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# CRITICAL ISOTROPIC DAMAGE DUE TO CRACKS CONCENTRATION

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**ABSTRACT**
This work is intended to determine the critical crack concentration by static loading of homogeneous isotropic solids. We use results from the linear fracture mechanics and we determine the limit cracks concentration on the basis of the crack resistance. Thus we use the critical stress intensity factor and the plastic limit stress in the end of a crack as material constants describing the brittle and ductile failure resistance. The obtained theoretical results are discussed and plotted in 2D and 3D graphics. We get a theoretical ground to explain the limited values of the critical cracks' concentration in real solids.
**Keywords:** critical crack concentration, isotropic solids, cracks concentration, damage

## 1. INTRODUCTION

It is clear that the well known Katchanov's damage parameter $D$, which varies from 0 (undamaged solid) to 1 (total damaged solid), can't reach the value of 1 because of the crack gathering. We propose here a simple method to determine the critical crack concentration (CCC), which is very important to determine the time-life of solids [1,2].

## 2. METHOD FOR DETERMINATION OF THE CCC

The fracture mechanics gives the following equation to the rivers displacement of the end of the crack $\delta_c$ in the plane case [3]

$$\delta = \frac{K^2}{E\sigma_0}, \tag{1}$$

where $K$ is the stress intensity factor, $E$ - the Young's module and $\sigma_0$ the plastic stress in the end of the crack. The Dugdale's expression to the critical crack rivers displacement in the same case is also well known [3]

$$\delta_c = \frac{8\sigma_0 l}{\pi E}\ln\left[\cos\frac{\pi\sigma_*}{2\sigma_0}\right]^{-1} \tag{2}$$

where $2l$ is the crack length and $\sigma_*$ is the external failure stress considering the plastic effects in the end of the crack.

Using the well known relation $\sigma = \frac{K}{\sqrt{l}}$ concerning central crack in the traction case, on the basis of equations (1) and (2) one receive the next limit failure curve, which determines the unstable crack state [3]

$$\frac{8}{\pi} ln\left[cos\frac{\pi\sigma_*}{2\sigma_0}\right]^{-1} = \left(\frac{\sigma_{lin}}{\sigma_0}\right)^2, \tag{3}$$

This limit curve is experimental approved in [4] for many different materials and stress states and can be applied for any required charges in 2D and 3D. In this case $\sigma_{lin}$ will be determined by the stress intensity factor for every possible charge and geometry.

The aim of this investigation is to determine the critical crack concentration in a given material under stress using solutions for the critical intensity factor and the pate to obtain the expression for the limit curve (3).

The expressions to the first stress intensity factor $K$ by normal external stresses for homogeneous dispersed cracks whose centers form triangles with equal sides are well known [5]

$$K = \sigma_{lin}\sqrt{l}(1+0.68\lambda^2+0.47\lambda^4+...) \quad (4)$$

Here $\lambda = 2l/L$, where $L$ represents the distance between the crack centers. It is sufficient to stop by the second member of the series since $\lambda \le 0.8$.

We will consider now the case of collinear cracks [5,6]

$$K = \sigma_{lin}\sqrt{l}\sqrt{\frac{2}{\pi\lambda}tg\frac{\pi\lambda}{2}} \quad (5)$$

This equation for $\lambda \le 0.8$ can be replaced by the expression [6]

$$K = \sigma_{lin}\sqrt{l}\sqrt{\sec\frac{\pi\lambda}{2}} \quad (6)$$

The crack concentration $D$ could be defined as $D = 2l/L$, thus the crack (damage) concentration and the parameter $\lambda$ coincide

$$D = \frac{2l}{L} = \lambda . \quad (7)$$

Introduce the relative intensity factors $Krel_i = \frac{K_i}{\sigma\sqrt{l}}$, where $K_i$ represent the intensity factors from (4, 5, 6) consecutively. Thus comparing (6) with (4, 5) – figure 1, we can conclude that the difference between these equations is negligible for $\lambda \le 0.8$ which corresponds to the limit real concentration case [2]. Note that the curve (6) is between the two others (5) and (4). On the other hand there exist respective solutions in the case of dispersed cracks whose centers form squares in the case of normal and tangential external stresses (concerning both stress intensity factors). All of them are similar and can be compared with equation (6). The real crack distribution is more complicated, but remains arbitrary and homogeneous. In this real case to determine the stress intensity factor we propose to use only the simple analytical expression (6).

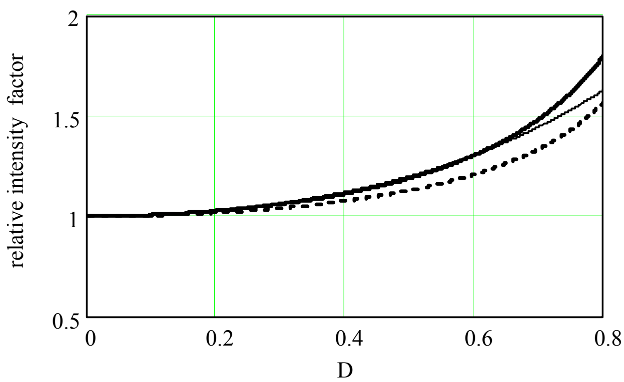


Figure 1. Relative stress intensity factor for different crack distributions using eq. 5, 6 and 4 respectively

The critical damage concentration $D^*$ can be received by equalizing the crack intensity factor $K$ and the critical crack intensity factor $K_c$ which can be experimentally received and which is determined for many materials [7]

$$K = K_c \quad (8)$$

Definitively on the basis of eqs (3, 6, 7, 8) we obtain the following expression concerning the critical crack concentration by arbitrary charge and geometry

$$D^* = \frac{2}{\pi}\arccos\left(\frac{8l}{\pi q}\ln\left(\sec\frac{\pi}{2}s\right)\right) \quad (9)$$

In equation (9) we have put

$$s = \frac{\sigma_*}{\sigma_o} \text{ and } q = \left(\frac{K_c}{\sigma_o}\right)^2 \quad (10)$$

where $\sigma_*$ is the nominal failure stress , $s$ is the failure stress-plastic stress (in the top of the cracks) ratio and $q$ will be called a crack resistance ratio (CRR). Equation (9) has been obtained by putting $D \to D^*$, which obviously should be carried out in the case of realized equality according to equation (8).

It follows from equations (9, 10) that the five parameters $D^*$, $\sigma_*$, $\sigma_0$, $K_c$ and $l$ are not independent. Using (9, 10) we can determine one of them knowing the other forth. Thus, the stress at the end of the crack $\sigma_0$ being sometimes difficult to obtain, we need to determine experimentally only the nominal destructive stress $\sigma_*$ , the critical stress intensity factor $K_c$, the cracks length $2l$ and the CCC.

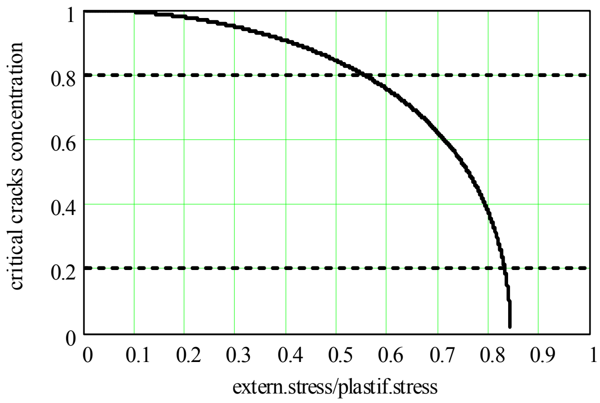


Figure 2. CCC as a function of the stress ratio *s*. Horizontal dotted lines – limit values of the CCC

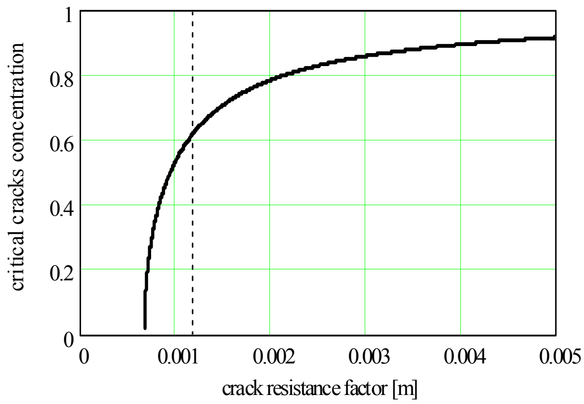


Figure 3. CCC as a function of the crack resistance ratio *q*. With dashed line – the CRR value - *q* for polyester resin

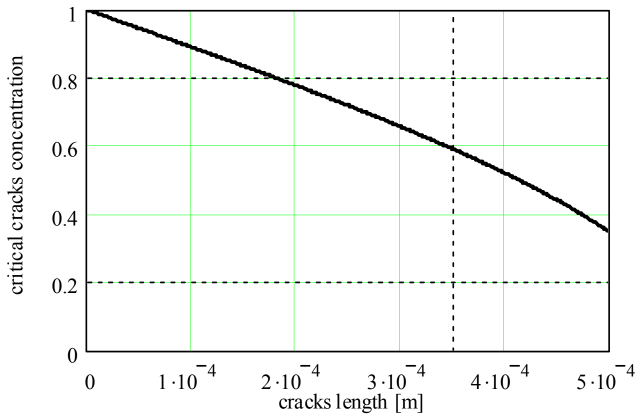


Fig.4 Influence of the cracks length on the CCC. Horizontal dotted lines – limit values of the CCC. Vertical dotted line - cracks lenght by CCC equal to 0.6 in the case of polyester resin according to figure 3

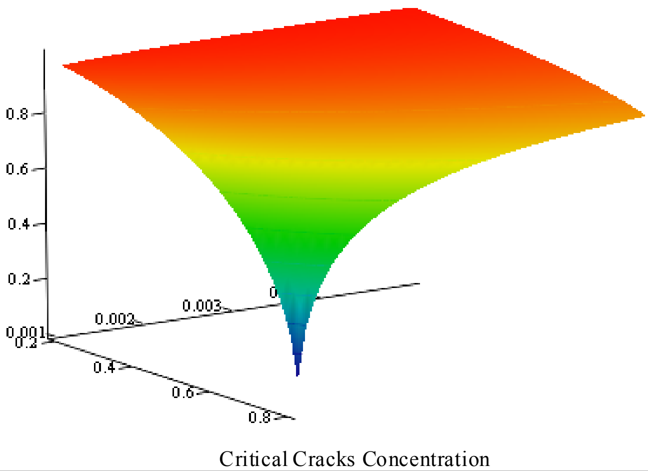


Figure 5. CCC as a function of the cracks' semi length $l$ and the critical stress intensity factor $K_c$

The limit curve (9) concerning the critical crack concentration (CCC) is plotted in figures 2, 3 and 4 as a function of the stress ratio *s*, the crack resistance ratio *q* and the cracks length respectively for polyester resin.

On the other hand equation (9) is a function of many variable parameters. We show bellow – figure 5 the CCC $D^*$ as a function of the critical stress intensity factor $K_c$ and the crack length respectively. This figure shows the influence of every of these factors on the critical damage concentration as 3D graphic.

## 3. CONCLUSION

Now we can make the following conclusions. The influence of the geometrical and physical parameters above mentioned is complicated and multidirectional. A greater damage concentration can be received only in the case of greater crack resistance and greater ductility. Reach the limit damage concentration $D^* \to 1$ is not possible because in this case equation (9) requires $\sigma_o \to \infty$, $l \to 0$ or $K_c \to \infty$. The investigations show [2, 8] that $D^*$ for many real materials is limited between 0.2 and 0.8.

In this work we have obtained useful relations to determine the critical crack concentration and to investigate the influence of the material parameters on the crack concentration.

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