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SOME DISADVANTAGES OF STANDARD BUCKLING ANALYSIS COMPARING TO INCREMENTAL GEOMETRIC NONLINEAR ANALYSIS

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Abstract:

Modeling of real behavior of structural systems requires complex assumptions, which have, as consequences, nonlinear stress-strain state. One of the causes of nonlinear behavior is geometric nonlinearity.
Determining of critical load parameter by standard bifurcation buckling solution is appropriate for solving stability problems if the longitudinal and lateral load ratio is small. In case that structure has large displacements, it is necessary to use incremental-iterative solution. The main advantage of this concept is possibility to verify changes in: loads, mechanical-reological properties of material, geometry of structural system, and modeling effect of those changes on structural behavior.

Keywords:

finite element method, geometric nonlinearity, buckling analysis

1. INTRODUCTION

A FEM linear model in some cases of numerical analysis of structural systems is not appropriate. Assumptions in linear analysis are simple and results may vary of close solution. Using simple FEM models, it is possible to achieve efficiency of analyses and calculations, but without adequate accuracy. Also, bearing capacity, adaptability and durability of structural system can be endangering.

For modelling of real behaviour of structural system nonlinear stress-strain relation should be taken into consideration. The nonlinear analysis is more complex than linear analysis because:

- during the load process deformations are not proportional,
- after the effect of load, model does not have original form,
- for finite values of displacements, deformations and stresses principle of superposition can not be applied and
- the stiffness matrix and load vector of loading can not be fully formed due to the fact that they depending on the final solution.

One of the causes of nonlinear behaviour is geometric nonlinearity, where relation between strains and displacement is nonlinear, but material has linear elastic properties. Geometric nonlinearity arises when deformations and/or displacements are large enough to significantly change geometry and position of the system. As a consequence of large deformation and/or displacement, relation deformation-displacement and equilibrium are nonlinear. Reaching the limit state in the geometric nonlinearity is loss of stability of structural system.

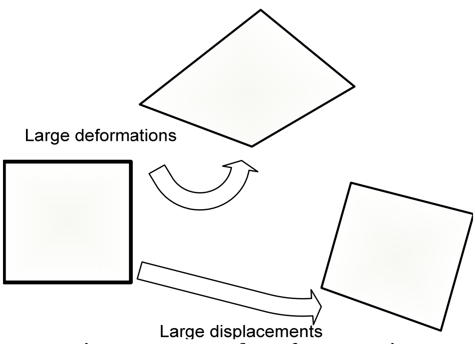

Figure 1. Examples of geometric nonlinearity: large displacements and large deformations

Problems of geometric nonlinearity can be classified as continual nonlinearity (smooth nonlinearities). The characteristic examples of smooth nonlinearity are nonlinear behavior due to large displacements and/or deformations, elasto-plastic material properties, reological properties of material, etc. Continual "smooth" functions are used for approximation of continual nonlinearity.

Figure 1 show examples of large deformations and large displacements. For large deformations, changes in shape are significant and for large displacements, changes in translational and/or rotational position are significant. Procedures for solving such problems are numerous and implemented in the general computer FEM software.

2. FEM MODELING OF GEOMETRIC NONLINEAR BEHAVIOR

For analyzing of geometric nonlinear phenomenon follow theories can be applied:

- general geometric nonlinear theory,
- geometric nonlinear theory in strict sense (second-order theory),
- "linearized" second-order theory and
- so-called "P-Δ" methods.

The greatest accuracy of numerical solution can be achieved using general nonlinear theory and the lowest accuracy is obtained by applying "P-Δ" methods.

Geometric nonlinear theory in strict sense assumed linear relations between displacements and deformations on deformed model of structures. This model is simple and because of quality approximation it can be used in modeling most of structural problems.

According to "linearized" second-order theory equilibrium equations are linear. The widest application of this theory is in analyzing of so-called "bifurcational stability" of construction although in some cases solution is not adequate. The main disadvantage of this theory is analyzing of undeformed structure, what is particularly significant in problems of stability of structural systems with large displacements.

The main problem in geometric nonlinear analysis is testing stability of structural system, i.e. determining of critical load. The critical load depends on the system topology (Figure 2), which may occur following cases:

- due to increasing of load, stiffness of system increases,
- after decreasing, stiffness of system is increasing, but it can be a point where buckling may occur and
- loss of stability is realized by suddenly transition to a new equilibrium branch ("snap-trough" effect).

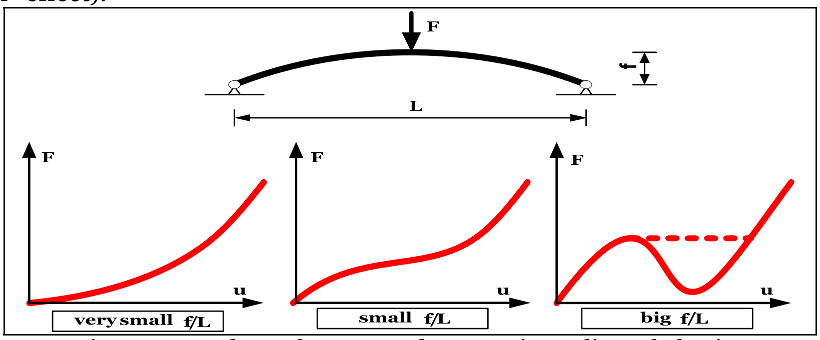

Figure 2. Topology of system and geometric nonlinear behavior

Determining of critical load parameter by standard bifurcation buckling solution is appropriate for solving stability problems if the longitudinal and lateral load ratio is small. In case that structure has large displacements, it is necessary to use incremental-iterative solution.

2.1. Linear buckling analysis

Verification of stability in geometric nonlinear analysis is determining of critical load for which the tangent stiffness matrix of FEM model becomes singular. The critical load is obtained by solving homogeneous problem according to "linearized" second-order theory:

$$\left([K_0]+\lambda[K_G]\right)=0 \qquad (1)$$

where:
$[K_o]$ – linear stiffness matrix,
$[K_G]$ – geometric stiffness matrix and
λ - factor of critical load.

Linear buckling analysis uses K_o and K_G based on the undeformed geometry of the structure, which is the main disadvantage of this theory. Because most buckling problems are nonlinear, the buckling analysis should be based on the tangent stiffness and incremental methods.

2.2. Incremental geometric nonlinear analysis

One of the methods for solving nonlinear problems is incremental methods. They use the tangent stiffness, which for single degree of freedom problem is the slope of the load versus displacement, $k_t=dF/du$. Tangent stiffness matrix is obtained as the sum of linear and geometric stiffness matrix of FEM model. Incremental methods are based on the approximation of total load on a range of smaller part – increments. The incremental loads are added successively and in each increment they are linear. The solution of nonlinear problem is obtained as sum of all linear incremental solutions. Better quality of approximations can be achieved using a number of increments, but numerical efficiency can be reduced.

In general case, nonlinear problem can be presented as:

$$[K_t]\{\Delta u\}+\lambda\{F\}=0 \tag{2}$$

$$\{P\}+\lambda\{F\}=0 \tag{3}$$

where: {P} – vector of generalized forces of FEM model and {F} – vector of load.

Incremental vector of displacements is obtained:

$$\{\Delta u\}_i = [K_t]^{1}\Delta\lambda_i\{F\} \tag{4}$$

where:

$$\{\Delta u\}_i = \{u\}_{i+1}-\{u\}_i \tag{5}$$

$$\{\Delta F\}_i = \{F\}_{i+1}-\{F\}_i \tag{6}$$

$$\Delta\lambda_i = \lambda_{i+1}-\lambda_i \tag{7}$$

The tangent matrix is formulated for the beginning of increment, and for the first increment is used linear matrix stiffness.

The incremental solution error is appearing because of linearization in each increment and it can be corrected by applying of some iterative procedures. The main advantages of incremental concept are step-by-step procedure which corresponds to the basic principles of FEM and possibility to verify changes in: loads, mechanical-reological properties of material, geometry of structural system, and modeling effect of those changes on structural behavior.

3. NUMERICAL EXAMPLES

As an illustration of previous consideration several numerical tests will be given. All examples are based on geometric nonlinear theory and "linearized" second-order theory. Figure 3 show FEM model of cantilever with rotation of fixed support.

For $\Delta\varphi=+45^o$ node 2 has displacements and according to "linearized" second-order theory it is: "v=L·Δφ" and "u=0". According to incremental solution the high solution accuracy is achieved: "v=L·sinΔφ" and "u=L-(cosΔφ-1)".

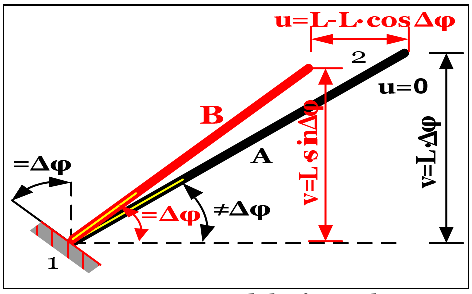

Figure 3. FEM model of cantilever - "linearized" second-order theory (A) and geometric nonlinear theory (B)

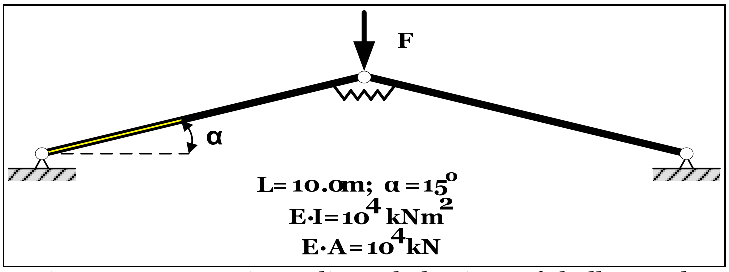

Figure 4. Geometric nonlinear behaviour of shallow arch

In the next numerical test (Figure 4) shallow arch is analyzed. For iterative correction of incremental solution three methods were analyzed: Newton-Raphson iteration (NR), modified Newton-Raphson iteration (MNR) and method of initial stiffness (MIS). The results are shown in Table 1.

Table 1. Load F for different methods of analysis

Load F (kN)	20		60		Fcr (kN)	
Fixed support ratio (%)	100	0	100	0	100	0
Linear theory	10.53	14.93	31.59	44.78	/	/
"Linearized" second-order theory	10.88	15.78	34.96	53.41	587.2	346.8
MIS	10.98	16.33	36.45	67.61	149.3	69.12
MNR	10.98	16.33	36.44	67.61	149.2	69.10
NR	10.98	16.33	36.44	67.65	149.2	69.10
Exact solution	10.99	16.35	36.55	68.60	149.1	69.09

The differences in solution (up to 25%) occur because of changes of tangent stiffness matrix due to correction of geometry system. Incremental-iterative solution with 20 increments provides greater accuracy than "linearized" second-order theory. Advantages of incremental solution occur when the ratio of critical load and applied load is high. Critical load is 69.09kN for fixed support ratio of 0% and

149.1kN for ratio of 100%. According to "linearized" second-order theory Fcr=346.75kN for 0% and Fcr=587.2kN for 100% ratio, which is too high error.

Third numerical test illustrate disadvantage of "linearized" second-order theory. Figure 5 show FEM model of frame with rigid and hinge interface condition between beam and column. The beam has 2I240 and column has I240 cross section. The material is assumed to remain linear elastic at all times, with E=210GPa. It will be analyzed critical load of a FEM model for "linearized" second-order theory (*) and for geometric nonlinear theory (**).

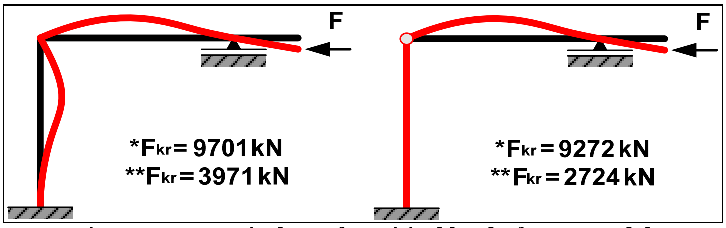

Figure 5. Numerical test for critical load of FEM model

The results of stability analysis of FEM models shows a difference in the buckling forms, as well as the value of critical force (Δ=244% for FEM model with rigid interface conditions and Δ**=340% for FEM model with hinge interface condition).

Next numerical test, Figure 6, show FEM model of frame structure loaded by two vertical forces. On response diagram "load-displacement", Figure 7, load parameter Ldp=1 is the value of critical load when the structure, after hardening, loses stability. Response diagram illustrate linear relation between load and displacement up to ~Pcr/2, which indicates an error if linear model is applied.

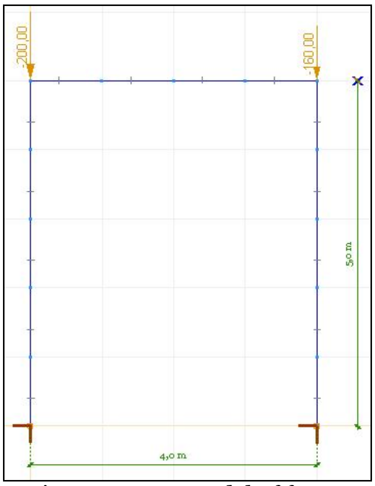

Figure 6. FEM model of frame structure

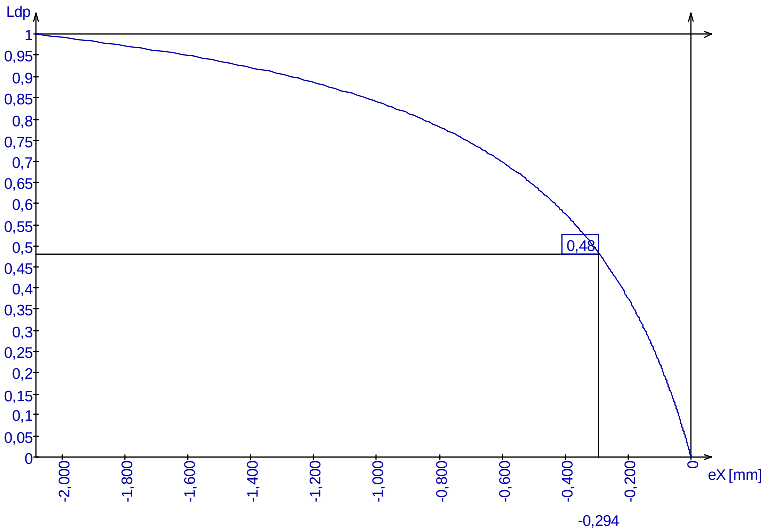

Figure 7. FEM model and response diagram of frame structure

4. CONCLUSIONS

In this paper geometric nonlinearity problems were analyzed. Determining of critical load and stability verification of the structure through the different concepts were presented. For adequate FEM numerical analysis of structural system advantages and disadvantages of implemented mathematical models must be emphasized, because the results can be different.

This paper emphasized advantages of incremental-iterative concept comparing to standard buckling analysis. The main disadvantage of incremental concept for adequate approximation is determining the size and number of increments. Difficulties to determine the solution exactly without analytical or experimental solutions is also disadvantage of incremental concept.

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