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# THE STUDY OF FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION

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**Abstract:** An exact solution of unsteady flow past an exponentially accelerated infinite vertical plate with variable temperature in the presence of variable mass diffusion has been studied. The plate temperature is raised linearly with time and species concentration level near the plate is also raised linearly with respect to time. The dimensionless governing equations are solved using Laplace-transform technique. The effect of velocity profiles are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number, '$a$' and time. It is observed that the velocity increases with increasing values of '$a$' or '$t$'.
**Keywords:** accelerated, vertical plate, exponential, heat transfer, mass diffusion.

## 1. INTRODUCTION

Process of involving coupled heat and mass transfer occur frequently in nature. It occurs not only due to temperature difference, but also due to concentration difference or the combination of these two and in different geophysical cases etc. In many process industries, such as extrusion of plastics in the manufacture of Rayon and nylon, purification of crude oil, pulp, paper industry, textile industry, the cooling of threads or sheets of some polymer materials is of importance in the production line.

Natural convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method by Gupta *et al* [2]. Kafousias and Raptis [4] extended this problem to include mass transfer effects subjected to variable suction or injection. Soundalgekar [7] studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh [6]. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [5]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [3]. Mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion was studied by Basant Kumar Jha *et al* [1].

It is proposed to study the effects of on flow past an exponentially accelerated vertical plate in the presence of variable temperature and mass diffusion. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

## 2. MATHEMATICAL ANALYSIS

Here the unsteady flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature and variable mass diffusion has been considered. The $x'$-axis is taken along the plate in the vertically upward direction and the $y$-axis is taken normal to the plate. At time $t' \le 0$, the plate and fluid are at the same temperature $T_\infty$ and concentration $C'_\infty$. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane and the temperature from the plate raised linearly with time $t$ and the mass is diffused from the plate to the fluid uniformly. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_P \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \tag{3}$$

with the following initial and boundary conditions:

$$u = 0,\ T = T_\infty,\ C' = C'_\infty \text{ for all } y, t' \le 0 \quad t' > 0:\ u = u_0 \exp(a't'),\ T = T_\infty + (T_w - T_\infty)At',$$
$$C' = C'_\infty + (C'_w - C'_\infty)At' \text{ at } y = 0,\ u \to 0,\ T \to T_\infty, \text{ as } y \to \infty \tag{4}$$

where $A = \frac{u^2{}_0}{\nu}$.

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0},\quad t = \frac{t'u^2{}_0}{\nu},\quad Y = \frac{yu_0}{\nu},\quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$
$$Gr = \frac{g\beta\nu(T - T_\infty)}{u_0{}^3},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\quad G_C = \frac{\nu g\beta^*(C'_w - C'_\infty)}{u^3{}_0}, \tag{5}$$
$$Pr = \frac{\mu C_p}{k},\quad a = \frac{a'\nu}{u_0{}^2},\ Sc = \frac{\nu}{D}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{8}$$

The initial and boundary conditions in non-dimensional quantities are

$$U = 0,\quad \theta = 0,\quad C = 0 \quad \text{for all } Y, t \le 0$$
$$t > 0:\quad U = \exp(at),\quad \theta = t,\quad C = t \quad \text{at } Y = 0 \tag{9}$$
$$U \to 0,\quad \theta \to 0,\quad C \to 0 \quad \text{as } Y \to \infty$$

The dimensionless governing equations (6) to (8) with the initial and boundary conditions (9) are tackled using Laplace transform technique.

$$\theta = t\left[(1 + 2\eta^2 Pr)\ erfc(\eta\sqrt{Pr}) - \frac{2}{\sqrt{\pi}}\eta\sqrt{Pr}\exp(-\eta^2 Pr)\right] \tag{10}$$

$$C = t\left[(1 + 2\eta^2 Sc)\ erfc(\eta\sqrt{Sc}) - \frac{2}{\sqrt{\pi}}\eta\sqrt{Sc}\exp(-\eta^2 Sc)\right] \tag{11}$$

$$U = \frac{\exp(at)}{2}\left[\exp(2\eta\sqrt{at})\,erfc(\eta + \sqrt{at}) + \exp(-2\eta\sqrt{at})\,erfc(\eta - \sqrt{at})\right]$$
$$+ \frac{Gr\,t^2}{6(Pr-1)}\Big[(3 + 12\eta^2 + 4\eta^4)\,erfc(\eta) - \frac{\eta}{\sqrt{\pi}}(10 + 4\eta^2)\exp(-\eta^2)$$
$$-\Big(3 + 12\eta^2 Pr + 4\eta^4 (Pr)^2\Big)\,erfc(\eta\sqrt{Pr}) + \frac{\eta\sqrt{Pr}}{\sqrt{\pi}}(10 + 4\eta^2 Pr)\exp(-\eta^2 Pr)\ \Big] \tag{12}$$
$$+ \frac{Gc\,t^2}{6(Sc-1)}\Big[(3 + 12\eta^2 + 4\eta^4)\,erfc(\eta) - \frac{\eta}{\sqrt{\pi}}(10 + 4\eta^2)\exp(-\eta^2)$$
$$-\ (3 + 12\eta^2 Sc + 4\eta^4 (Sc)^2)\,erfc(\eta\sqrt{Sc}) + \frac{\eta\sqrt{Sc}}{\sqrt{\pi}}(10 + 4\eta^2 Sc)\exp(-\eta^2 Sc)\ \Big]$$

where, $\eta = Y / 2\sqrt{t}$.

### 3. RESULTS AND DISCUSSION

For physical understanding of the problem numerical computations are carried out for different physical parameters *a*, *Gr*, *Gc*, *Sc* and *t* upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number *Pr* is chosen such that they represent air (*Pr* = 0.71). The numerical values of the velocity are computed for different physical parameters like *a*, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The velocity profiles for different ( *a* = 0.2, 0.5, 0.8 ), *Gr* = *Gc* = 5, *Sc* = 0.6, *Pr* = 0.71 and *t* = 0.2 are studied and presented in figure 1. It is observed that the velocity increases with increasing values of *a*. Figure 2 demonstrates the effects of different thermal Grashof number (*Gr* = 2, 5) and mass Grashof number (*Gc* = 5,10) on the velocity when *a* = 0.5, *Pr* = 0.71, *Sc* = 0.6 and *t* = 0.2. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

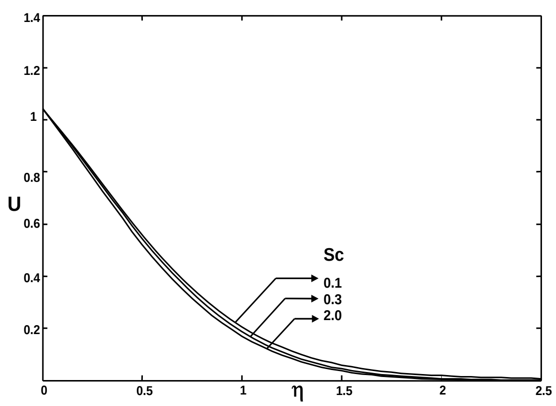


Figure 1 Velocity profiles for different a

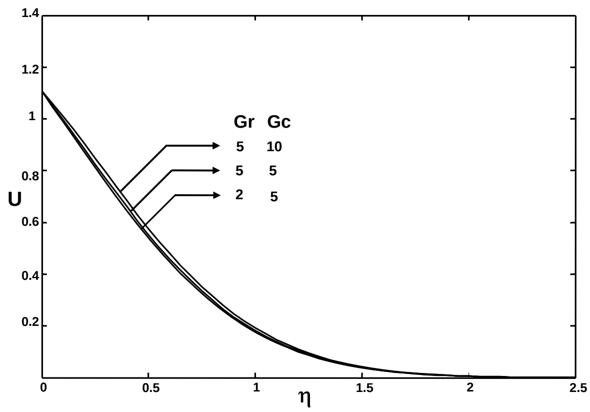


Figure 2 Velocity profiles for different Gr and Gc

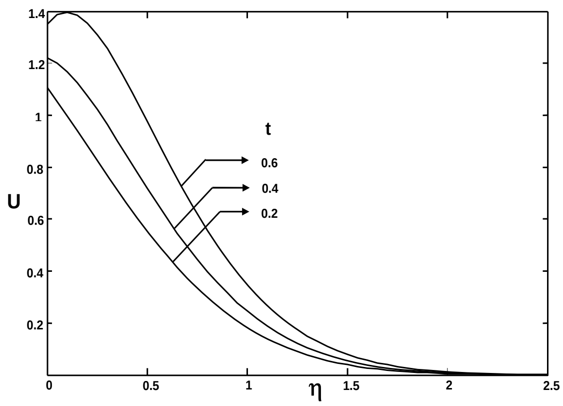


Figure 3 Velocity profiles for different t

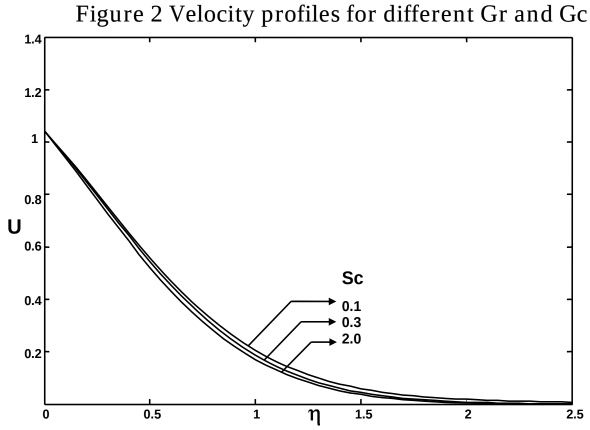


Figure 4 Velocity profiles for different Sc

The velocity profiles for different values of ( t = 0.2, 0.4, 0.6) and *a* = 0.5, *Pr* = 0.71, *Sc* = 0.6, *Gr* = *Gc* = 5 are shown in figure 3. In this case, the velocity increases gradually with respect to time *t*. The effect of velocity for different values of the Schmidt number (Sc =0.1, 0.3, 2.0) are shown in figure 4. The trend shows that the velocity increases with decreasing Schmidt number. It is observed that the relative variation of the velocity with the magnitude of the time and the Schmidt number.

### 4. CONCLUSIONS

The theoretical solution of flow past an exponentially accelerated infinite vertical plate in the presence of variable temperature and uniform mass diffusion has been studied. The dimensionless governing equations are solved by the usual Laplace - transform technique. The effect of different parameters like thermal Grashof number, mass Grashof number, Schmidt number, *a*, Sc, and *t* are studied graphically. The conclusions of the study are as follows:

- The velocity increases with increasing thermal Grashof number or mass Grashof number.
- The velocity increases with increasing values of *a* and time *t*, but the trend is just reversed with respect to the Schmidt number.

**Nomenclature**

$A, a, a'$ Constants

$C'$ species concentration in the fluid $kg\ m^{-3}$

$C$ dimensionless concentration

$C_p$ specific heat at constant pressure $J.kg^{-1}.k$

$D$ mass diffusion coefficient $m^2.s^{-1}$

$G_c$ mass Grashof number

$Gr$ thermal Grashof number

$g$ acceleration due to gravity $m.s^{-2}$

$k$ thermal conductivity $W.m^{-1}.K^{-1}$

$Pr$ Prandtl number

$Sc$ Schmidt number

$T$ temperature of the fluid near the plate $K$

$t'$ time $s$

$u$ velocity of the fluid in the $x'$-direction $m.s^{-1}$

$u_0$ velocity of the plate $m.s^{-1}$

$u$ dimensionless velocity

$y$ coordinate axis normal to the plate $m$

$Y$ dimensionless coordinate axis normal to the plate

**Greek symbols**

$\beta$ volumetric coefficient of thermal expansion $K^{-1}$

$\beta^*$ volumetric coefficient of expansion with concentration $K^{-1}$

$\mu$ coefficient of viscosity $Ra.s$

$\nu$ kinematic viscosity $m^2.s^{-1}$

$\rho$ density of the fluid $kg.m^{-3}$

$\tau$ dimensionless skin-friction $kg.m^{-1}.s^2$

$\theta$ dimensionless temperature

$\eta$ similarity parameter

erfc complementary error function

**Subscripts**

$w$ conditions at the wall

$\infty$ free stream conditions

**REFERENCES**


[1] *BASANT KUMAR JHA, RAVINDRA PRASAD and SURENDRA RAI, Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux, Astrophysics and Space Science, Vol. 181, pp.125-134, 1991.*

[2] *GUPTA A.S., POP, I. and SOUDALGEKAR, V.M., Free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid, Rev. Roum. Sci. Techn.-Mec. Apl., Vol. 24, pp. 561-568, 1979.*

[3] *HOSSAIN M.A. and SHAYO, L.K., The skin friction in the unsteady free convection flow past an accelerated plate, Astrophysics and Space Science, Vol.125, pp.315-324,1986.*

[4] *KAFOUSIAS N.G. and RAPTIS A.A., Mass transfer and free convection effects on the flow past an accelerated vertical infinite plate with variable suction or injection, Rev. Roum. Sci. Techn.-Mec. Apl., Vol. 26, pp. 11-22, 1981.*

[5] *SINGH A.K. and NAVEEN KUMAR, Free convection flow past an exponentially accelerated vertical plate, Astrophysics and Space Science, Vol. 98, pp.245 - 258, 1984.*

[6] *SINGH A.K. and SINGH J., Mass transfer effects on the flow past an accelerated vertical plate with constant heat flux, Astrophysics and Space Science, Vol.97, pp. 57-61,1983.*

[7] *SOUNDALGEKAR .V.M , Effects of mass transfer on the flow past an accelerated vertical plate, Letters in Heat and Mass Transfer, Vol.9, pp. 65-72, 1982.*


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