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FORMATION OF INVOLUTE MESHING BY SPUR GEARS, MODIFIED IN TWO DIRECTIONS

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ABSTRACT:
A generalized model of symmetric involute meshing is proposed, where the pinion and the gear are cut by two different rack-cutters. The essence of gears modified in two-directions, got by the combination of the traditional shift of the rack-cutter in radial direction and the proposed tangential shift of teeth profiles is explained. The basic equation of the involute meshing is determined, corresponding to the generalized model. All possible designs of meshing between the rack-cutter and the gear are determined, differing in regard to the radial and tangential modification. A classification of involute gears, cut by a rack-cutter, is proposed.

KEYWORDS:
Involute gears, Involute meshing, Rack-cutter, Radial modification, Tangential modification

1. INTRODUCTION

Many basic advantages of the involute meshing, connected most often with the possibilities for a geometric influence on the gear strength and the design of the gear drive in a randomly designated centre distance are due to the *radial modification* of gears [6], [8]. The radial modification of the pinion and the gear, determined by the radial shift coefficients x_1 and x_2, influences essentially the geometry and quality indices of the gear drive. For the determination of x_1 and x_2 in [7] are proposed a great number of *blocking contours* of gear drives of different teeth number of the pinion and the gear. The boundaries of these plain contours determine the field of the tolerable value of the coefficients of radial shift.

In the author's opinion the area of application of the traditional cylindrical gear drives can be additionally enlarged, if simultaneously with the radial modification is realized also the so called *tangential modification* of the gears [1-5]. In this case the advantages are related to the abilities for the increase of the volume strength of teeth and avoiding their sharpening, especially by the design of gears of a small teeth number. In this connection the present work relates to the definition of the geometry of gears of cylindrical involute gear drives, modified in radial and tangential direction.

2. A GENERALIZED MODEL OF INVOLUTE MESHING

The generalized model, with which the symmetric involute meshing is designed, is explained with the help of Fig. 1, where the common connection between the *basic rack profile* (BRP) is shown, rack-cutters, the cut of the pinion and the gear and the obtaining of the involute meshing. In accordance with the proposed generalized model, initially the geometric shape of BRP is assigned, with which the geometry of two different rack-cutters RQ1 and RQ2 is determined (Fig. 1a). Then, with the rack-cutter RQ1 is cut the pinion G1 (Fig. 1b), and with the rack-cutter RQ2 – the gear G2 (Fig. 1c). Subsequently with the pinion and gear is formed the generalized model of involute meshing (Fig. 1d).

2.1 Basic profile of the rack

The geometry of the basic rack profile (Fig. 1a) is determined simply from the following *five independent parameters* : module m, profile angle α, deptn coefficient of the addendum part of tooth h_a^*, coefficient of bottom clearance c^* and coefficient of the radius of fillet ρ^*. Despite the module is a scale factor, and the teeth pitch p and the other linear parameters of BRP are determined by equations

$$p = \pi m, \qquad h_a = h_a^* m, \qquad c = c^* m, \qquad \rho = \rho^* m. \tag{1}$$

In the general case using BRP is determined the geometry of two different rack-cutters. With the one are cut the pinion teeth, and with the other – the gear teeth. The location of the opposite profiles in rack-cutters RQ1 and RQ2 is determined by the tangential shift at a distance of $X_{\tau 1}$ and $X_{\tau 2}$ to the position which they have in BRP. Then the tangential shifts are positive, if on the datum (middle) line of the rack-cutter the tooth space is greater than the tooth thickness. In the opposite case the values of the tangential shifts are negative. In the presence of a tangential shift of profiles it is expedient, the coefficient ρ^* to be chosen within the range $0 < \rho^* \leq c^* / (1 - \sin\alpha)$.

2.2 Radial and tangential modified gears

The geometry of teeth of the pinion and the gear is determined by their meshing with the respective rack-cutters (Fig. 1b, Fig. 1c). In the general case the relative location of the rack-cutter to the cut gear is obtained by a shift of the rack and its side profiles in radial and tangential direction, respectively.

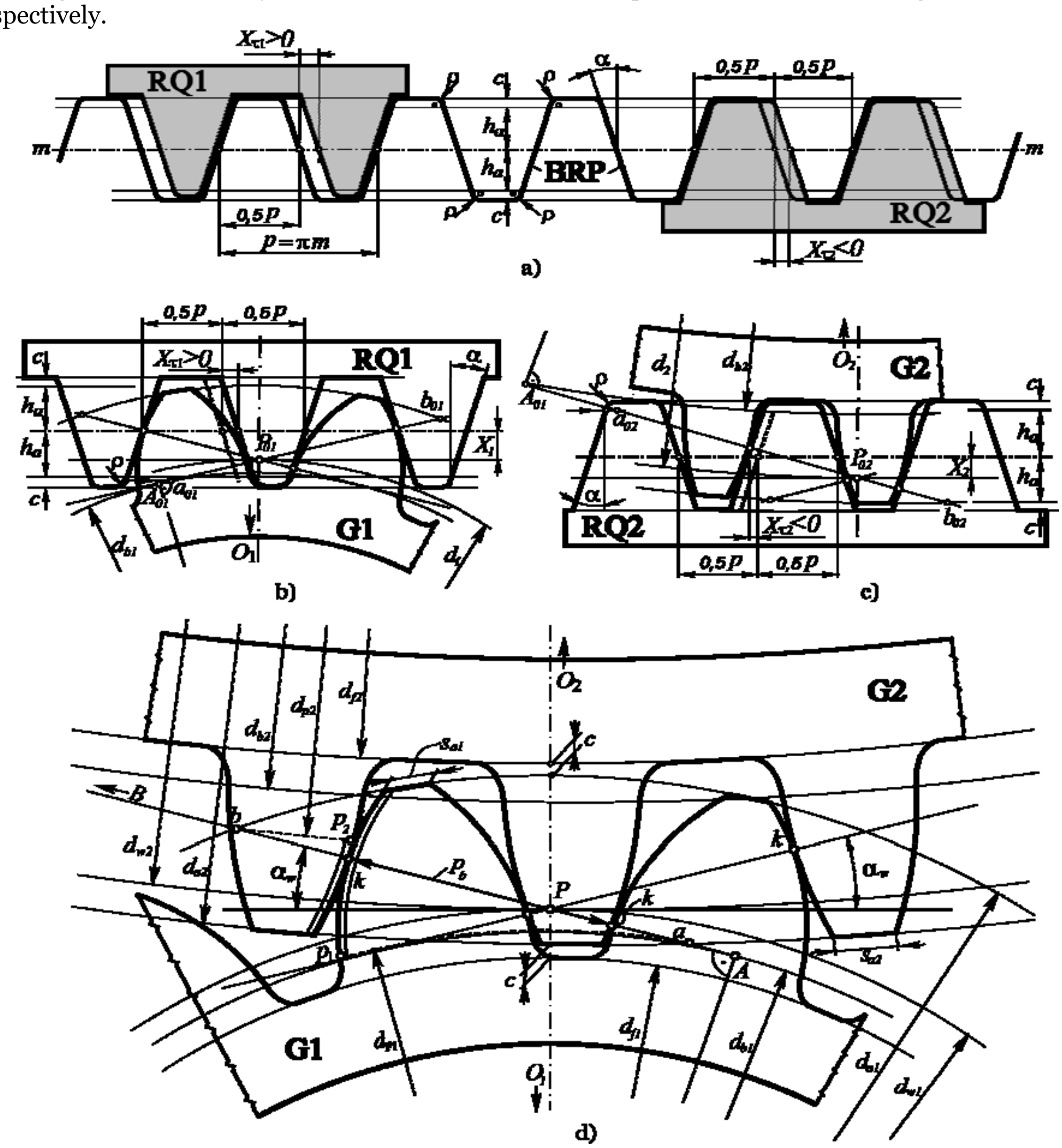

Figure 1. Formation of involute meshing: a) basic rack profile-BRP;
b) meshing of RG1 and G1; b) meshing of RG2 and G2; d) involute meshing of G1 and G2

The value of the radial shifts (X_1, X_2) and tangential shifts ($X_{\tau 1}$, $X_{\tau 2}$) are determined by equations

$$X_1 = x_1 m, \quad X_2 = x_2 m, \quad X_{\tau 1} = x_{\tau 1} m, \quad X_{\tau 2} = x_{\tau 2} m, \tag{2}$$

where x_1, x_2, $x_{\tau 1}$ and $x_{\tau 2}$ are the coefficients of the respective modification. The radial and tangential modification of gears directly influences the thickness and shape of the cut teeth. From the choice of x_1, x_2, $x_{\tau 1}$ and $x_{\tau 2}$ to a great extend depend the geometry of gears, quality indices of the gear meshing and carrying capacity of the gear drive. Because of this the choice of optimal values for the coefficients of radial and tangential shifts is one of the basic stages of the geometric design of the gear drive.

2.3 Variants of the involute meshing

Variants of the involute meshing are shown on Fig. 2, where the sums of coefficients $x_1 + x_2$ and $x_{\tau 1} + x_{\tau 2}$ act as basic differentiating indications.

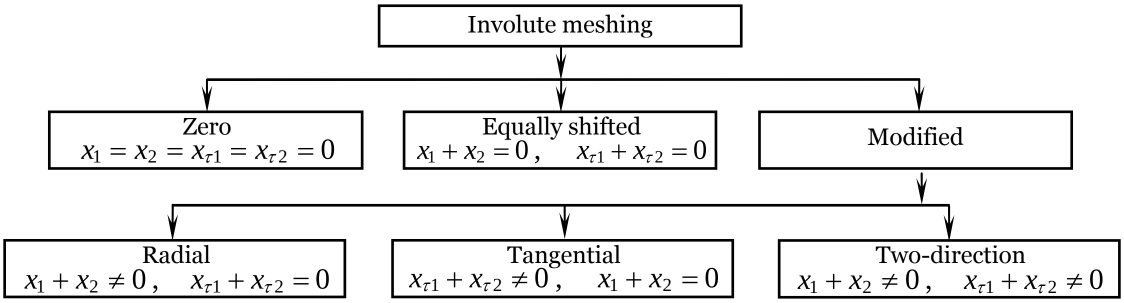

Figure 2. Variants of involute meshing

Out of all variants of the symmetric involute meshing the most general is the case, where the gears are modified simultaneously in two directions – radial and tangential. In this case, corresponding to the generalized model of involute meshing,

$$x_1 + x_2 \neq 0, \qquad x_{\tau 1} + x_{\tau 2} \neq 0. \tag{3}$$

The picture of meshing of the generalized model is shown in Fig. 1d. At any other variant at least one of the differentiating conditions (3) is transformed to an equation. In this way the geometric design of each variant, including the traditional symmetric meshing, is determined as a private case of the generalized model of meshing. The case, where

$$x_1 + x_2 \neq 0, \qquad x_{\tau 1} = x_{\tau 2} = 0 , \tag{4}$$

corresponds to the traditional involute meshing, modified in radial direction.

3. BASIC EQUATION OF THE SYMMETRIC INVOLUTE MESHING

With the basic equation of meshing different problems of the design of involute gear drives are solved. When the pinion and the gear form meshing without a clearance (Fig. 1d), the pitch circles of gears of diameters d_{w1} and d_{w2} contact the pitch point P. Because these circles roll one by one without sliding, the tooth thickness on the pitch circle of the one gear is equal to the width of the tooth space on the pitch circle of the other gear. After equalizing these values and the respective transformations, the basic equation of the symmetric involute meshing is determined as follows

$$x_{\tau 1} + x_{\tau 2} + 4(x_1 + x_2)\tan\alpha = 2(z_1 + z_2)(\mathrm{inv}\,\alpha_w - \mathrm{inv}\,\alpha), \tag{5}$$

where α_w is the pressure angle of the gear drive, and z_1 and z_2 – teeth number of the pinion and the gear.

Using the basic equation of meshing the pressure angle α_w of the gear drive (at specified number of teeth z_1, z_2, profile angle α of BRP and coefficients of radial and tangential shift x_1, x_2, $x_{\tau 1}$, $x_{\tau 2}$) is determined.

The basic equation of meshing is used also when design of the involute gear drive in a specified centre distance a_w. In this case the problem is solved starting with preliminary determination of the pressure angle α_w from equation

$$\alpha_w = \arccos[m(z_1 + z_2)\cos\alpha / 2a_w], \tag{6}$$

then from the basic equation of meshing is determined the relation between the summarized coefficients of shift $x_{\tau\Sigma}$ and x_Σ ($x_\Sigma = x_1 + x_2$, $x_{\tau\Sigma} = x_{\tau 1} + x_{\tau 2}$). After that, after clarifying the coefficients x_1, x_2, $x_{\tau 1}$ and $x_{\tau 2}$, the geometry of gears and gear drive is determined.

4. CLASSIFICATION OF INVOLUTE GEARS

The radial and tangential modification directly influences the teeth thickness s on the reference circle (of radius r_1 and r_2) of the gear (Fig. 1). By two-direction modification the thickness is determined by the formula

$$s = m(\pi/2 + x_\tau + 2x\tan\alpha), \tag{7}$$

whence it is seen that the positive values of x and x_τ increase the teeth thickness s at the expense of the width of the tooth spaces e on the reference circle. Depending on the value of the respective shift and comparing s and e on Fig. 3, a new classification of involute gears is proposed, where the known variants are given in the left side.

In this classification the gear is *zero* (фиг. 4), when both coefficients of shift x and x_τ have zero values. If only one of these coefficients is different from zero, the gear is *modified*. The modified gears divide in three types: *radial modified,tangential modified and two- direction modified*.

For the radial modified gears is characteristic that the tangential shift is equal to zero, while for the tangential modified gears the radial shift is zero. The radial modified gears, as is known divide in *positive* (Fig. 5a), if $x>0$ and *negative* (Fig. 5b) at $x<0$. Analogously the tangential modified gears on their side at $x_\tau>0$ are positive (Fig. 6a), and at $x_\tau<0$ – negative (Fig. 6b).

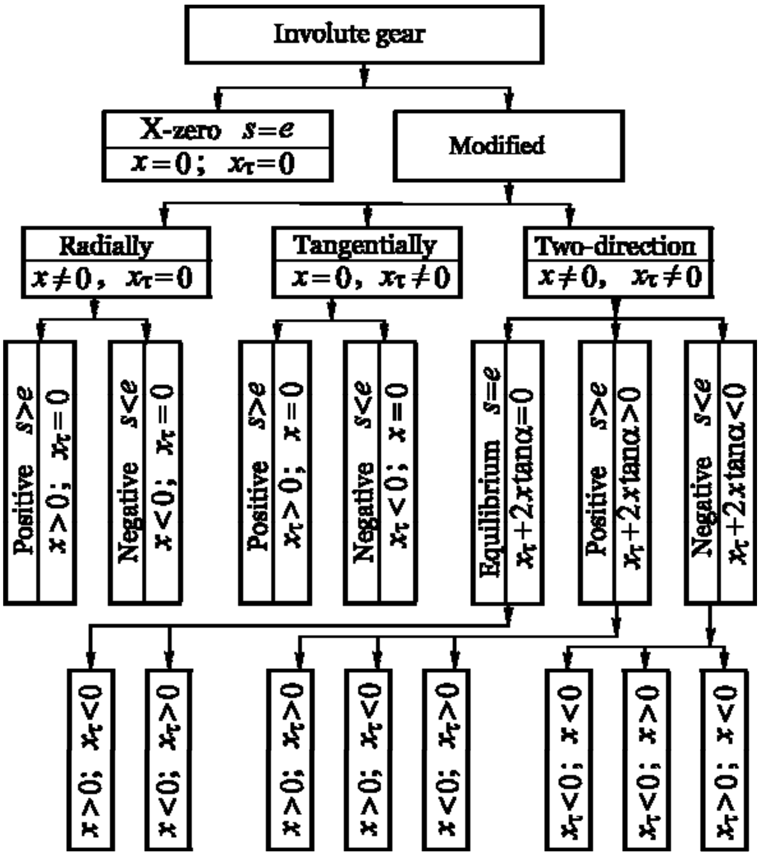

Figure 3. Classification of involute gears

Wider variety of different gears is obtained at simultaneous modification in radial and tangential direction. In this case of two-direction modification, it is expedient that the differentiating indication to be determined under the condition where the tooth thickness and the width of tooth space on the reference circle have equal values. Then, taking into account that $s=0{,}5p=0{,}5\pi m$ at $s=e$, from equation (7) is determined an additional differentiating condition

$$x_\tau + 2x\tan\alpha = 0, \qquad (8)$$

whose satisfaction determines a new type of involute gears, called *equilibrium modified*. To a certain extent these gears (Fig. 7) are similar to the zero one (Fig. 4), as in both cases $s=e$. The differences between the zero and equilibrium modified gears are more essential regarding the teeth shape. On Fig. 7 are shown two characteristic cases of equilibrium modified gears. In the first case (Fig. 7a), the radial shift is positive, and the tangential one – negative. In the second case (Fig. 7b) the signs of the radial and tangential shifts are exchanged. As a result, despite the equality of teeth thickness on the reference circles, in both cases the gear teeth have different geometric shape.

The two-direction modified gears also divide in positive and negative. Since for a specified gear the radial and tangential shift may be of different signs, its type (positive or negative) is determined by the predominating influence of the respective shift in equation (8). The gear is *two-direction positive*, if $x_\tau + 2x\tan\alpha > 0$ and *two-direction negative* at $x_\tau + 2x\tan\alpha < 0$. For each of these gears are characteristic three variants, shown on Fig. 8 and Fig. 9.

From Fig. 3 it is seen that the total number of all variants in the proposed classification is *thirteen*. For a comparison, with the traditional modification the variants are only *three* (zero, radial positive gears and radial negative gears)

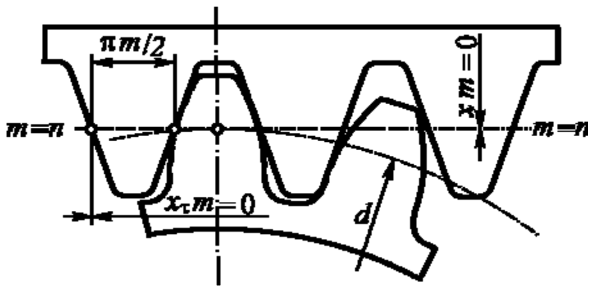

Figure 4. X-zero gear

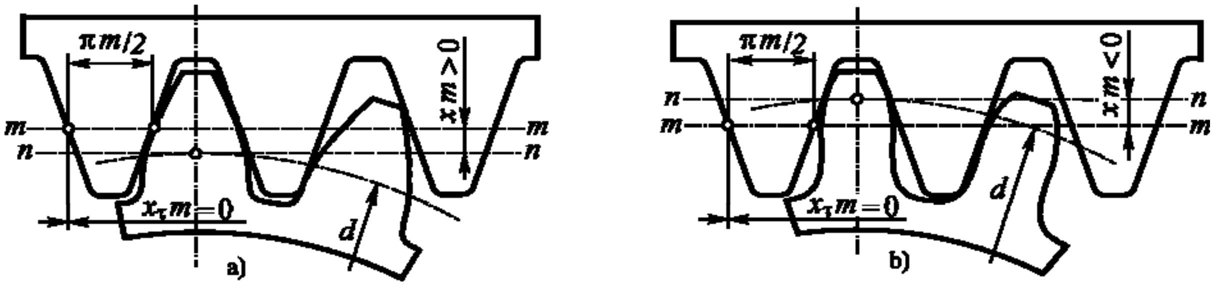

Figure 5. Radial modified gears: a) positive ($x>0$, $x_\tau=0$); б) negative ($x<0$, $x_\tau=0$)

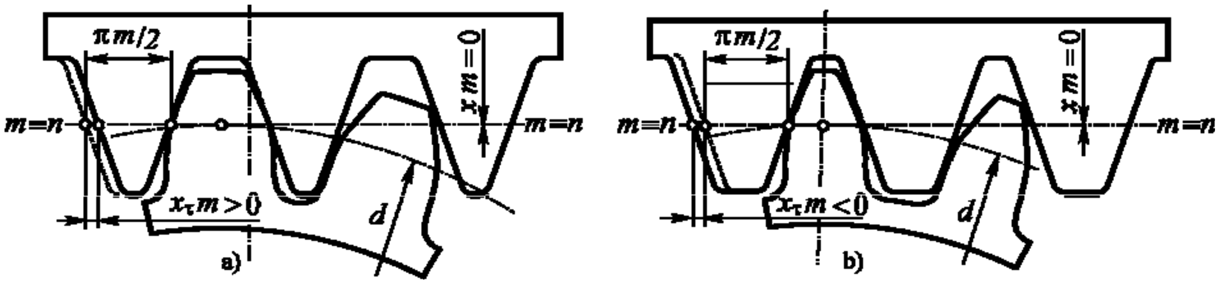

Figure 6. Tangential modified gears: a) positive ($x_\tau>0$, $x=0$); b) negative ($x_\tau<0$, $x=0$)

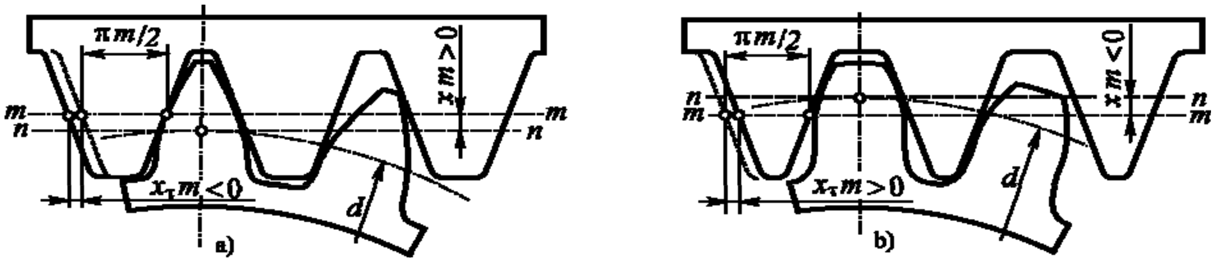

Figure 7. Equilibrium modified gears: a) $x>0$, $x_\tau<0$, $2x\,\mathrm{tg}\,\alpha=-x_\tau$; b) $x<0$, $x_\tau>0$, $x_\tau=-2x\,\mathrm{tg}\,\alpha$

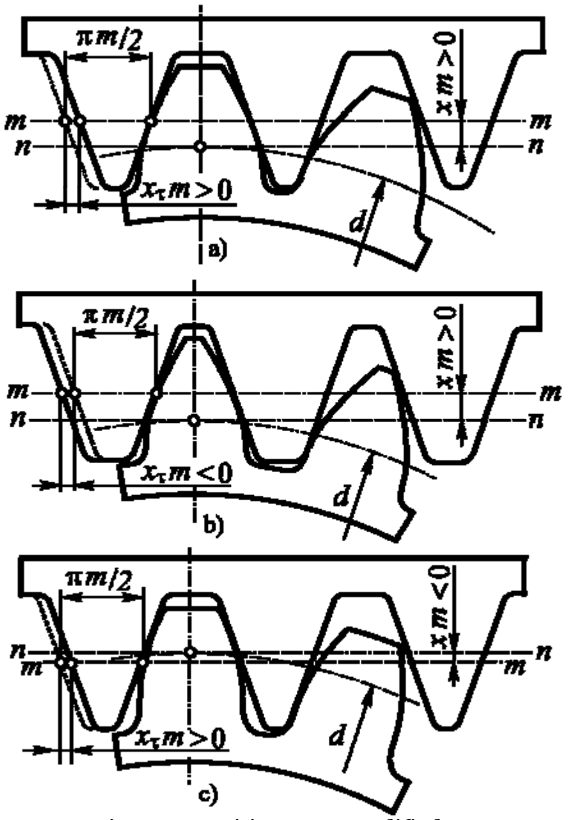

Figure 8. Positive gears modified

a) $x>0$, $x_\tau>0$; b) $x>0$, $x_\tau<0$; c) $x<0$, $x_\tau>0$

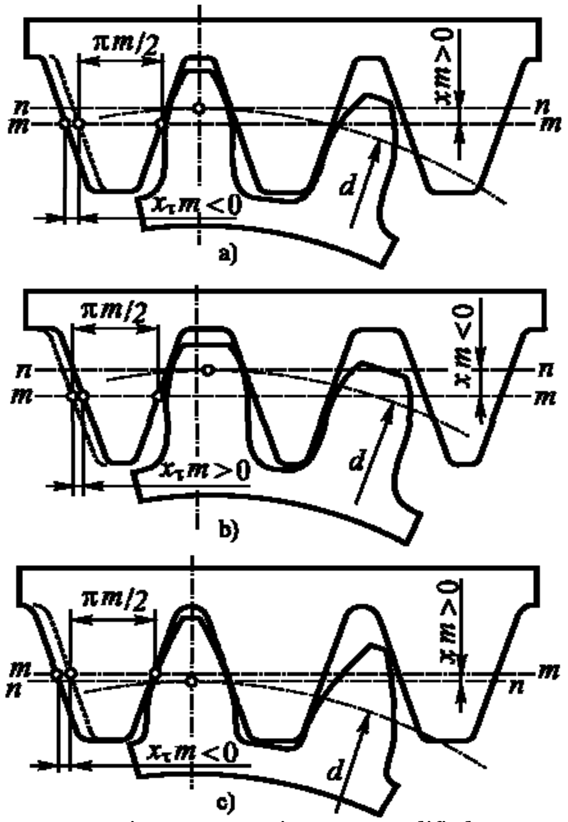

Figure 9. Negative gears modified

a) $x<0$, $x_\tau<0$; b) $x<0$, $x_\tau>0$; c) $x>0$,

5. CONCLUSION

The proposed two-direction modification of the cylindrical involute gears shows an additional way for geometric influence on the efficiency of the designed gear drive. The two-direction modification broadens additionally the area, in which the gear drives of involute meshing may exist. The combination of the radial and tangential modification increases to a great extent the variants of gears. The obtained thirteen different designs of modification and the proposed classification of gears contribute to the correct choice of the respective shift (radial and tangential) by the geometric design of involute gear drives.

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