (\underline{Z}_{Tn} - \underline{Z}_{nn})\\ \underline{Z}_{RS} - \underline{Z}_{Sn} - (\underline{Z}_{Rn} - \underline{Z}_{nn}) & \underline{Z}_{SS} - \underline{Z}_{Sn} - (\underline{Z}_{Sn} - \underline{Z}_{nn}) & \underline{Z}_{ST} - \underline{Z}_{Rn} - (\underline{Z}_{Tn} - \underline{Z}_{nn})\\ \underline{Z}_{RT} - \underline{Z}_{Tn} - (\underline{Z}_{Rn} - \underline{Z}_{nn}) & \underline{Z}_{ST} - \underline{Z}_{Tn} - (\underline{Z}_{Sn} - \underline{Z}_{nn}) & \underline{Z}_{TT} - \underline{Z}_{Rn} - (\underline{Z}_{Tn} - \underline{Z}_{nn})\end{bmatrix} \quad (13)$$

If we apply to the matrix (13) a transform of the type (7) we get the sequence impedance matrix whose elements have the following significance:

$$\begin{aligned}
\underline{Z}^{+0} &= \underline{Z}_p^0 - \underline{Z}_m^0 & \underline{Z}^{-0} &= \underline{Z}_p^+ - \underline{Z}_m^0\\
\underline{Z}^{++} &= \underline{Z}_p^+ + 2\underline{Z}_m^+ & \underline{Z}^{-+} &= \underline{Z}_p^+ - \underline{Z}_m^+ - 3\underline{Z}_{mn}^+\\
\underline{Z}^{+-} &= \underline{Z}_p^- - \underline{Z}_m^- - 3\underline{Z}_{mm}^- & \underline{Z}^{0+} &= \underline{Z}_p^+ - \underline{Z}_m^+ - 3\underline{Z}_{mn}^+\\
\underline{Z}^{--} &= \underline{Z}_p^- + 2\underline{Z}_m^- & \underline{Z}^{0-} &= \underline{Z}_p^- - \underline{Z}_m^- - 3\underline{Z}_{mn}^-
\end{aligned} \quad (14)$$

$$\underline{Z}^{00} = \underline{Z}_p^0 + 2(\underline{Z}_m^0 - 3\underline{Z}_{mn}^0) + 3\underline{Z}_{nn}$$

where $Z_p^0$, $Z_p^+$, $Z_p^-$ are the "symmetric components" of the proper impedance; and $Z_m^+$, $Z_m^-$, $Z_m^0$ are the "symmetric components" of the mutual impedance between phases, so:

$$\begin{bmatrix}\underline{Z}_m^+\\ \underline{Z}_m^-\\ \underline{Z}_m^0\end{bmatrix} = [\underline{T}]\cdot\begin{bmatrix}\underline{Z}_{RR} - \underline{Z}_{Rn} - (\underline{Z}_{Rn} - \underline{Z}_{nn})\\ \underline{Z}_{SS} - \underline{Z}_{Sn} - (\underline{Z}_{Sn} - \underline{Z}_{nn})\\ \underline{Z}_{TT} - \underline{Z}_{Tn} - (\underline{Z}_{Tn} - \underline{Z}_{nn})\end{bmatrix} \quad (15)$$

and $Z_{mn}^{+}$, $Z_{mn}^{-}$, $Z_{mn}^{0}$ are the symmetric components of the mutual impedance between phases and neuter, so:

$$\begin{bmatrix} \underline{Z}_{mn}^{+} \\ \underline{Z}_{mn}^{-} \\ \underline{Z}_{mn}^{0} \end{bmatrix} = [\underline{T}] \cdot \begin{bmatrix} \underline{Z}_{Rn} \\ \underline{Z}_{Sn} \\ \underline{Z}_{Tn} \end{bmatrix} \tag{16}$$

If the element from figure 2 presents cyclic symmetry, by applying the transform (7) for the sequence impedance matrix the following expression is obtained:

$$[\underline{Z}_{Scv}] = \begin{bmatrix} \underline{Z} + a^2\underline{Z}' + a\underline{Z}'' & 0 & 0 \\ 0 & \underline{Z} + a\underline{Z}' + a^2\underline{Z}'' & 0 \\ 0 & 0 & \underline{Z} + \underline{Z}' + \underline{Z}'' - 3(\underline{Z}'_n - \underline{Z}_n) \end{bmatrix} \tag{17}$$

c) The three-phase element (open-air electrical line) with protective conductor linked to the ground.

For the three-phase element with protective conductor linked to the ground at the two ends and presented in fig. 3.9 the following relations can be written:

$$\begin{bmatrix} \Delta\underline{U}_f \\ \Delta\underline{U}_C \end{bmatrix} = [\underline{Z}_f] \cdot \begin{bmatrix} \underline{I}_f \\ \underline{I}_C \end{bmatrix} \tag{18}$$

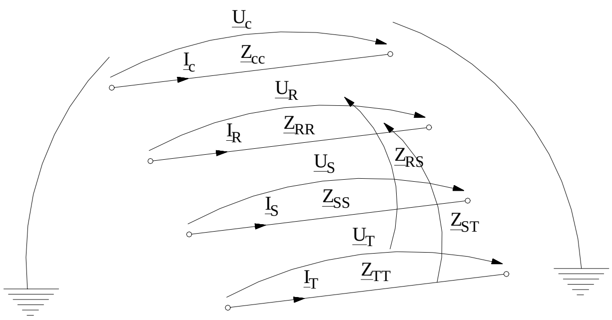


Figure 3. Element of three-phase system with protective conductor linked to the ground

The phase impedance matrix $[\underline{Z}_f]$ has the form:

$$[\underline{Z}_f] = \begin{bmatrix} \underline{Z}_{RR} & \underline{Z}_{RS} & \underline{Z}_{RT} & \underline{Z}_{Rc} \\ \underline{Z}_{SR} & \underline{Z}_{SS} & \underline{Z}_{ST} & \underline{Z}_{Sc} \\ \underline{Z}_{TR} & \underline{Z}_{TS} & \underline{Z}_{TT} & \underline{Z}_{Tc} \\ \underline{Z}_{cR} & \underline{Z}_{cS} & \underline{Z}_{cT} & \underline{Z}_{cc} \end{bmatrix} \tag{19}$$

where $Z_{cc}$ is the phase proper impedance of the protective conductor; $Z_{ci}$(1=1,2,3)- the mutual impedance between the phase conductors and the protective conductor. If we admit a cyclic symmetry for the phase conductors and a total one as compared to the protective conductor, the phase impedance matrix receives the form:

$$[\underline{Z}_f] = \begin{bmatrix} \underline{Z} & \underline{Z}' & \underline{Z}'' & \underline{Z}'_c \\ \underline{Z}'' & \underline{Z} & \underline{Z}' & \underline{Z}'_c \\ \underline{Z}' & \underline{Z}'' & \underline{Z} & \underline{Z}'_c \\ \underline{Z}'_c & \underline{Z}'_c & \underline{Z}'_c & \underline{Z}_c \end{bmatrix} \tag{20}$$

If we consider that the voltage of the protective conductor is zero, we find out from the last line of the system, the current's expression through the protective conductor, thus:

$$\underline{I}_c = -\frac{\underline{Z}'_c}{\underline{Z}_c} \cdot (\underline{I}_R + \underline{I}_S + \underline{I}_T) \tag{21}$$

By giving up the last line and column from the matrix's expression $[\underline{Z}_f]$ this one becomes:

$$[\underline{Z}_f] = \begin{bmatrix} \underline{Z} & \underline{Z}' & \underline{Z}'' \\ \underline{Z}'' & \underline{Z} & \underline{Z}' \\ \underline{Z}' & \underline{Z}'' & \underline{Z} \end{bmatrix} - \frac{(\underline{Z}'_C)^2}{\underline{Z}_C} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{22}$$

and in sequence components:

$$[\underline{Z}_{Scv}]=\begin{bmatrix}\underline{Z}+a^2\underline{Z}'+a\underline{Z}'' & 0 & 0\\ 0 & \underline{Z}+a\underline{Z}'+a^2\underline{Z}'' & 0\\ 0 & 0 & \underline{Z}+\underline{Z}'+\underline{Z}''-3\underline{Z}'^2_C/\underline{Z}_C\end{bmatrix} \qquad (23)$$

It is ascertained that the presence of the supplementary conductor linked to the ground changes only the form of the zero sequence impedance.

## 4. ABOUT THE CIRCULATION OF POWERS IN THREE-PHASE NETWORKS WITH UNBALANCED CONSUMERS

The non-symmetric situation caused by some unbalanced consumers determines the increase of power losses. The increase of losses in non-symmetric situation can be based on supplementary power circulation of negative and zero sequence in the supplying network.

Thus if we consider an electric subsystem formed of an ideal generator (G) that debits through a symmetric and linear network (R) over two consumers, one balanced (E) and other unbalanced (D), figure 4.

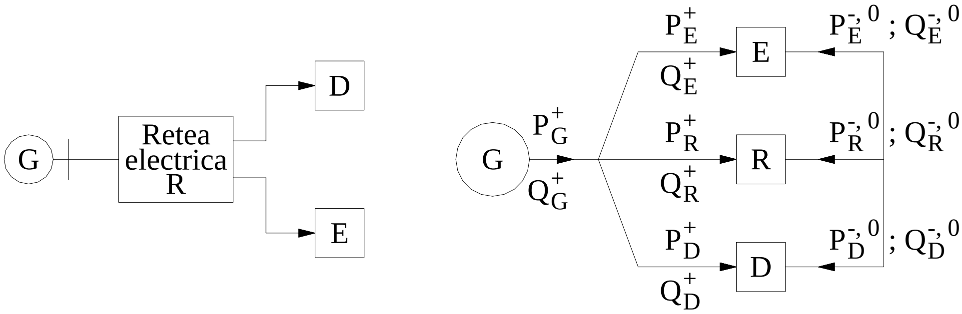


Figure 4. Electric system schematic representation. The sequence powers circulation in system

The power debited by generator and received by network and absorbed by the two consumers expressed in symmetric components can be written as follows:

$$\underline{S}_k = \underline{S}_k^+ + \underline{S}_k^- + \underline{S}_k^0$$
$$k = G,R,E,D \qquad (24)$$

The preservation theories of the complex apparent powers lead to the following preservation relations:

$$\underline{S}_G = \underline{S}_R + \underline{S}_E + \underline{S}_D$$
$$\underline{S}_{G\alpha} = \underline{S}_{R\alpha} + \underline{S}_{E\alpha} + \underline{S}_{D\alpha} \qquad (25)$$

The previous relations show that the element which introduces the non-symmetry is the unbalanced consumer. Consequently the electric generator debits powers to two consumers only on positive sequence, and the unbalanced consumer is the one that receives on positive sequence a higher power than it is necessary. A part from it is converted in powers of negative and zeros sequence that it is injected again in network, and at the balanced consumer, supplementing this way the power losses.

## 5. EXPERIMENTAL MEASUREMENTS

It is presented one of the most representative experimental measurements realized with ACE 2000 in a transformation post RAOTL, from Oradea, departure for electrical alimentation of tram repressor station. The results are presented in table 1.

Table 1. The analyze of power components for PT RAOTL

| Harmonic | P | $P^+$ | $P^-$ | $P^0$ |
|---|---|---|---|---|
| 1 | 53980.9 | 54234.9 | -17.9 | -271.8 |
| 3 | -203.6 | -1.4 | -6.2 | 208.3 |
| 5 | -320.1 | -4.0 | -315.7 | 8.4 |
| 7 | -241.9 | -248.6 | 1.2 | -7.9 |

In the next table there is the total non-symmetries coefficient, for several points in electrical networks.

Table 2. Experimental values that demonstrate presence of the non-symmetries situation

| Measuring point | Non-symmetries | |
|---|---|---|
| | $K_{nsU}$ [%] | $K_{nsI}$ [%] |
| Centru Redresor | 2.1 | 4.,9 |
| Centru Zamfirescu | 2.2 | 8.2 |
| P.T. Accesorii | 0.,82 | 6.6 |
| P.T. Cuptoare | 0.79 | 4.9 |

## 6. CONCLUSIONS

Active power received by unbalanced tree-phase electrical receptor is only positive sequence active power ($P^+$), an important result for obtaining power factor in non-symmetrical situation ($k_p=P^+/S$).

For a three-phase element, the matrix $[\underline{Z}_{Scv}]$ receives particular forms if we consider that the network's system's elements dispose of certain symmetry.

The positive and negative sequence impedance did not change their form for the three-phase element with neuter conductor or for the three-phase element with protective conductor linked to the ground, only the impedance of zero sequence has a modified expression as compared to the one from the element without neuter conductor.

**References:**

[1.] Antoniu, I. S. - "Chestiuni speciale de electrotehnică." Editura Academiei, 1956;

[2.] Buta, A. - "Transportul şi distribuţia energiei electrice." Universitatea Tehnică, Timisoara, 1991;

[3.] Matica, L.M. - "Contribuţii privind analiza sistemelor trifazate în regimuri sinusoidal nesimetric şi periodic nesinusoidal." Teză de doctorat, Timişoara, 2001;

[4.] Matica, R.I. - "Contribuţii privind îmbunătăţirea performanţelor reţelelor de medie tensiune având neutrul tratat cu bobină de stingere." Teză de doctorat, Timişoara,2002

[5.] Sora, C. - "Bazele electrotehnicii." Editura Didactică şi pedagogică, Bucureşti, 1982;