ANNALS of Faculty Engineering Hunedoara – International Journal of Engineering
ISSN: 1584 - 2673

A CAD STUDY ON GENERATING OF 2D SUPERSHAPES IN DIFFERENT COORDINATE SYSTEMS

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ABSTRACT:
The objective of this paper is to develop a computer programme for generating of 2D supershapes in different coordinate systems, based on descriptive geometry and computer programming. The mathematical methods and graphical algorithms permit a CAD representation of complex curves. The Maple programming language permits to obtain conclusions for shape and profile of 2D supershapes, but also facilitate the new ideas in shape design. Results from this study are applied in geometric constructions and computer aided design used in sculpture design.
KEYWORDS: Sculpture design, 2D supershapes, CAD

1. THEORETICAL CONSIDERATIONS

The combination of computer, math, art provides a wide visual and artistic inspiration for many artists.

The computer used as a virtual prototyping tool, for geometric modeling and graphical rendering allow to obtain new artistic shapes, that can be explored in interactive ways, thus creating new artistic works in virtual form.

The supershapes, has been proposed by by Johan Gielis in 1997 [1], to describe numerous complex curves and shapes that are found in nature.

Supershapes have been widely studied in different fields such as biological structures, computer graphics, computer vision and sculpture design [2, 3, 4].

For the sake of simplicity, geometric objects are represented in cartesian (rectangular) coordinate system.

In polar coordinates, the radius of a supershape is defined as:

$$r(\phi)=\frac{1}{\left[\left(\left|\frac{1}{a}\cos\left(\frac{m}{4}\phi\right)\right|\right)^{n2}+\left(\left|\frac{1}{b}\sin\left(\frac{m}{4}\phi\right)\right|\right)^{n3}\right]^{1/n1}},\ \text{with } n_i\in R^+, m\in R^+, a\in R_o^+, b\in R_o^+. \quad (1)$$

where a and b are positive real numbers but not zero; m and n_i are positive real numbers.

Parameters a and b control the scale, m represents the number of symmetry axes, and coefficients $n1$, $n2$ and $n3$ control the shape.

For $n2 \neq n3$, asymmetrical shapes can be obtained.

2. COMPUTER PROGRAMME

Using the Maple programming language next procedure was written for generation of 2D supershapes, in different coordinate systems [5].

Programme:

```
> restart:
> with(plots):
```

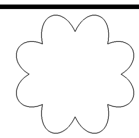
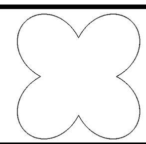
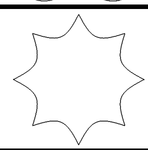
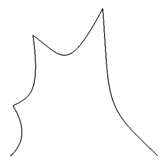
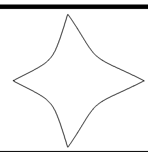
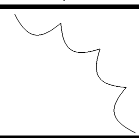
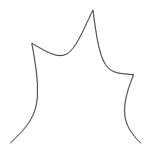
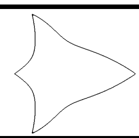
```
> C:=[bipolar, cardioid, cartesian, cassinian, elliptic, hyperbolic, invcassinian, invelliptic, logarithmic, logcosh, maxwell, parabolic, polar, rose]:
> a:=1: b:=1: m:=8: n1:=1: n2:=1: n3:=1: coordinate_system:=C[3]: nops(C):
> r:=evalf((abs(1/a*cos(m*phi/4*Pi/180))^n2+abs(1/b*sin(m*phi/4*Pi/180)) ^n3)^(-1/n1)):
> x:=r*(cos(phi*Pi/180)):
> y:=r*(sin(phi*Pi/180)):
> plot([x,y,phi=-180..180], numpoints=200, thickness=1, coords= coordinate_system);
```

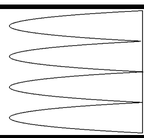
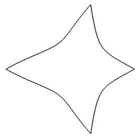
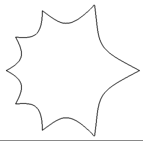
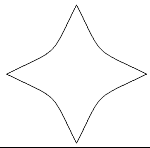
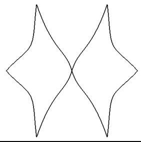
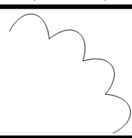
3. GRAPHICAL REPRESENTATIONS IN DIFFERENT COORDINATE SYSTEMS

Let's consider the particular 2D supershape with next parameters: $a = 1$, $b = 1$, $m = 8$, $n1 = 1$, $n2 = 1$, $n3 = 1$.

The graphical representations of this supershape in different coordinate systems are given in Table 1.

Table 1. graphical representations of this supershape in different coordinate systems

No.	Type of coordinate system	The equations for conversion to cartesian coordinates: $(u, v) \rightarrow (x, y)$	Graphics in 2D
1	Bipolar	$\begin{cases} x = \sinh(v) / (\cosh(v) - \cos(u)) \\ y = \sin(u) / (\cosh(v) - \cos(u)) \end{cases}$	
2	Cardioid	$\begin{cases} x = (1/2) \cdot (u^2 - v^2)/(u^2 + v^2)^2 \\ y = u \cdot v/(u^2 + v^2)^2 \end{cases}$	
3	Cartesian	$\begin{cases} x = u \\ y = v \end{cases}$	
4	Cassinian-oval (cassian)	$\begin{cases} x = a \cdot \sqrt{2}/2 \cdot ((e^{2 \cdot u} + 2 \cdot e^u \cdot \cos(v) + 1)^{1/2} + \\ + e^u \cdot \cos(v) + 1)^{1/2} \\ y = a \cdot \sqrt{2}/2 \cdot ((e^{2 \cdot u} + 2 \cdot e^u \cdot \cos(v) + 1)^{1/2} - \\ - e^u \cdot \cos(v) - 1)^{1/2} \end{cases}$	
5	Elliptic	$\begin{cases} x = \cosh(u) \cdot \cos(v) \\ y = \sinh(u) \cdot \sin(v) \end{cases}$	
6	Hyperbolic	$\begin{cases} x = ((u^2 + v^2)^{1/2} + u)^{1/2} \\ y = ((u^2 + v^2)^{1/2} - u)^{1/2} \end{cases}$	
7	Inverse Cassinian-oval	$\begin{cases} x = a \cdot \sqrt{2}/2 \cdot ((e^{2 \cdot u} + 2 \cdot e^u \cdot \cos(v) + 1)^{1/2} + e^u \cdot \cos(v) + 1)^{1/2} / \\ /(e^{2 \cdot u} + 2 \cdot e^u \cdot \cos(v) + 1)^{1/2} \\ y = a \cdot \sqrt{2}/2 \cdot ((e^{2 \cdot u} + 2 \cdot e^u \cdot \cos(v) + 1)^{1/2} - e^u \cdot \cos(v) - 1)^{1/2} / \\ /(e^{2 \cdot u} + 2 \cdot e^u \cdot \cos(v) + 1)^{1/2} \end{cases}$	
8	Inverse elliptic (invelliptic)	$\begin{cases} x = a \cdot \cosh(u) \cdot \cos(v) / (\cosh(u)^2 - \sin(v)^2) \\ y = a \cdot \sinh(u) \cdot \sin(v) / (\cosh(u)^2 - \sin(v)^2) \end{cases}$	

9	Logarithmic	$\begin{cases} x = a/\pi \cdot \ln(u^2 + v^2) \\ y = 2 \cdot a/\pi \cdot \arctan(v/u) \end{cases}$	
10	Logcosh	$\begin{cases} x = a/\pi \cdot \ln(\cosh(u)^2 - \sin(v)^2) \\ y = 2 \cdot a/\pi \cdot \arctan(\tanh(u) \cdot \tan(v)) \end{cases}$	
11	Maxwell	$\begin{cases} x = a/\pi \cdot (u + 1 + e^u \cdot \cos(v)) \\ y = a/\pi \cdot (v + e^u \cdot \sin(v)) \end{cases}$	
12	Parabolic	$\begin{cases} x = (u^2 - v^2)/2 \\ y = u \cdot v \end{cases}$	
13	Polar	$\begin{cases} x = u \cdot \cos(v) \\ y = u \cdot \sin(v) \end{cases}$	
14	Rose	$\begin{cases} x = ((u^2 + v^2)^{1/2} + u)^{1/2} / (u^2 + v^2)^{1/2} \\ y = ((u^2 + v^2)^{1/2} - u)^{1/2} / (u^2 + v^2)^{1/2} \end{cases}$	

4. CONCLUSIONS

In this paper are proposed contributions concerning CAD generating of 2D supershapes in different coordinate systems, based on descriptive geometry and computer programming. Using different coordinate systems in representation of the 2D supershapes interesting 2D shapes can be obtained.

The Maple programming language permits to obtain conclusions for shape and profile of 2D supershapes, but also facilitate the new ideas in shape design.

ACKNOWLEDGEMENTS

This work has partly been funded by the Romanian Ministry of Education, Research and Youth, through The National University Research Council, Grant PN–II–ID–PCE–2007–1, code ID_1107, 2007 – 2009.

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