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MODELING OF THE ENERGY ENTITIES FUNCTIONING USING THE MARKOV CHAINS METHOD

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Abstract:
In this paper the authors present a functioning of the energy entities modelling method using Markov chains. It is modelled the functioning of an electric supply loop for a 0.4kV voltage consumer by successive reducing of the electric supply scheme and taking into consideration the two possible states for each loop element: failure and repair rates.
Keywords:
Markov chains, state transition rates matrix, failure rate, repair rate

1. INTRODUCTION

The Markov processes represent a particular case of the stochastic processes, whose characteristically property is that they have no memory. The final states of the systems that are modelled by Markov chains are connected only with the transition probability from an *i* state (at *t* time) to a *j* state (at *t+1* time) and they have no connection with the initial state. The evolution of a Markov process is influenced only by its current state. If a Markov process has a finite number of states, then it is a Markov chain.

In energy systems field this modelling method is used only when the independence of the containing elements assumption cannot be admitted.

In this paper the behaviour of the elements is modelled by an exponential distribution function of the stochastic variable, described by the relation:

$$f(t) = \lambda \cdot e^{-\lambda t} \quad , \tag{1}$$

The probability density is given by the relation:

$$F(t) = 1 - e^{-\lambda t} \quad , \tag{2}$$

2. PROBLEM FORMULATION

The evolution in time of an energy entity through the various states that can appear after the failure and repairing of the containing elements can be assimilated with a continuous time Markov chain [1]. The determining of the state probabilities for a given reference period is made by solving the differential equations system, written in the following matrix form:

$$P'(t) = P(t) \times Q \quad , \tag{3}$$

where

$P(t)$ represents the state transition functions matrix at t moment.

Q represents the state transition rates matrix.

The state transition functions p_{ij} are the elements of the state transition functions matrix. They depend only on the difference between two moments of time and they have the properties:

$$p_{ij} \in [0,1] \quad , \tag{4}$$

$$\sum_{j=1}^{m} p_{ij} = 1 \quad , \tag{5}$$

The state transition functions are used in directly form very rarely. A continuous time Markov chain is usually characterized by the state transition rates [2, 6]. In functioning of the energy entity modeling is using the state transition rates matrix $Q = [q_{ij}]$, $i, j = 1,2....$ with the following property:

$$\sum_{j=1}^{m} q_{ij} = 0 \qquad , \tag{6}$$

The Q matrix form is given by the relation:

$$Q = \begin{vmatrix} q_{11} & q_{21} & \cdots & q_{n1} \\ q_{12} & q_{22} & \cdots & q_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ q_{1n} & q_{2n} & \cdots & q_{nn} \end{vmatrix}, \tag{7}$$

The elements $q_{ij} \geq 0$ of the state transition state matrix Q indicate the probabilities that the system to pass from *i* state in *j* state, $i, j = \overline{1,n}$,where *n* is the number of the possible states for the studied installation. As it is shown in [1], is using the following notations for the reliability parameters in energy installations:

λ [h^{-1}] is the failure rate of an energy entity element

μ [h^{-1}] is the repair rate of an energy entity element

ν [h^{-1}] is the replacement rate of an energy entity element.

In situation when the system passing from *i* state in *j* state is made by failure of an element with λ failure rate, then $q_{ij} = \lambda$.

In situation when the system passing from *i* state in *j* state is made by repairing of an element with μ repair rate, then $q_{ij} = \mu$.

In situation when the system passing from *i* state in *j* state is made by repairing of an element with ν replacement rate, then $q_{ij} = \nu$.

Taking into consideration a system composed by *n* elements, each element having two possible states (failure and repairing), the total number of the possible states of the installation is 2^n. From here is resulting that to determine the state probabilities P_i, $i = \overline{1,n}$ is necessary to solve an 2^n equations system.

If the reference time is ample $(t \to \infty)$, the (1) differential equations system becomes an algebraic equations system written in the following form:

$$P \times Q = 0 \qquad , \tag{8}$$

The functioning of the electric supply scheme modelling implicates the reducing of its structure.

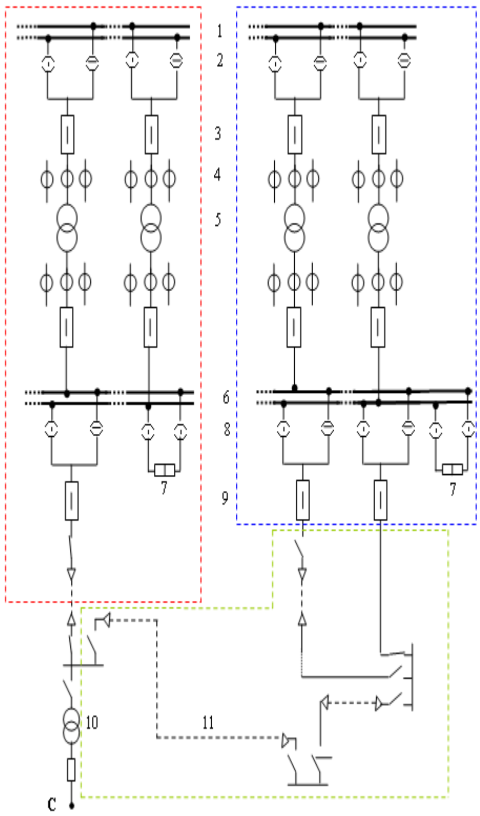

Fig.1. Electric supply scheme where C consumer is connected
Legend fig. 1: 1-110kV electric line, 2-110kV disconnector, 3-110kV circuit breaker, 4-110kV current transformers, 5-110/20kV power transformers, 6-20kV electric bar 7-20kV transversal couple, 8-20kV disconnector 9-20kV circuit breaker, 10-20/0,4kV power transformer 11-20kV underground cable

After successive reducing of the scheme in fig.1, the authors obtained the block diagram in fig.2. The situation is analysed taking into consideration the followings:

- the containing elements of the scheme can be failed only when the entity is functioning.
- the state with all three failed elements is not existing, because the element no.3 cannot fail if the elements no.1 and no.2 are failed, and also, the elements no.1 and no.2 cannot fail if the elements no.3 is failed.

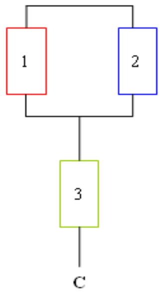

Fig.2. Block diagram obtained by reducing the electric scheme

Table 1

Possible states	Functioning elements	Failed elements
1	1, 2, 3	-
2	2, 3	1
3	1, 3	2
4	1, 2	3
5	3	2, 1
6	2	3, 1
7	1	2, 3

In table 3 are presented all possible states of the entity.

The set of a Markov chain possible states, together with the state transition state matrix Q are defining an oriented graph. The points of this oriented graph represent the system states. If the q_{ij} rate is different from 0 value, then it is existing an oriented arc from i state to j state [6]. The figure 3 represents the Markov chain transition graph for the studied entity.

Taking into consideration that the studied system has 7 different states and the Markov chain transition graph (fig.3), the Q matrix is given by the relation:

$$Q = \begin{vmatrix} -\lambda_1-\lambda_2-\lambda_3 & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_3 & 0 & 0 \\ \mu_1 & -\mu_1 & 0 & 0 & 0 & 0 & 0 \\ \mu_2 & 0 & -\mu_2-\lambda_1-\lambda_3 & 0 & \lambda_1 & 0 & \lambda_3 \\ \mu_3 & 0 & 0 & -\mu_3-\lambda_1-\lambda_2 & 0 & \lambda_1 & \lambda_2 \\ 0 & \mu_2 & \mu_1 & 0 & -\mu_1-\mu_2 & 0 & 0 \\ 0 & \mu_3 & 0 & \mu_1 & 0 & -\mu_1-\mu_3 & 0 \\ 0 & 0 & \mu_3 & \mu_2 & 0 & 0 & -\mu_2-\mu_3 \end{vmatrix} \quad (9)$$

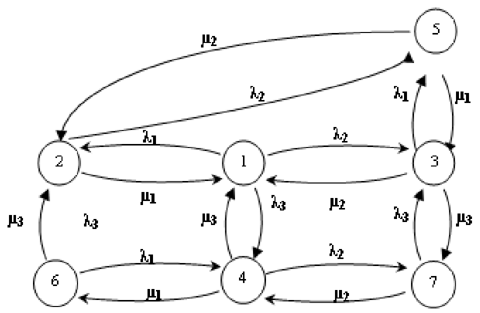

Fig.3. Markov chain transition graph

In order to solve the equation system (8), to avoid the unique banal solution, one of the system equations is replaced with the condition $\sum_{i=1}^{n} P_i = 1$ (the sum of the states probability on one line of the matrix is 1). It means that system states realise a complete events set.

The equation system (8) becomes:

$$\begin{cases} P_1 \cdot q_{11} + P_2 \cdot q_{21} + \ldots + P_7 \cdot q_{71} = 0 \\ P_1 \cdot q_{12} + P_2 \cdot q_{22} + \ldots + P_7 \cdot q_{72} = 0 \\ \ldots\ldots\ldots\ \ldots\ldots\ldots\ \ldots\ldots\ldots\ \ldots\ldots\ldots\ \ldots\ldots\ldots \\ P_1 \cdot q_{17} + P_2 \cdot q_{27} + \ldots + P_7 \cdot q_{77} = 0 \end{cases} \quad (10)$$

P_i represent the elements of the state transition functions matrix. The solution of the equations system (10) is the state vector $P = [P_1 \ldots P_7]$. Replacing the state transition rates q_{ij} with the failure and the repair rates, the equations system (10) becomes:

$$\begin{cases} -P_1(\lambda_1+\lambda_2+\lambda_3)+P_2\mu_1+P_3\mu_2+P_4\mu_3=0 \\ P_1\lambda_1-P_2\mu_1+P_5\mu_2+P_6\mu_3=0 \\ P_1\lambda_2-P_3(\mu_2+\lambda_1+\lambda_3)+P_5\mu_1+P_7\mu_3=0 \\ P_1\lambda_3-P_3(\mu_3+\lambda_1+\lambda_2)+P_6\mu_1+P_7\mu_2=0 \\ P_3\lambda_1-P_5(\mu_1+\mu_2)=0 \\ P_4\lambda_1-P_6(\mu_1+\mu_3)=0 \\ P_3\lambda_3+P_4\lambda_2-P_7(\mu_2+\mu_3)=0 \\ P_1+P_2+P_3+P_4+P_5+P_6+P_7=0 \end{cases} \quad (11)$$

3. PROBLEM SOLUTION

After the electric scheme from figure 1 is reduced and using the reliability parameters (failure rate λ and repair rate μ) for the containing components (electric bar, disconnectors, circuit breakers, transversal couples, current transformers, power transformers, underground lines) [1], it can be computed the reliability indicators from block diagram presented in figure 2.

Table 2

	Element 1	Element 2	Element 3
$\lambda[h^{-1}]$	$0.63 \cdot 10^{-4}$	$0.0126 \cdot 10^{-4}$	$0.045 \cdot 10^{-4}$
$\mu[h^{-1}]$	$129.94 \cdot 10^{-4}$	$140.52 \cdot 10^{-4}$	$699 \cdot 10^{-4}$

The values of the failure rate λ and repair rate μ for the element from figure 2 are presented in table 2.

4. OBTAINED RESULTS

The solution of the equation system (11) is the probability state vector $P=[P_1 \ldots P_7]$:

$$P_1 = 0.995;\ P_2 = 0.0048;\ P_3 = 0.0001135253;\ P_4 = 0.000064;\ P_5 = 0.000000432;\ P_6 = 0.00000031;\ P_7 = 0.000000005 \quad (12)$$

The probability state vector serves in determining the reliability indicators:

- success probability

$$P = \sum_{i \in S} P_i = 0.995 \quad (13)$$

- unsuccessful probability

$$Q = \sum_{i \in R} P_i = 1 - P = 0.005 \quad (14)$$

- medium success duration in 1 year (8760 h)

$$M[\alpha(T)] = P \times T = 8716.2h \quad (15)$$

- medium unsuccessful duration in 1 year (8760 h)

$$M[\beta(T)] = Q \times T = 43.8h \quad (16)$$

- failure total number in 1 year (8760 h)

$$M[\nu(T)] = \left[\sum_{i \in S} \left(P_i \times \sum_{j \in R} q_{ij} \right) \right] \times T = 394{,}17 \quad (17)$$

- medium functioning time

$$M[T_f] = \frac{M[\alpha(T)]}{M[\nu(T)]} = 22222h \quad (18)$$

- medium unfunctioning time

$$M[T_d] = \frac{M[\beta(T)]}{M[\nu(T)]} = 14{,}28h \quad (19)$$

In relations (12)-(19) there were noted: S-success states set, $P_7 = 0.000000005$ and R-unsuccessful states set.

5. CONCLUSION

The computing of the state probabilities based on solving the equation system (8) is useful in all cases when it's making a quantitative analysis of the energy installation reliability, indifferent if the containing elements of the system are dependent or independent. The advantages of the Markov chain modelling are:

- the energy system that is analysed can be modelled with dependent elements.
- It can be analysed each possible state of the system, using Markov chain transition graph, and the reliability parameters (λ, μ and ν)
- the resulting data, reliability indicators can give important informations about the system functioning.

This modelling method is limited in situations when implicates a lot of calculation, the reason for reducing the computing scheme. In situation when the results do not demand accuracy, it is indicated to use the binomial method, in witch the element can be considered independent.

REFERENCES:

[1.] ENEL Distribuţie Banat SA , „Lucrarea nr.216T/2002 – Alimentare cu energie electrică SC Alin Trans SRL Deva", 2002.
[2.] Iosifescu M., „Lanţuri Markov finite şi aplicaţii", Editura Tehnică Bucureşti 1977.
[3.] Osaci M., Panoiu M., Heput T., "Numerical Stochastic Model for the Magnetic Relaxation Time of the Fine Particle System with Dipolar Interaction", *Applied Mathematical Modelling*, vol.30, 2006, pag.545-553.
[4.] *** PE 013 „Prescripţie energetică privind metodele şi elementele de calcul al siguranţei în funcţionare a instalaţiilor energetice", ISPE Bucuresti 1994.
[5.] Panoiu M., Panoiu C., Iordan A., Rob R., "Simulation Results Regarding High Power Loads Balancing", 12th WSEAS International Conference of SYSTEMS, *New Aspects of SYSTEM, 2008*, pag.614-619.
[6.] Petrişor E., „Simulare Monte Carlo", Editura Politehnica Timişoara 2006.
[7.] Rafiroiu M., „Modele de simulare în construcţii", Editura Facla Timişoara 1982.
[8.] Săcuiu I., Zorilescu D., „Numere aleatoare", Editura Academiei 1978.