ANNALS of Faculty Engineering Hunedoara – International Journal of Engineering
ISSN: 1584 - 2665

EFFECT OF TRANSVERSE SURFACE ROUGHNESS ON THE PERFORMANCE OF A CIRCULAR STEP BEARING LUBRICATED WITH A MAGNETIC FLUID

Rakesh M. PATEL[1], G. M. DEHERI[2], H. C. PATEL[3]

[1.] Department of Mathematics, Gujarat Arts and Science College, Gujarat State, INDIA
[2.] Department of Mathematics, Sardar Patel University, Gujarat State, INDIA
[3.] Department of Mathematics, Government Engineering College, Patan, Gujarat State, INDIA

ABSTRACT:
An attempt has been made to study and analyze the performance of a magnetic fluid based transversely rough circular step bearing. The associated Reynolds' equation is stochastically averaged with respect to the random roughness parameter. This averaged Reynolds' equation is then solved with suitable boundary conditions to obtain the pressure distribution. This is then used to calculate the load carrying capacity leading to the computation of response time. Results are presented graphically. This investigation suggests that the performance of the bearing system registers an enhancement due to the presence of magnetic fluid lubricant. It is observed that the radii ratio has a moderately adverse effect on the performance of the bearing system. It is seen that the effect of transverse roughness is adverse in general. However, negatively skewed roughness tends to improve the performance of the bearing system. This positive effect gets further enhanced especially when negative variance is involved. In addition, this study makes it clear that the performance of the bearing system can be improved substantially by choosing suitable values of the magnetization parameter and radii ratio. Besides, the bearing with magnetic field can support a load even when there is no flow.
Keywords: Circular step bearing, magnetic fluid, roughness, Reynolds equation, load carrying capacity

1. INTRUDUCTION

The performance of a thrust bearing was analyzed by Elwell and Sternlicht (1) theoretically as well as experimentally. Dowson (2) studied the inertia effects in hydrostatic thrust bearings. The behavior of an externally pressurized oil lubricated rectangular thrust bearing was subjected to investigation by Majumdar and Ghosh (3). Stiffness and damping characteristic of compensated thrust bearings were considered by Ghosh and Majumdar (4). Further, Majumdar (5) studied the performance of an oil lubricated circular step bearing. Lin (6) dealt with the performance of an externally pressurized circular step bearing lubricated with couple stress fluids.

Verma (7) and Bhat and Deheri (8) discussed the performance of a squeeze film behavior between porous annular plates and observed that its performance with magnetic fluid lubricant was much better than with conventional lubricant. Further, Bhat and Deheri (9) considered a magnetic fluid based squeeze film in curved porous circular plates. Deheri, Patel and Patel (10) studied the configuration of Majumdar (5) in the presence of a magnetic fluid lubricant by improving the analysis of Majumdar (5). At the same time Deheri and Patel (11) analyzed the performance of a magnetic fluid based squeeze film between porous circular disks with sealed boundary and concluded that by sealing the boundary and choosing a proper value of magnetization parameter the performance of the bearing system could be enhanced considerably.

Most of the theoretical studies of bearing lubrication have more or less explicitly assumed that the bearing surfaces can be represented by the smooth mathematical planes. However, it has been recognized that this might be an unrealistic assumption particularly, in bearings working with small film thicknesses. Various devices such as postulating a sinusoidal variation in film thickness (Burton (12)) have been introduced in order to seek more realistic representation of engineering rubbing surfaces. But this method is perhaps more appropriate in an analysis of the influence of waviness rather than roughness. Tzeng and Saibel (13) introduced stochastic concepts and succeeded in conducting an analysis of a two dimensional inclined slider bearing with one dimensional roughness in the direction transverse to the sliding direction. However, bearing surfaces having received some run in and wear seldom exhibit a type of roughness approximated by this model. The effect of surface roughness was studied by many investigators (Davies (14); Michell (15); Tonder (16); Christensen and Tonder ((17);(18);(19)); Berthe and Godet (20)). Christensen and Tonder ((17);(18);(19)) proposed a comprehensive general analysis both for transverse as well as longitudinal surface roughness. Christensen and Tonder's approach formed the basis of the analysis to study the effect of surface roughness in a number of investigations (Ting (21); Prakash and Tiwari (22); Prajapati ((23); (24)); Guha (25); Gupta and Deheri (26); Andharia, Gupta and Deheri ((27); (28)). Recently, Patel and Deheri (29) investigated the behavior of a magnetic fluid based squeeze film between porous circular plates with a concentric circular pocket. Lin and Chiang (30) investigated the effects of surface roughness and rotational inertia on the optimal stiffness of thrust bearings.

Here we propose to study the configuration of Deheri, Patel and Patel (10) by taking the bearing surfaces to be transversely rough.

2. ANALYSIS

Usually, in order to get the load capacity, flow requirement and frictional power loss the following assumptions are considered.

(i) The recess is sufficiently deep so that the pressure in it is uniform.

(ii) The bearing admits low rotational velocity and its effect are neglected for the pressure development.

The configuration of the bearing system is shown above; wherein, a thrust load w is applied and the bearing supports the load without metal to metal contact. The load w is supported by the fluid within the pocket and land. The fluid escapes radially through the restriction by a land or sill around the recess. A magnetic fluid based film is formed with film thickness h. We assume axially symmetric flow of magnetic fluid under an oblique magnetic field $\bar{H}$ whose magnitude H is a function of r vanishing at $r = r_i$ and r_o. The bearing surfaces are assumed to be transversely rough. The thickness $h(x)$ of the lubricant film is defined as

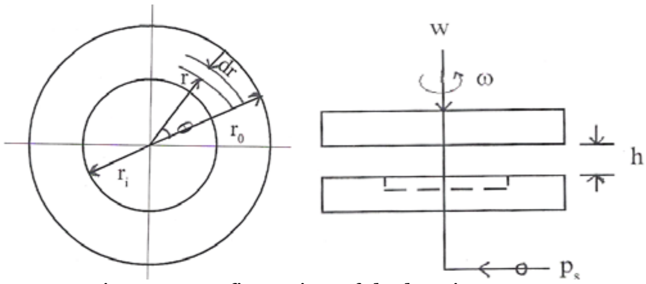

Figure 1. Configuration of the bearing system

$$h(x) = \bar{h}(x) + h_s$$

where $\bar{h}(x)$ is the mean film thickness and h_s is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. h_s is considered to be stochastic in nature and governed by the probability density function $f(h_s)$, $-c \le h_s \le c$ where c is the maximum deviation from the mean film thickness. The mean α, standard deviation σ and parameter ε which is the measure of symmetry of the random variable h_s are defined by the following relations;

$$\alpha = E(h_s),$$

$$\sigma^2 = E[(h_s - \alpha)^2]$$

and

$$\varepsilon = E[(h_s - \alpha)^3]$$

where E denotes the expected value defined by

$$E(R) = \int_{-c}^{c} Rf(h_s)dh_s$$

The associated Reynolds equation (Majumdar (5), Deheri, Patel and Patel (10)) gives the pressure induced flow for a circular step bearing as

$$Q = -\frac{h^3 + 3\sigma^2 h + 3h^2\alpha + 3h\alpha^2 + 3\sigma^2\alpha + \alpha^3 + \varepsilon}{12\eta} 2\pi r \frac{d}{dr}\left(p - 0.5\mu_0 \bar{\mu} H^2\right) \quad (1)$$

where μ_0 is the permeability of free space, $\bar{\mu}$ is magnetic susceptibility, η is the absolute viscosity of the fluid and

$$H^2 = (r - r_i)(r_o - r); \; r_i \le r \le r_o .$$

By making use of the boundary conditions

$$p(r_o) = 0 \text{ and } p(r_i) = p_s$$

one gets the governing equation for the film pressure p as

$$p = 0.5\mu_0\bar{\mu}H^2 + p_s \frac{\ln\left(\frac{r}{r_o}\right)}{\ln\left(\frac{r_i}{r_o}\right)} \tag{2}$$

wherein

$$p_s = \frac{6Q\eta}{\pi(h^3 + 3\sigma^2 h + 3h^2\alpha + 3h\alpha^2 + 3\sigma^2\alpha + \alpha^3 + \varepsilon)}\ln\left(\frac{r_o}{r_i}\right) \tag{3}$$

Introduction of the non-dimensional quantities

$$\mu^* = -\frac{h^3\mu_0\bar{\mu}}{\eta\dot{h}} \qquad R = \frac{r}{r_o} \qquad \alpha^* = \frac{\alpha}{h}$$

$$\sigma^* = \frac{\sigma}{h} \qquad \varepsilon^* = \frac{\varepsilon}{h^3} \qquad k = \frac{r_i}{r_o}$$

$$P_s = \frac{p_s}{1 + 3\sigma^{*2} + 3\alpha^* + 3\alpha^{*2} + 3\sigma^{*2}\alpha^* + \alpha^{*3} + \varepsilon^*}$$

paves the way for the expression of the dimensionless pressure

$$P = -\frac{h^3 p}{\eta\dot{h}\pi r_o^2} = \frac{\mu^*(R-k)(1-R)}{2\pi} + \frac{P_s\ln(R)}{\ln(k)} \tag{4}$$

The load carrying capacity obtained by integrating the pressure, takes the form

$$w = \frac{\pi\mu_0\bar{\mu}}{12}(r_o + r_i)(r_o - r_i)^3 + \frac{\pi p_s(r_o^2 - r_i^2)}{2\ln(r_o/r_i)} \tag{5}$$

Now the non-dimensional load carrying capacity turns out to be

$$W = -\frac{h^3 w}{\eta\dot{h}\pi^2 r_o^4} = \frac{\mu^*(1+k)(1-k)^3}{12\pi} + \frac{P_s(1-k^2)}{2\ln(1/k)} \tag{6}$$

In fine, the response time Δt, to reach a film thickness h_2 at t_2 starting from an initial film thickness h_1 at t_1, is expressed in non-dimensional form as

$$\Delta T = \frac{W\ h^2\ \Delta t}{\eta\pi^2 r_o^4} = -h^2\, W\int_{\bar{h}_1}^{\bar{h}_2}\frac{1}{\bar{h}^3 + 3\sigma^{*2}\bar{h} + 3\bar{h}^2\alpha^* + 3\bar{h}\alpha^{*2} + 3\sigma^{*2}\alpha^* + \alpha^{*3} + \varepsilon^*}d\bar{h} \tag{7}$$

where $\bar{h}1 = \frac{h_1}{h}$ and $\bar{h}2 = \frac{h_2}{h}$.

3. RESULTS AND DISCUSSION

Setting the roughness parameters to be zero this present study reduces to the analysis of Deheri Patel and Patel (10). Further, taking the magnetization parameter zero it leads to the study of Majumdar (5). Equations (4), (6) and (7) present respectively, the pressure distribution, load carrying capacity and the response time. It is clearly seen that these performance characteristics depend upon several parameters such as magnetization μ^*, standard deviation σ^*, variance α^*, skewness ε^* and radii ratio k. It is needless to say that these performance characteristic depend upon supply pressure also. It is observed that $w \propto p_s$ and

$$p_s \propto \frac{Q}{(h^3 + 3\sigma^2 h + 3h^2\alpha + 3h\alpha^2 + 3\sigma^2\alpha + \alpha^3 + \varepsilon)}.$$

Therefore, for a constant flow rate the load capacity increases as stochastically averaged film thickness decreases. Hence the bearing is self compensating provided the flow rate is treated as constant. Furthermore, from equations (4) and (6) it is established that the dimensionless pressure and the non-dimensional load carrying capacity get increased by

$\frac{\mu^*(R-k)(1-R)}{2\pi}$ and $\frac{\mu^*(1+k)(1-k)^3}{12\pi}$ respectively. In addition, the response time is increased by $\frac{\mu^*(1+k)(1-k)^3}{24\pi}$.

Thus, it is easily suggested that the magnetization parameter has an overall positive effect on the performance of the bearing system.

The variation of load carrying capacity with respect to magnetization parameter μ* is presented in Figures (1 - 4) for different values of standard deviation σ*, variance α*, measure of symmetry ε* and the radii ratio k respectively. It is seen that the standard deviation has a considerably adverse effect while the load carrying capacity increases due to negatively skewed roughness. Identical is the case of negative variance. These figures also indicate that the negative effect induced by the standard deviation and the radii ratio can be compensated to certain extent in the case of negatively skewed roughness when negative variance occurs.

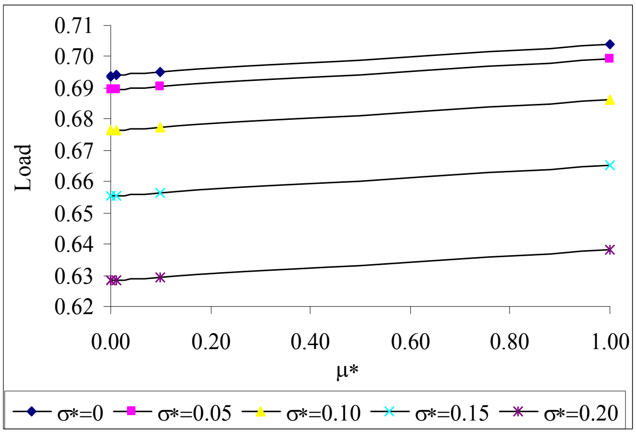

Figure: 1 Variation of load carrying capacity with respect to μ* and σ*

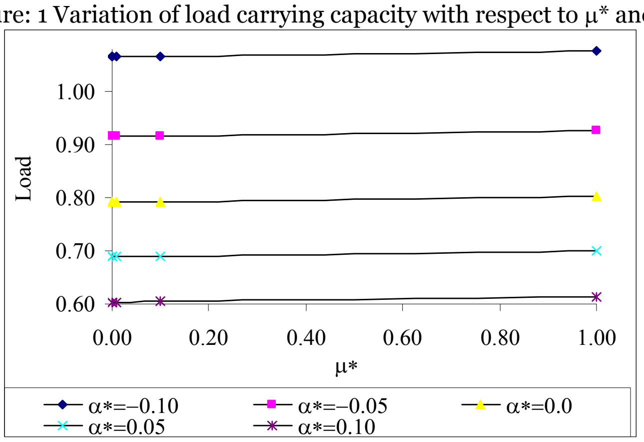

Figure: 2 Variation of load carrying capacity with respect to μ* and α*

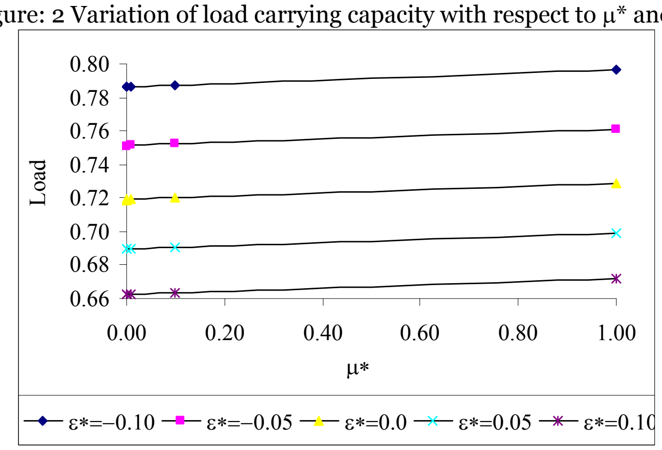

Figure: 3 Variation of load carrying capacity with respect to μ* and ε*

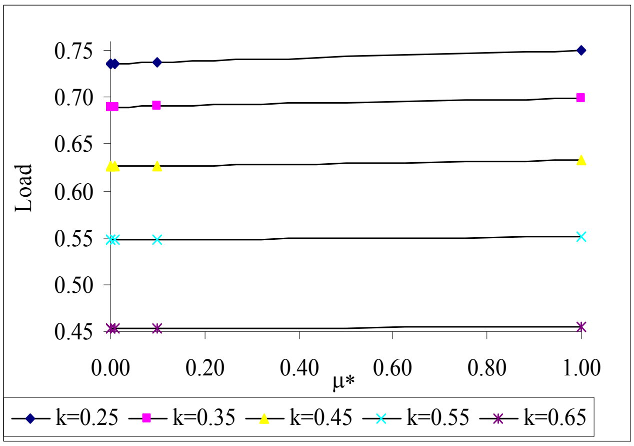

Figure: 4 Variation of load carrying capacity with respect to μ* and k

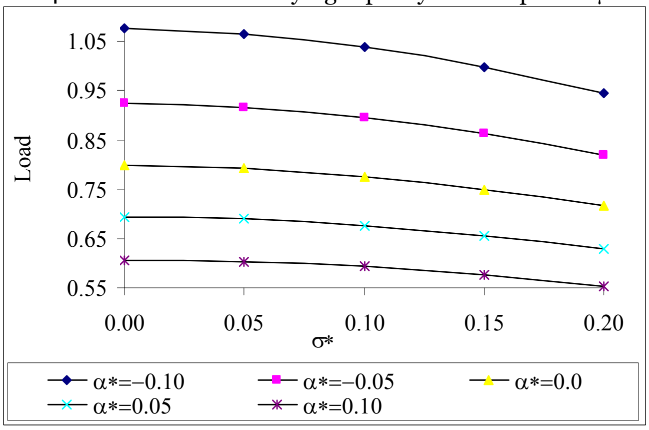

Figure: 5 Variation of load carrying capacity with respect to σ* and α*

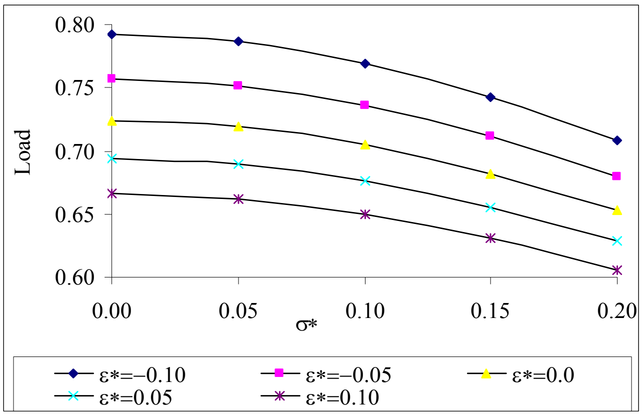

Figure: 6 Variation of load carrying capacity with respect to σ* and ε*

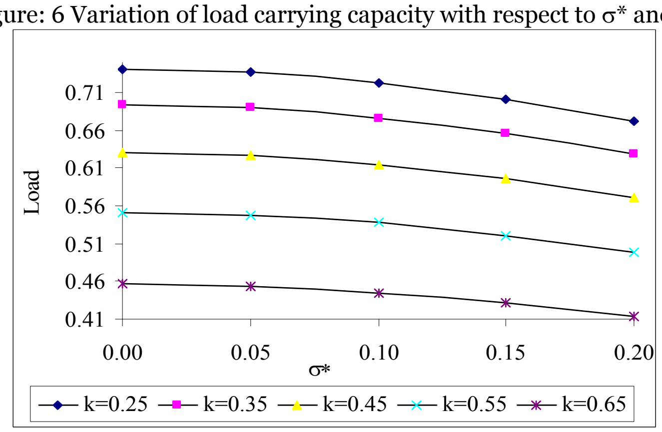

Figure: 7 Variation of load carrying capacity with respect to σ* and k

In Figures (5 - 7) we have the variation of the load carrying capacity with respect to the standard deviation associated with roughness for various values of α*, ε* and radii ratio k respectively. It is easily seen that positively skewed roughness and variance positive tend to decrease load carrying capacity substantially, which in turn, makes it clear that transverse roughness adversely affects the bearing system. This adverse effect is more pronounced when large values of radii ratio are involved.

Figures (8 - 9) describe the variation of load carrying capacity with respect to α* for various values of skewness and radii ratio respectively. It is clear from these figures that the bearing system registers an enhanced performance for a moderate value of radii ratio in the case of negatively skewed roughness which is seen from Figure: 10. Further, the combined effect of negatively sewed roughness and negative variance is considerably positive which goes a long way in minimizing the negative effect of the standard deviation and radii ratio.

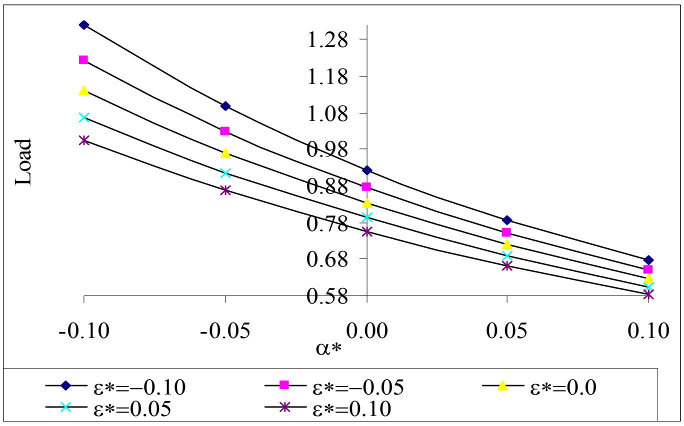

Figure: 8 Variation of load carrying capacity with respect to α* and ε*

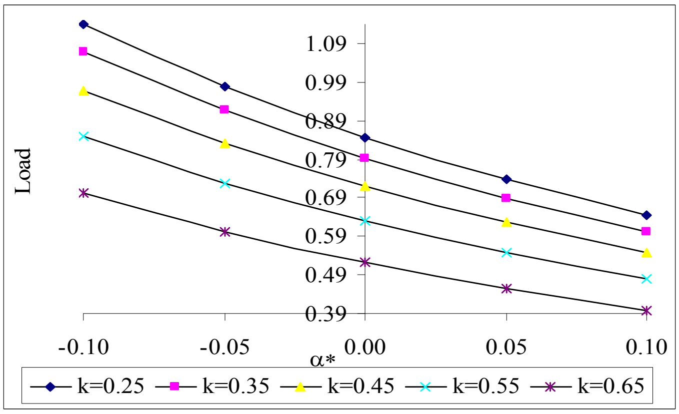

Figure: 9 Variation of load carrying capacity with respect to α* and k

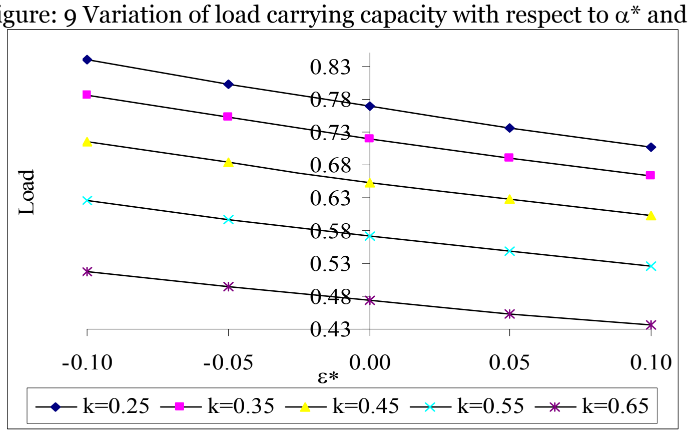

Figure: 10 Variation of load carrying capacity with respect to ε* and k

This investigation also underlines that the supply pressure may play a crucial role in augmenting the performance of the bearing system in the case of negatively skewed roughness when variance turns out to be negative. Therefore, this study makes it mandatory that the roughness must be give due respect while designing the bearing system especially, from longevity point of view.

Nomenclature

h	Uniform film thickness
k	Radii ratio
p	Pressure distribution
P	Dimensionless pressure
p_s	The Supply pressure
P_s	Dimensionless supply pressure
w	Load carrying capacity
W	Load carrying capacity in dimensionless form
Δt	Response time
ΔT	Dimensionless response time
σ	Standard deviation
α	Variance
ε	Measure of symmetry
σ^*	Standard deviation in dimensionless form
α^*	Non dimensional variance
ε^*	Dimensionless measure of symmetry

ACKNOWLEDGEMENT
Two of the authors, R. M. Patel and G. M. Deheri thank UGC for the funding of U. G. C. major research project (U. G. C. F. No. 32-143/2006 (SR) – "Magnetic fluid based rough bearings") under which this study has been conducted.

REFERENCES

[1] *Elwell, R.C. and Sternlicht, B., Theoretical and experimental analysis of hydrostatic thrust bearings. J.Basic Engg., Trans.ASME, D, 82, 1960, 505-512.*
[2] *Dowson, D., Inertia effects in hydrostatic thrust bearings, J. Basic Engg., Trans. ASME, D, 83, 1961, 227.*
[3] *Majumdar, B.C. and Ghosh, B., Load and flow parameters of externally pressurized oil-lubricated rectangular thrust bearings, M.E. Division, The Instn. of Engrs. (1), 50, 1970, 253-257.*
[4] *Ghosh, M.K. and Majumdar, B.C., Dynamic stiffness and damping characteristics of compensated hydrostatic thrust bearings, J.Lub. Tech., Trans. ASME, F, 104, 1982, 491-496.*
[5] *Majumdar, B.C., Introduction to Tribology of Bearings. Wheeler Publisher, Wheeler Co. Ltd., 1985.*
[6] *Lin, J.R., Static and Dynamic Characteristics of Externally Pressurized Circular Step Thrust Bearings Lubricated with Couple Stress Fluids, Tribology International, 32, 1999, 207-216.(SCI)(NSC-88-2212-E-0253-001 V).*
[7] *Verma, P. D. S., Magnetic fluid based squeeze film, International Journal of Engineering Sciences, 24, 1986, 395-401.*
[8] *Bhat, M. V., Deheri, G. M., Squeeze film behavior in porous annular disks lubricated with magnetic fluid, Wear, 151, 1991, 123-128.*
[9] *Bhat, M. V., Deheri, G. M., Magnetic fluid based squeeze film in curved porous circular disks, Journal of Magnetism and Magnetic Material, 127, 1993, 159-162.*
[10] *Deheri, G. M., Patel, H. C. and Patel, R. M., Performance of magnetic fluid based circular step bearings, MECHANIKA, 1(57), 2006, 22-27.*
[11] *Deheri, G. M. and Patel, R. M., Squeeze film based magnetic fluid in between porous circular disks with sealed boundary, Int. J. of Applied Mechanics and Engineering, 11(4), 2006, 803-812.*
[12] *Burton R.A., Effect of two dimensional sinusoidal roughness on the load support characteristics of a lubricant film, J. Basic Engg. Trans.ASME, 85, 1963, 258-264.*
[13] *Tzeng S.T. and Saibel E., Surface roughness effect on slider bearing lubrication, Trans. ASME, J. Lub. Tech., 10, 1967, 334 -338.*
[14] *Davis M.G., The Generation of pressure between rough lubricated, moving deformable surfaces, Lub. Engg., 19, 1963, 246.*
[15] *Michell A.G.M., Lubrication, its principle and practice, Blackie, London, 1950, 317.*
[16] *Tonder K.C., Surface distributed waviness and roughness, First World Conference in Industrial Tribology, New Delhi, A3, 1972, 128.*
[17] *Christensen H. and Tonder K.C., Tribology of rough surfaces: Stochastic models of hydrodynamic lubrication, SINTEF Report No. 10/69-18, 1969.a.*
[18] *Christensen H. and Tonder K.C., Tribology of rough surfaces: Parametric study and comparison of lubrication models, SINTEF Report No. 22/69-18, 1969.b.*
[19] *Christensen H. and Tonder K.C., Tribology of rough surfaces: A stochastic model of mixed lubrication, SINTEF Report No. 18/ 70-21, 1970.*
[20] *Berthe D. and Godet M., A more general form of Reynolds equation – Application to rough surfaces, Wear, 27, 1973, 345-357.*
[21] *Ting L.L., Engagement behaviour of lubricated porous annular disks Part I: Squeeze film phase, surface roughness and elastic deformation effects, Wear, 34, 1975, 159-182.*

[22] *Prakash J. and Tiwari K., Lubrication of a porous bearing with surface corrugations, J. Lub. Tech., Trans. ASME, 104, 1982, 127-134.*

[23] *Prajapati B.L., Behaviour of squeeze film between rotating porous circular plates: Surface roughness and elastic deformation effects, Pure and Appl. Math. Sci., 33(1-2), 1991, 27-36.*

[24] *Prajapati B.L., Squeeze film behaviour between rotating porous circular plates with a concentric circular pocket: Surface roughness and elastic deformation effects, Wear, 152, 1992, 301-307.*

[25] *Guha S.K., Analysis of dynamic characteristics of hydrodynamic journal bearings with isotropic roughness effects, Wear, 167, 1993, 173 -179.*

[26] *Gupta J.L. and Deheri G.M., Effect of roughness on the behaviour of squeeze film in a spherical bearing, Tribology Transcations, 39, 1996, 99-102.*

[27] *Andharia P.I., Gupta J.L. and Deheri G.M., Effect of transverse surface roughness on the behaviour of squeeze film in a spherical bearing, International Journal of Applied Mechanics and Engineering, 4, 1999, 19-24.*

[28] *Andharia P.I.,Gupta J.L. and Deheri G.M., Effect of longitudinal surface on hydrodynamic lubrication of slider bearings, Proc. Tenth International Conference on Surface Modification Technologies, The Institute on Materials, 1997, 872-880.*

[29] *Patel, R. M., Deheri, G. M., Magnetic fluid based squeeze film behavior between rotating porous circular plates with a concentric circular pocket and surface roughness effects, Int. J. of Applied Mechanics and Engineering, 8(2), 2003, 271-277.*

[30] *Lin, J.R. and Chinang, C.F., Effects of Surface Roughness and Rotational Inertia on the optimal Stiffness of Hydrostatic Thrust Bearings, Int. J. of Applied mechanics and Engineering, 7(4), 2002, 1247-1261.*

ANNALS OF FACULTY ENGINEERING HUNEDOARA
– INTERNATIONAL JOURNAL OF ENGINEERING

Faculty of Engineering Hunedoara,
5, Revolutiei, 331128, Hunedoara,
ROMANIA
http://annals.fih.upt.ro