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FIRST ORDER CHEMICAL REACTION ON EXPONENTIALLY ACCELERATED ISOTHERMAL VERTICAL PLATE WITH MASS DIFFUSION

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ABSTRACT
Exact solution of unsteady flow past an exponentially accelerated infinite isothermal vertical plate has been presented in the presence of homogeneous chemical reaction of first order. The plate temperature is raised to T_w and species concentration level near the plate is also raised to C'_w. The dimensionless governing equations are solved using Laplace-transform technique. The velocity and concentration fields are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number, a and time. It is observed that the velocity increases with increasing values of a or t. But the trend is just reversed with respect to K
Keywords:
accelerated, isothermal, vertical plate, exponential, chemical reaction, homogeneous

1. INTRODUCTION

The Effect of a chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of the order n, if the reaction rate if proportional to the n^{th} power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself.

Chambre and Young[1] have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Das *et al*[2] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das *et al*[3]. The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level.

Gupta[4] studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis[5] extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar[6]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [7]. Basant Kumar Jha *et al* [8] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion.

It is proposed to study the effects of on flow past an exponentially accelerated infinite isothermal vertical plate in the presence of chemical reaction of first order. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

2. MATHEMATICAL ANALYSIS

Here the unsteady flow of a viscous incompressible fluid past an exponentially accelerated infinite isothermal vertical plate with uniform mass diffusion, in the presence of homogeneous chemical reaction of first order is studied. Consider the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞. The x'-axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to the plate. At time $t' \le 0$, the plate and fluid are at the same temperature T_∞. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane and the temperature from the plate is raised to T_w and and the concentration level near the plate are also raised to C'_w. It is also assumed that there exists first order chemical reaction between the fluid and the species concentration. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \quad (1) \qquad \rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} \quad (2) \qquad \frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - K_l C' \quad (3)$$

with the following initial and boundary conditions:

$$\begin{array}{lllll} & u = 0, & T = T_\infty, & C' = C'_\infty & \text{for all } y, t' \le 0 \\ t' > 0: & u = u_0 \exp(a't'), & T = T_w, & C' = C'_w & \text{at } y = 0 \\ & u \to 0 & T \to T_\infty, & C' \to C'_\infty & \text{as } y \to \infty \end{array} \quad (4)$$

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0}, t = \frac{t'u_0^2}{\nu}, Y = \frac{yu_0}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3},), C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gc = \frac{\nu g\beta^*(C'_w - C'_\infty)}{u_0^3}), \quad (5)$$

$Pr = \dfrac{\mu C_p}{k}, a = \dfrac{a'\nu}{u_0^2}, K = \dfrac{\nu K_l}{u_0^2}, Sc = \dfrac{\nu}{D}$ in equations (1-4), leads to $\dfrac{\partial U}{\partial t} = Gr\theta + Gc\,C + \dfrac{\partial^2 U}{\partial Y^2}$ (6);

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \ (7); \qquad \frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - K\,C \ (8).$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{lllll} & U = 0, & \theta = 0, & C = 0 & \text{for all } Y, t \le 0 \\ t > 0: & U = \exp(at), & \theta = 1, & C = 1 & \text{at } Y = 0 \\ & U \to 0, & \theta \to 0, & C \to 0 & \text{as } Y \to \infty \end{array} \quad (9)$$

3. METHOD OF SOLUTION

The solutions are obtained for hydrodynamic flow field in the presence of first order chemical reaction. The equations (6) to (8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\begin{aligned} U =\ & \frac{\exp(at)}{2}\left[\exp(2\eta\sqrt{at})\,erfc(\eta + \sqrt{at}) + \exp(-2\eta\sqrt{at})\,erfc(\eta - \sqrt{at})\right] \\ & - \frac{Grt}{(1 - Pr)}\left[(1 + 2\eta^2)\,erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right] + \frac{Gc}{c(1 - Sc)}\,erfc(\eta) \\ & - \frac{Gc\exp(ct)}{2c(1 - Sc)}\left[\exp(2\eta\sqrt{ct})\,erfc(\eta + \sqrt{ct}) + \exp(-2\eta\sqrt{ct})\,erfc(\eta - \sqrt{ct})\right] \\ & + \frac{Gr\,t}{(1 - Pr)}\left[(1 + 2\eta^2 Pr)\,erfc(\eta\sqrt{Pr}) - \frac{2\eta\sqrt{Pr}}{\sqrt{\pi}}\exp(-\eta^2 Pr)\right] \\ & - \frac{Gc}{2c(1 - Sc)}\left[\exp(2\eta\sqrt{KtSc})\,erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\,erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \\ & + \frac{Gc\exp(ct)}{2c(1 - Sc)}\left[\begin{array}{l}\exp(2\eta\sqrt{Sc(K + c)t})\,erfc(\eta\sqrt{Sc} + \sqrt{(K + c)t}) \\ + \exp(-2\eta\sqrt{Sc(K + c)t})\,erfc(\eta\sqrt{Sc} - \sqrt{(K + c)t})\end{array}\right] \end{aligned} \quad (10)$$

$$\theta = erfc(\eta\sqrt{Pr}) \quad (11);$$

$$C = \frac{1}{2}\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \quad (12)$$

where, $c = \frac{KSc}{1-Sc}$ and $\eta = \frac{Y}{2\sqrt{t}}$.

4. RESULTS AND DISCUSSIONS

For physical understanding of the problem numerical computations are carried out for different physical parameters a, K, Gr, Gc, Sc and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number Pr are chosen such that they represent air($Pr = 0.71$). The numerical values of the velocity are computed for different physical parameters like a, chemical reaction parameter, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The effect of velocity for different time ($t = 0.2, 0.4, 0.6, 0.8$), $K = 2, a = 0.5, Gr = Gc = 2$ are shown in Figure 1. In this case, the velocity increases gradually with respect to time t. Figure 2 illustrates the effect of velocity for different values of the chemical reaction parameter ($K = 0.2, 2, 5$), $Gr = 5, Gc = 10$ and $t = 0.2$. The trend shows that the velocity increases with decreasing chemical reaction parameter.

The velocity profiles for different ($a = 0, 0.2, 0.5, 0.8$) and $Gr = Gc = 2$ at $t = 0.2$ are studied and presented in Figure 3. It is observed that the velocity increases with increasing values of a. Figure 4. demonstrates the effect velocity fields for different thermal Grashof number ($Gr = 2, 5, 10$), mass Grashof number ($Gc = 5, 10$), $a = 0.5$ and $t = 0.2$. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

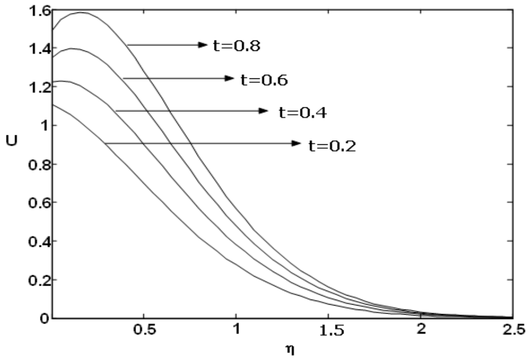

Figure 1. Velocity profiles for different values of t

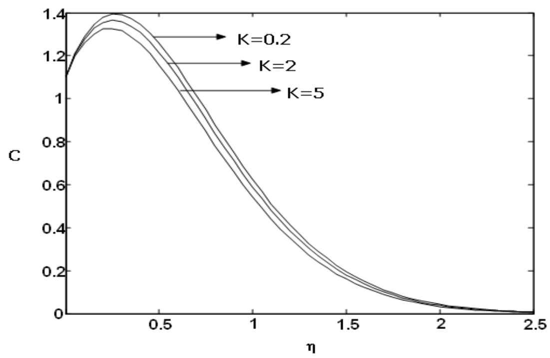

Figure 2. Velocity profiles for different values of K

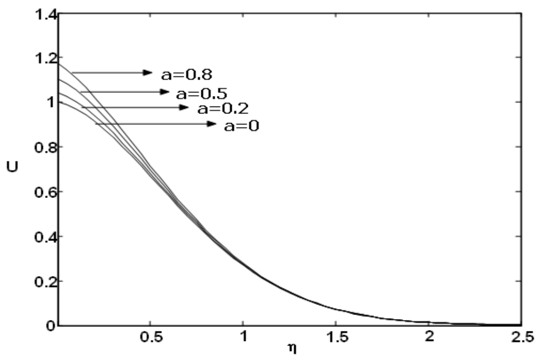

Figure 3. Velocity profiles for different values of a

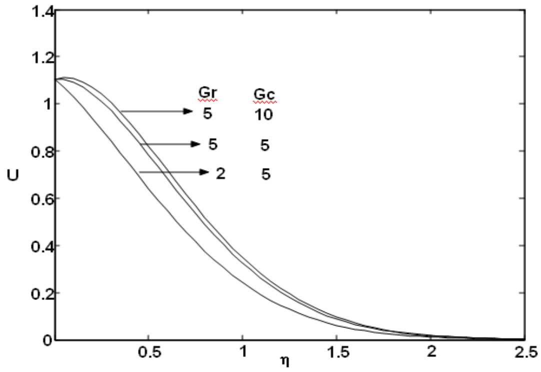

Figure 4. Velocity profiles for different values of Gr,Gc

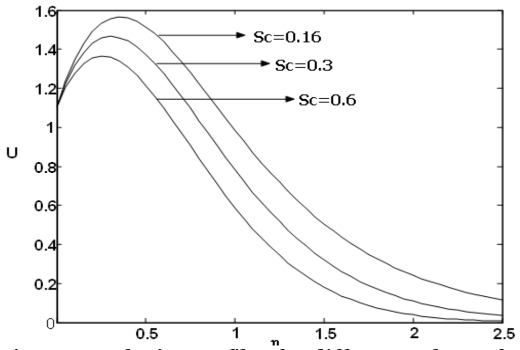

Figure 5. Velocity profiles for different values of Sc

Figure 5 represents the effect of velocity profiles for +different Schmidt number $(Sc = 0.16, 0.3, 0.6), Gr = 5, Gc = 10, a = 0.5, K = 0.2$ and $t = 0.2$. The trend shows that the velocity increases with decreasing Schmidt number or time. It is observed that the relative variation of the velocity with the magnitude of the time and the Schmidt number.

5. CONCLUSION

An exact analysis of mass transfer effects on unsteady flow past an exponentially accelerated infinite isothermal vertical plate in the presence of chemical reaction of first order. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like chemical reaction parameter, thermal Grashof number, mass Grashof number, a and t are studied graphically. The conclusions of the study are as follows:

1. The velocity decreases with increasing with decreasing values of the chemical reaction parameter K or Schmidt number Sc.
2. Velocity increases with increasing thermal Grashof number or mass Grashof number.
3. It is observed that the velocity increases with increasing values of a.

Nomenclature

C' concentration kg.m^{-3}
C dimensionless concentration
D mass diffusion coefficient
g acceleration due to gravity
Gr thermal Grashof number
Gc mass Grashof number
k thermal conductivity of the fluid W.m^{-1}.K^{-1}
Pr Prandtl number
q_r radiative heat flux in the y-direction
Kl chemical reaction parameter
K dimensionless chemical reaction parameter
Sc Schmidt number
T temperature K
t' time
t dimensionless time
u_0 amplitude of the oscillation
u velocity component in x-direction
U dimensionless velocity component in x-direction
x spatial coordinate along the plate
y spatial coordinate normal to the plate
Y dimensionless spatial coordinate normal to the plate

Greek Symbol

α thermal diffusivity
β coefficient of volume expansion
β^* volumetric coefficient of expansion with concentration
η similarity parameter
μ coefficient of viscosity
ν kinematic viscosity
ρ density of the fluid
ωt phase angle
σ electric conductivity
θ dimensionless temperature

Subscripts

ω conditions on the wall
∞ free stream conditions

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