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TRANSMISSION OF RADIO WAVE PULSE THROUGH AN ISOTROPIC IONOSPHERE

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ABSTRACT:
To describe the characteristics of a signal pulse and channel model for a radio wave communication through ionosphere, different electron density profiles for the ionospheric layer have been proposed earlier. In most cases the electron density variation was assumed to be abrupt. In this paper we consider a more general sech^2 profile for a horizontally stratified ionosphere. This profile allows completely analytic treatment, and hence we obtain expressions for transmission distance and rise time of a pulse propagating through ionosphere. The result shows that rise time can be a real measure of the degree of distortion of the received pulses.
KEYWORDS:
Electromagnetic waves, Ionosphere, Radio wave propagation.

1. INTRODUCTION

Research on propagation of electromagnetic waves through ionosphere is of great interest to communication researchers. Radio waves in the ionosphere are subject to some attenuation and distortion due to the presence of electrons and ions. The electrons being distributed irregularly within the ionosphere give rise to the reflecting property of the ionosphere. Hence the study of propagation of radio waves through the ionosphere necessitates the investigation of electron concentration variation within the ionosphere. Different electron density profiles for the ionospheric layer have been proposed earlier. In most cases the electron density transition was assumed to be abrupt i.e. a free space was assumed below the ionosphere. But in reality, electron concentration has a gradual transition from the ground i.e. there are finite non zero electron concentration even at ground. Such a realistic sech^2 distribution profile can be used to study the accurate propagation characteristics of radio waves.

The present paper deals with the propagation of radio wave packets or pulses through isotropic, horizontally stratified ionosphere. We assume the ionosphere to be collisionless and ignored the effect of earth's magnetic field. Considering the sech^2 model for electron distribution, briefly we review in Section 2 the transmission distance of a radio wave packet for oblique incidence, following the presentation in [1]. In Section 3, we have considered an ionospheric transfer function and studied the shape of a radio wave pulse received at the receiver after reflection from the ionosphere. Here we have seen that for faithful reproduction of pulse, rise time should be less than the time period of the signal. In Section 4 we have discussed the variation of rise time as we increase the pulse frequency towards MUF. We find out explicit relations for rise time in terms of the parameters of sech^2 profile, following the steps given in [2]. We conclude our results in Section 5.

2. TRANSMISSION PATH AT OBLIQUE INCIDENCE

2.1 General Formulae

To find out an expression for transmission distance of a wave packet from the transmitter to the receiver after reflection at the ionosphere, a Cartesian coordinate system (x, y, z) is used with the directions x, y horizontal, and z vertically upward. The electron concentration N is assumed to vary along z direction only. Let the direction cosines along x, y, z are l_x, l_y, l_z respectively. We assume the incident wave as a plane wave travelling obliquely upward. When this wave propagates through a medium of varying electron concentration, we can write the wave normal or refractive index vector at each height of the medium as

$$\hat{\mu} = [l_x, l_y, q(z)] \tag{1}$$

where q(z) satisfies the Booker quartic equation. It is useful to imagine the ionosphere as a number of thin discrete homogeneous strata. Thus for a given plane wave in any one stratum, l_x and l_y do not change following Snell's law. For isotropic medium, q(z) can be expressed as

$$q^2 = \mu^2 - l_x^2 - l_y^2 \tag{2}$$

and the expression of phase path is

$$P = l_x x + l_y y + \int_0^z q\,dz \tag{3}$$

Thus the ray tracing equations are

$$x = l_x \int_0^z \frac{dz}{q} \quad , \quad y = l_y \int_0^z \frac{dz}{q} \tag{4}$$

Without any loss of generality we can assume that the wave is lying in the x-z plane and that the wave starts from a transmitter at origin (x, y, z) = (0, 0, 0). If the wave is incident obliquely making an angle θ with the vertical, we can write $l_x = \sin\theta$, $l_y = 0$. Thus equation (2) and (4) can be written in this case as

$$q^2 = \mu^2 - \sin^2\theta \tag{5}$$

$$x = \sin\theta \int_0^z \frac{dz}{q} \quad , \quad y = 0 \tag{6}$$

On the downward part of the ray after reflection at a point $z = z_r$, where q = 0, the ray remains in the x-z plane. So the horizontal distance after which the wave again reaches the ground is given by

$$D = \sin\theta \int_\Omega \frac{dz}{q} \tag{7}$$

where Ω is the contour for complex z which starts and end at z = 0 and circumvents the reflection point z_r.

2.2 Calculation for sech² distribution

A more realistic electron concentration profile, where the transitions are believed to be gradual, is the sech² distribution profile given as

$$N = N_m \operatorname{sech}^2\left[\frac{(h - h_m)}{a}\right] \tag{8}$$

where N_m is the maximum electron concentration at height $h = h_m$ above the ground and 'a' is the thickness of the layer. Here we changed the variable from z to h for our calculation purpose. Assuming $\xi = (h - h_m)/a$, we can have the refractive index expression for the profile as

$$\mu = \left[1 - \frac{f_p^2}{f^2}\operatorname{sech}^2(\xi)\right]^{1/2} \tag{9}$$

where f_p is called the penetration frequency. Now equation (5) can be written as

$$q^2 = \cos^2\theta - \frac{f_p^2}{f^2}\operatorname{sech}^2(\xi) \tag{10}$$

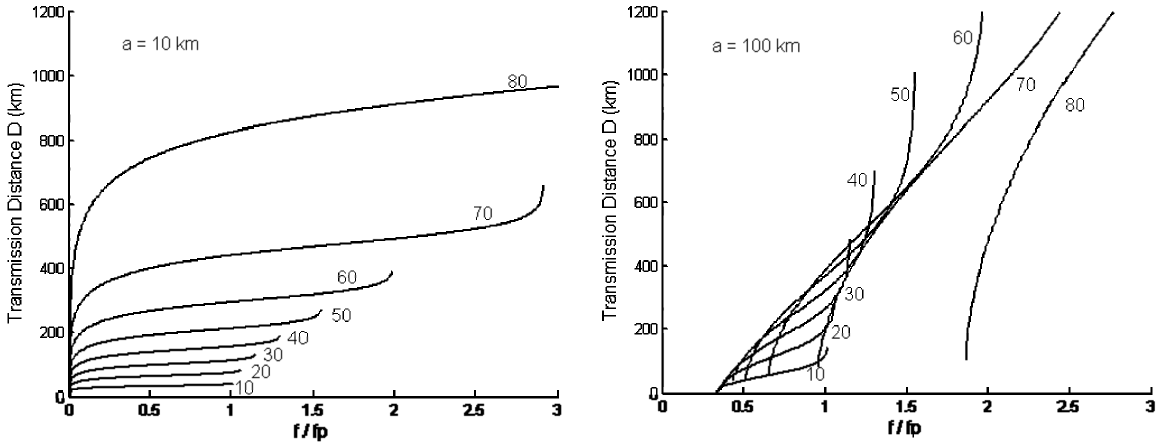
Since for this profile we have a nonzero electron concentration below the ionosphere, even at ground, we use equation (10) for the entire range in equation (6) and calculated the expression for x as

$$x = \frac{1}{2} a\tan\theta \left[\log\left(\frac{\cos\theta\sqrt{\frac{f_p^2}{f^2} - \cos^2\theta + q^2} + \frac{f_p}{f} q}{\cos\theta\sqrt{\frac{f_p^2}{f^2} - \cos^2\theta + q^2} - \frac{f_p}{f} q} \right) - \log\left(\frac{\cos\theta\sqrt{\frac{f_p^2}{f^2} - \cos^2\theta + q_0^2} + \frac{f_p}{f} q_0}{\cos\theta\sqrt{\frac{f_p^2}{f^2} - \cos^2\theta + q_0^2} - \frac{f_p}{f} q_0} \right) \right] \qquad (11)$$

where $q_0 = q(h=0) = \sqrt{\cos^2\theta - \frac{f_p^2}{f^2}\mathrm{sech}^2\left(\frac{h_m}{a}\right)}$

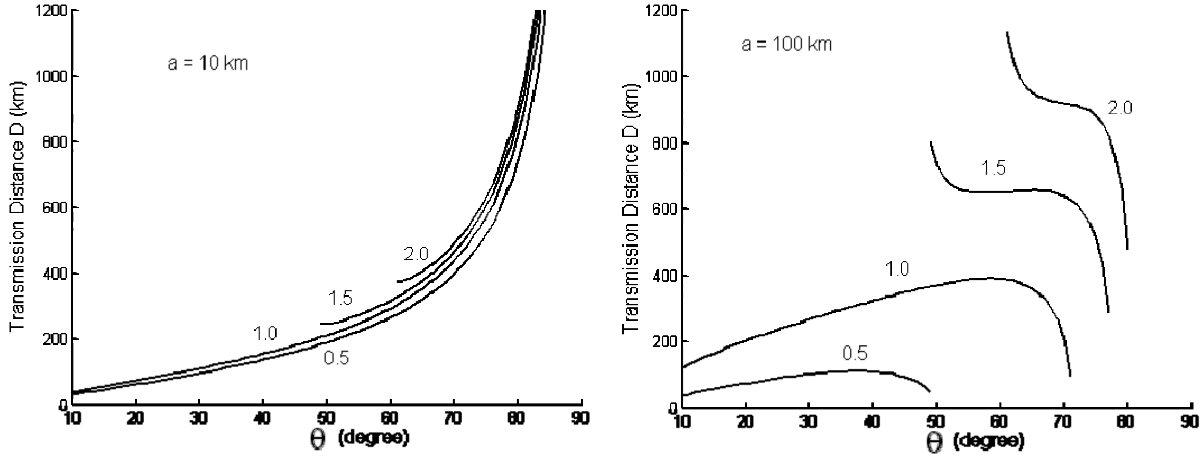
Now at the reflection point i.e. at the top of the trajectory $dx/dh \to \infty$ i.e. q = 0, and at that point x = ½ D. Thus from equation (11) we find

$$D = a\tan\theta \log\left[\frac{\frac{f}{f_p}\cos\theta\tanh\left(\frac{h_m}{a}\right) - \left(\frac{f^2}{f_p^2}\cos^2\theta - \mathrm{sech}^2\left(\frac{h_m}{a}\right)\right)^{1/2}}{\frac{f}{f_p}\cos\theta\tanh\left(\frac{h_m}{a}\right) + \left(\frac{f^2}{f_p^2}\cos^2\theta - \mathrm{sech}^2\left(\frac{h_m}{a}\right)\right)^{1/2}} \right] \qquad (12)$$



(a) (b)

Figure 1. Variation of transmission distance d (in km) against f / f_p for different values of θ (degree). (a) for a = 10 km; and (b) for a = 100 km



(a) (b)

Figure 2. Variation of transmission distance d (in km) against θ (degree) for different values of f / f_p. (a) for a = 10 km; (b) for a = 100 km

From equation (12) it is evident that D is a function of frequency f and angle of incidence θ. Figure 1(a) and (b) shows the variation of D with frequency ratio f / f_p for different angle of incidence. The curves are drawn for layer thickness, a = 10 km and 100 km, the highest value corresponding to normal F layer and lower value for E layer. We assume that the height of the lower layer of ionosphere from ground is 80 km. From Figure 1(a) it is clear that when $f <= f_p$, reflection from E layer occurs for all θ, but when $f > f_p$ then reflection will not be found for lower θ i.e. as we move towards the normal incidence. From the plot in Figure 1(b) it is evident that no F layer reflection occurs when $f << f_p$. The same conclusion can be drawn from the plots of D as a function of θ for various values of f / f_p, which is shown in Figure 2(a) and (b). From both the figures we also can say that as we go beyond the penetration frequency f_p, the possibility of reflection decreases, and for $f = 2f_p$ reflection will be possible for a short range of θ (~ 65° to 80°).

3. SHAPE OF THE RECEIVED PULSE

Let we consider a carrier $\cos(2\pi f_0 t)$ is amplitude modulated with pulse m(t) of period $T >> f_0^{-1}$. So the modulated signal is

$$S(t) = \mathrm{Re}\,[m(t)\exp(2\pi\, i f_0 t)] \tag{13}$$

In terms of Fourier transform expression the signals m(t) and S(t) are given by

$$m(t) = \int_{-\infty}^{\infty} M(f)\exp(2\pi i f t)\,df \tag{14}$$

$$S(t) = \mathrm{Re}\int_{-\infty}^{\infty} M(f-f_0)\exp(2\pi i f t)\,df \tag{15}$$

The location of the pulse at any time t can be expressed by the group path P' as

$$P' = c\,t(z) = \frac{d(fP)}{df} \tag{16}$$

Using equation (3) the equation (16) can be written as

$$P' = l_x x + l_y y + \int_0^z \frac{\partial(fP)}{\partial f}\,dz \tag{17}$$

Now after a propagation delay P/c the received signal expression is as follows

$$S_r(t) = \mathrm{Re}\int_{-\infty}^{\infty} M(f-f_0)\exp(2\pi i f t)\exp(-2\pi i f\, P/c)\,df \tag{18}$$

cf. equation (11.119) of [1].

Extracting the high frequency carrier term from equation (18) and introducing a new measure of time $\tau = t - P'/c$ and also substituting $\varphi = f - f_0$, the received pulse can be written as

$$m_r(\tau) = \int_{-\infty}^{\infty} M(\varphi)\exp\left[\frac{2\pi i}{c}\{f_0 P(f_0) - (\varphi+f_0)P(\varphi+f_0) + \phi P'(f_0)\}\right]\exp(2\pi i\varphi\tau)\,d\varphi \tag{19}$$

In another way, if we consider g(t) as an ionospheric transfer function, then the expression for received pulse can be found as

$$m_r(\tau) = \int_{-\infty}^{\infty} M(\varphi)G(\varphi)\exp(2\pi i\varphi\tau)\,d\varphi \tag{20}$$

where

$$g(t) = \int_{-\infty}^{\infty} G(\varphi)\exp(2\pi i\varphi t)\,d\varphi \tag{21}$$

Comparing equation (19) and (20), an expression for the transfer function is given as

$$G(\varphi) = \exp\left[\frac{2\pi i}{c}\{f_0 P(f_0) - (\varphi+f_0)P(\varphi+f_0) + \varphi P'(f_0)\}\right] \tag{22}$$

If it is assumed that f is closer to f_0 i.e. φ is small, then Taylor series expansion can be used in equation (22). Hence neglecting the higher order terms from the series and substituting $P_1^/ = dP^/\!/df$ & $P_2^/ = d^2P^/\!/df^2$ one can find

$$G(\varphi) = \exp\left[-\frac{\pi i}{c}(\varphi^2 P_1^/(f_0) + \frac{1}{3}\varphi^3 P_2^/(f_0)\right] \tag{23}$$

Two special cases are now considered:
(1) If φ is sufficiently small, the first term in equation (23) dominates over the second. In this case the transfer function is

$$G_1(\varphi) = \exp(-\frac{\pi i \varphi^2}{f_1^2}), \tag{24}$$

$$g_1(t) = \frac{1}{t_1}\exp(-\frac{1}{4}\pi i + \frac{\pi i t^2}{t_1^2}), \tag{25}$$

characterized by the time

$$t_1 = f_1^{-1} = \left(P_1^/ / c\right)^{1/2} \tag{26}$$

Hence the propagation channel acts like a low pass filter with cut-off frequency f_1 or rise time t_1.
(2) When the term $P_2^/$ dominates over the first term, the transfer function is

$$G_2(\varphi) = \exp\left(-\frac{\pi i \varphi^3}{f_2^3}\right), \tag{27}$$

$$g_2(t) = 2\left(\frac{\pi^2}{3}\right)^{1/3}\frac{1}{t_2}\mathrm{Ai}\left[-2\left(\frac{\pi^2}{3}\right)^{1/3}\frac{t}{t_2}\right] \tag{28}$$

So, now the channel is characterized by the time

$$t_2 = f_2^{-1} = \left(P_2^/ / c\right)^{1/3} \tag{29}$$

Hence the propagation channel acts like a low pass filter with cut-off frequency f_2 or rise time t_2. In this case the transfer function is represented by Airy function. If the rise time tends to zero, it can be found from equation (21) that the transfer function tends to a delta function. Thus we can conclude that, for zero rise time the received pulse will be undistorted. So smaller rise time gives better reproduction of the received pulse.

To verify this statement, we analyze both cases stated above for a triangular pulse of the form:

$$\begin{aligned} m(t) &= A\left(1+\frac{2}{T}t\right) \qquad -T/2 \le t \le 0 \\ &= A\left(1-\frac{2}{T}t\right) \qquad 0 \le t \le T/2 \end{aligned} \tag{30}$$

We obtain explicit expressions for the received pulse $m_r(\tau)$ using the equation (20). The expressions are not given here but the results are shown in Figure 3, which illustrates the absolute value of received pulse when m(t) is convoluted with the transfer function $g_1(t)$, (left panel) for different values of t_1 / T . The same pulse when convoluted with $g_2(t)$ are shown in right panel of Figure 3 for different values of t_2 / T. From these illustrations, it is apparent that the pulse will be clearly distinguishable if the rise time for the first case $|t_1| \le T / 2$ and for the second case also $|t_2| \le T / 2$. The consequences of this result are that firstly, smaller rise time gives better reproduction of the signal and secondly, as we increase the frequency of the transmitted signal rise time will decrease.

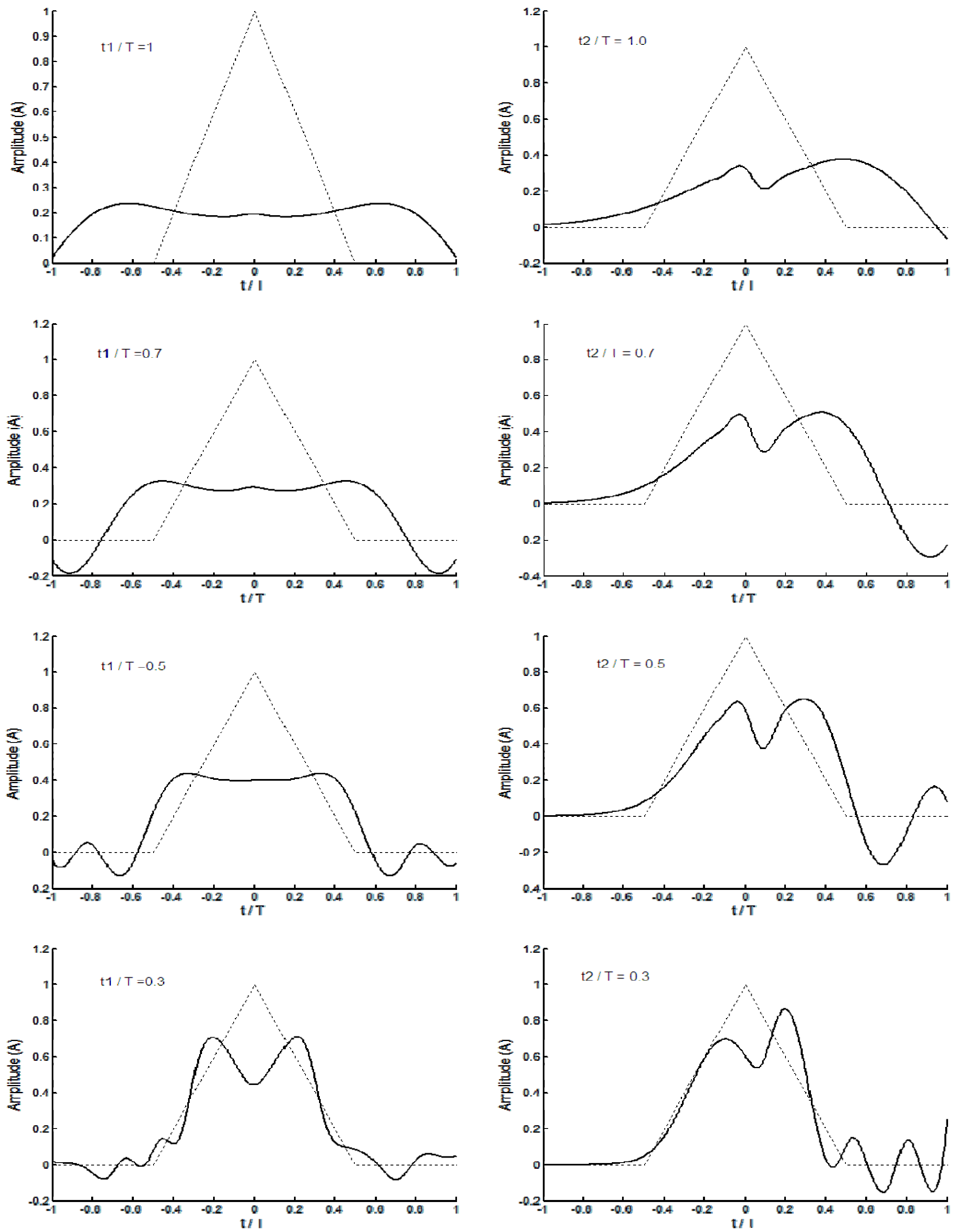


Figure 3. The dotted curves are for initial triangular pulse m(t) of period t and the solid lines are the real part of the received pulse $m_r(t)$. Left panel shows the received pulse for transfer function $g_1(t)$ for a number of values of t_1 / t. Right panel shows the received pulse for transfer function $g_2(t)$ for a number of values of t_2 / t

4. ESTIMATION OF RISE TIME

In this section we use our results of Section 2 for sech² profile to find out the variation of rise time with frequency of transmission. First we find out explicit expressions for $P_1^{/}$ and

$P_2^{/}$ considering the sech² profile, then we use these expressions in the formula given below to calculate the rise time. We choose the same coordinate system and same direction of propagation as described in Section 2.

Using the equations (6) & (7) the equation (17) for group path at the receiver can be written as

$$P^{/} = D\sin\theta + \cos^2\theta\int_{\Omega}\frac{dz}{q} \tag{31}$$

Now using the equation (10) for sech² profile we verified that this profile satisfies the Breit and Tuve's theorem

$$P^{/} = D/\sin\theta \tag{32}$$

Using equation (32) and the relations $P_1^{/} = dP^{/}/df$ & $P_2^{/} = d^2P^{/}/df^2$ one can finds that $P_1^{/}$ and $P_2^{/}$ can be written as

$$P_1^{/} = \frac{\cos^2\theta D\frac{\partial D}{\partial f}}{\sin\theta D - f\sin^3\theta\frac{\partial D}{\partial f}} \tag{33}$$

$$P_2^{/} = \frac{\cos^2\theta}{\sin\theta}\left[\frac{\partial^2 D}{\partial f^2} + 3\left(\frac{\sin\theta}{P^{/}}\frac{\partial D}{\partial f}\right)^2\left(D - f\frac{\partial D}{\partial f}\right)\right]\left[\frac{D}{D - f\sin^2\theta\frac{\partial D}{\partial f}}\right]^3 \tag{34}$$

cf. equation (43) and (51) of [2].

For our calculation of rise time we use a new variable $k = f/f_P$. Now from the equation (12) for D, we obtain the derivatives as follows

$$\frac{\partial D}{\partial k} = a\sin\theta\sinh\left(\frac{2h_m}{a}\right)\left[\frac{1-2\cos^2\theta k^2[k^2\cos^2\theta - \mathrm{sech}^2(\frac{h_m}{a})]^{-1/2}}{(1-k^2\cos^2\theta)}\right] \tag{35}$$

$$\frac{\partial^2 D}{\partial k^2} = a k\cos\theta\sin 2\theta\sinh\left(\frac{2h_m}{a}\right)\left[\begin{array}{l}1-[k^2\cos^2\theta - \mathrm{sech}^2(\frac{h_m}{a})]^{-1/2} - \\ [k^4\cos^4\theta - \mathrm{sech}^2(\frac{h_m}{a})][k^2\cos^2\theta - \mathrm{sech}^2(\frac{h_m}{a})]^{-3/2}\end{array}\right] \tag{36}$$

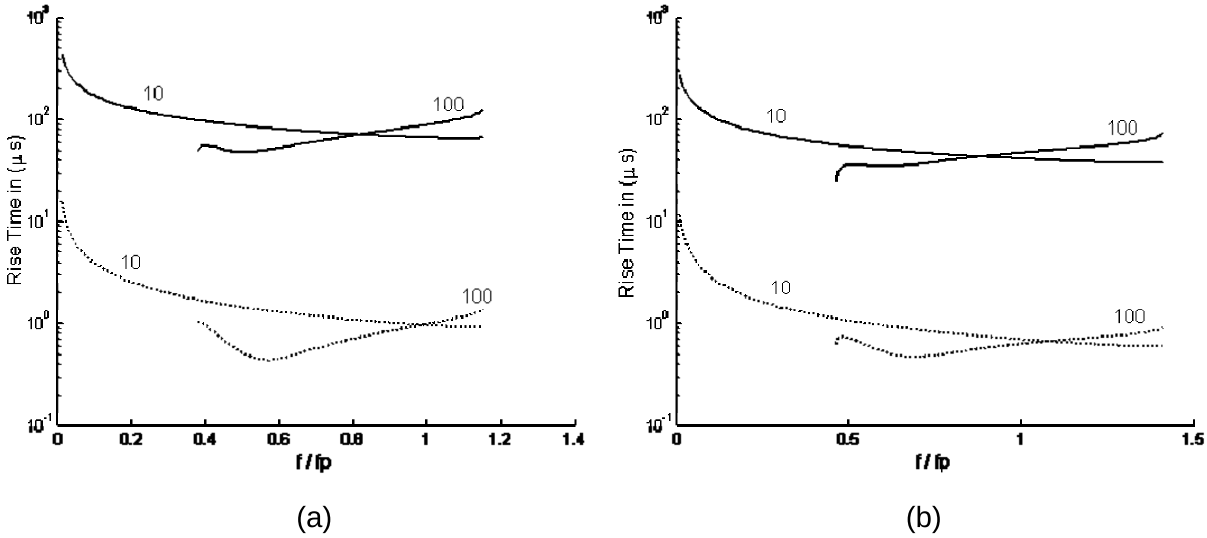


Figure 4. The absolute value of rise times (in μs): t_1 (solid line) and t_2 (dotted line) as a function of f / f_p for angle of incidence θ = 30° in fig (a) and 45° in fig (b) for sech² profile. The profile used here assumed to have fp = 5 MHz

Hence changing the variable f in equations (33) and (34) to k and then substituting equations (35) and (36) we find explicit relations for $P_1^{/}$ and $P_2^{/}$. Then from the relations (26) and (29), expressions for the rise time t_1 and t_2 are obtained in terms of the parameters of the $sech^2$ profile. These expressions are not given here but we confine ourselves to numerical solutions for those expressions.

Our results are shown in Figure 4 for two angles of incidence θ. In both cases we calculated rise times t_1 and t_2 in μs for a = 10 km and 100 km. as shown in figure. It is clear that variation of rise time with frequency is almost similar for different angle of incidence. The rise time t_2 is always smaller than t_1. Also both rise times for lower thickness (a = 10 km) compared to the higher one are larger in the low frequency range, and tends to infinity as frequency tends to zero. As we move towards the penetration frequency, rise times for lower thickness decreases below that for higher thickness. For a = 10 km, rise time decreases gradually as we increase f, i.e. quality of received pulse increases for increase in the frequency. But for a = 100 km a minima is clearly visible at $f \approx 0.6f_p$. Hence, if the ionosphere is characterized by the transfer function $g_2(t)$ and if the transmitted pulse frequency is close to $0.6f_p$ then we have a better reproduction of the pulse at the receiver. Discontinuities in the curves are clear as we move close to MUF. This problem arises since the ray theory becomes invalid at this limit and we need full wave theory to find out the rise time at that point.

5. CONCLUSION

The propagation path of a plane wave pulse through an isotropic ionosphere and the characteristics of a received pulse after reflection have been discussed by using a more realistic $sech^2$ electron density profile for the ionosphere. We briefly review the analytic solution for transmission distance of the pulse incident obliquely. It is shown that for $f \leq f_p$, reflection occurs for all θ, but when $f > f_p$ reflection will not be found as we move towards the normal incidence. No F layer reflection occurs when $f \ll f_p$. Also as we go beyond the penetration frequency f_p, the possibility of reflection decreases. As an example: for $f = 2f_p$ reflection will be possible for a short range of θ (~ 65° to 80°). Afterwards, we analyze a simple communication problem considering two specific channel transfer function and the result shows that smaller rise time gives better reproduction of the signal. Finally, an explicit relation of rise time is obtained using the $sech^2$ profile and it is shown that for thin layer, quality of received pulse increases with the increase of frequency. But for thick layer better reproduction will be possible around $0.6f_p$. To summarize, for realistic electron concentration variation, best suitable frequency for communication is around $0.6f_p$ for any angle of incidence. But, as we move towards / beyond penetration frequency, possibility of communication is limited for a short range of angle of incidence at the cost of quality degradation of the received pulse. The analytic solutions obtained in this paper are of importance to characterize transionospheric channel model for communications.

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