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FREE CONVECTION FLOW OF A DUSTY GAS PAST A SEMI-INFINITE VERTICAL PLATE WITH HEAT FLUX

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ABSTRACT:
A numerical solution for the flow of fluid with dusty particles past a semi-infinite vertical plate with heat flux is obtained by an implicit finite difference method, which is unconditionally stable. Gas velocity, dust-velocity, temperature, skin friction and Nusselt number are shown graphically. It is observed that the velocity of the dusty-gas and dust particles decreases with increasing mass concentration of the dust. The temperature decreases as Gr increases. Local skin friction decreases as κ increases and local Nusselt number increases by decreasing value of mass concentration of dust and increasing value of Gr.
Key words: Dusty gas, finite –difference, heat flux, skin friction, Nusselt number.

1. INTRODUCTION

Two-dimensional free convection flow past a semi-infinite plate with different boundary conditions has attracted the attention of many researchers. However, in nature, the fluid in pure form is rarely available. Air and water contains impurities like dust particles and foreign bodies. This is connected with a wide range of natural occurring phenomena and practical applications. The study of the flow of dusty fluids is of practical importance, particularly through packed beds, sedimentation, environmental pollution, chemical reactors, combustion systems, pneumatic transport, and centifugal separation of particles.

The unsteady natural convection flow past a semi-infinite vertical plate was first solved by Hellums and Churchill (1962) using explicit finite difference scheme. As explicit finite difference scheme has its own deficiencies, a more efficient implicit finite difference method has been solved by Soundalgekar and Ganesan (1981). Saffmanz (1962) has formulated the basic equations for the flow of dusty fluid. Michael and Miller (1966) studied the flow of a dusty gas past an impulsively started horizontal plate using the momentum equations given by saffman (1962) and solved by Laplace transform technique. Micheal (1968) considered the effect on the steady flow past a sphere of uniform upstream distribution of dust particles having a small relaxation time. Soundalgekar and Gokhale (1984) studied the flow of a dusty gas past an impulsively started infinite vertical plate by employing an implicit finite difference technique. Ganesan (2004) studied the unsteady free convection flow of a dusty gas past a semi- infinite inclined plate with constant heat flux.

An explicit finite difference solution of flow of a dusty gas past a uniformly accelerated horizontal plate in a viscous incompressible gas was presented by Das et al (1992). Due to the importance of dusty viscous flows in other technological fields various studies have appeared in the literature. The effects of heat transfer on the flow of dusty gas past a semi-infinite isothermal vertical plate with variable temperature have not yet received the attention in the literature.

2. MATHEMATICAL ANALYSIS

A problem of transient, laminar, two-dimensional flow of a dusty gas past a semi-infinite vertical plate with constant heat flux is considered here. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, the plate starts moving impulsively in the vertical direction with constant velocity u_o against gravitational field, the plate and the dusty gas are assumed to be at the same temperature T'_∞. At time $t' > 0$, the heat assumed to be supplied at a constant rate from the plate. The effect of viscous dissipation effects is assumed to be negligible in the energy equation. It is assumed that the dust particles are uniform in size and shape and they are uniformly distributed. The number density of the dust particles is constant through out the motion.

The governing equations of the flow based on Saffman model of dusty viscous incompressible fluid with usual Boussinesq's approximation are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T'-T'_\infty)+\nu\frac{\partial^2 u}{\partial y^2}+\frac{K_1 n_o}{\rho}(u_p-u) \tag{2}$$

$$m\left(\frac{\partial u_p}{\partial t'}+u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)=K_1(u-u_p) \tag{3}$$

$$\rho\, C_p\left(1+\frac{mn_o}{\rho}\frac{C_{pL}}{C_p}\right)\left(\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}\right)=\kappa\frac{\partial^2 T'}{\partial y^2} \tag{4}$$

If the dust particles are spheres of radius d, then Stokes resistance coefficient $k_1 = 6\pi\mu d$. Equation of continuity for dust

$$\frac{\partial N_o}{\partial t'}+\frac{\partial (N_o u_p)}{\partial x}+\frac{\partial (N_o v_p)}{\partial y}=0 \tag{5}$$

By our assumption, the number density N_o of dust particles is constant through out the motion. Hence Equation (5) reduces to

$$\frac{\partial u_p}{\partial x}+\frac{\partial v_p}{\partial y}=0 \tag{6}$$

The initial and boundary conditions are as follows

$$\begin{aligned} &t'\le 0:\ u=0,\ \ v=0,\ \ u_p=0,\ \ v_p=0,\ \ T'=T'_\infty \ \text{ for all x and y}\\ &t'>0:\ u=u_o,\ \ v=0,\ \ u_p=0,\ \ v_p=0,\ \ \frac{\partial T'}{\partial y}=-\frac{q_w}{k}\ \text{ at } y=0\\ &u=0,\ \ u_p=0,\ \ T'=T'_\infty \ \text{ at } x=0\\ &u\to 0,\ \ u_p\to 0,\ \ T'\to T'_\infty \ \text{ as } y\to\infty \end{aligned} \tag{7}$$

On introducing the following non-dimensional quantities:

$$\begin{gathered} X=\frac{xu_o}{\nu},\ Y=\frac{yu_o}{\nu},\ U=\frac{u}{u_o},\ V=\frac{v}{u_o},\ U_p=\frac{u_p}{u_o},\ V_p=\frac{v_p}{u_o},\\ t=\frac{t'u_o^2}{\nu},\ T=\frac{T'-T'_\infty}{T'_w-T'_\infty},Gr=\frac{\nu g\beta(T'_w-T'_\infty)}{u_o^3},\ f=\frac{mn_o}{\rho}\\ Pr=\frac{\nu}{\alpha},\ h=\frac{fC_p}{C_{pL}},\ \lambda=Pr(1+h),\ K=\frac{\nu K_1}{mu_o^2} \end{gathered} \tag{8}$$

Equations (1),(6),(2),(3) and (4) are reduced to the following non-dimensional form

$$\begin{aligned} \frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y} &= 0\\ \frac{\partial U_p}{\partial X}+\frac{\partial V_p}{\partial Y} &= 0\\ \frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y} &= Gr\,T+\frac{\partial^2 U}{\partial Y^2}+fK(U_p-U)\\ \frac{\partial U_p}{\partial t}+U_p\frac{\partial U_p}{\partial X}+V_p\frac{\partial U_p}{\partial Y} &= K(U-U_p)\\ \frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y} &= \frac{1}{\lambda}\frac{\partial^2 T}{\partial Y^2} \end{aligned} \tag{9}$$

The corresponding initial and boundary conditions in non-dimensional quantities are

$t\le 0$: U = 0, V= 0, U_p = 0, V_p = 0, T = 0 for all x and y

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$$t>0:\quad U = 1,\ V = 0,\ U_p = 0,\ V_p = 0,\ \frac{\partial T}{\partial Y} = -1 \quad \text{at} \quad Y = 0 \tag{10}$$
$$U = 0,\quad U_p = 0,\quad T = 0 \qquad \text{at} \quad X = 0$$
$$U \to 0,\ U_p \to 0,\ T \to 0 \qquad \text{as} \quad Y \to \infty$$

3. NUMERICAL TECHNIQUE

An implicit finite difference scheme of Crank-Nicolson type has been used to solve the governing non-dimensional equations (9) under the conditions (10). The method of solving the above finite difference equations using the Crank-Nicolson type has been discussed by Soundalgekar and Ganesan (1981).

Here the region of integration is considered as a rectangle with sides X_{max} (=1) and Y_{max} (=14), where Y_{max} corresponds to $Y = \infty$ which lies very well outside both the momentum and thermal boundary layers. The maximum of Y was chosen as 14 after some preliminary investigations so that the last two of the boundary conditions (10) are satisfied with in the tolerance limit 10^{-5}. After experimenting with a few set of mesh sizes, the appropriate mesh sizes $\Delta X = 0.05$, $\Delta Y = 0.25$ with time step $\Delta t = 0.01$ are considered for calculation.

4. RESULTS AND DISCUSSION

The velocity profiles for the dusty gas are shown in Figure 1. The velocity of the dusty gas increases with time. It is observed that due to the presence of dust particles in the fluid, time taken to reach the steady state increases. However due to the presence of dust particles, the velocity of the gas decreases since these dust particles oppose the motion of the gas.

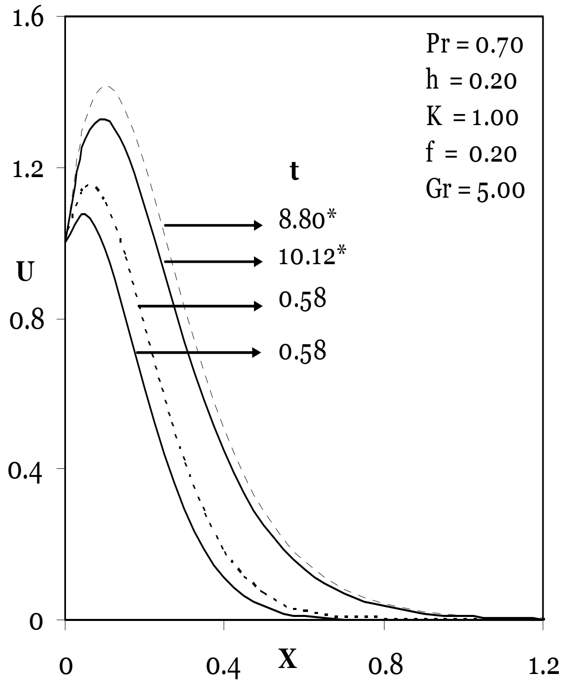

Figure 1. Transient velocity profiles at X = 1.0

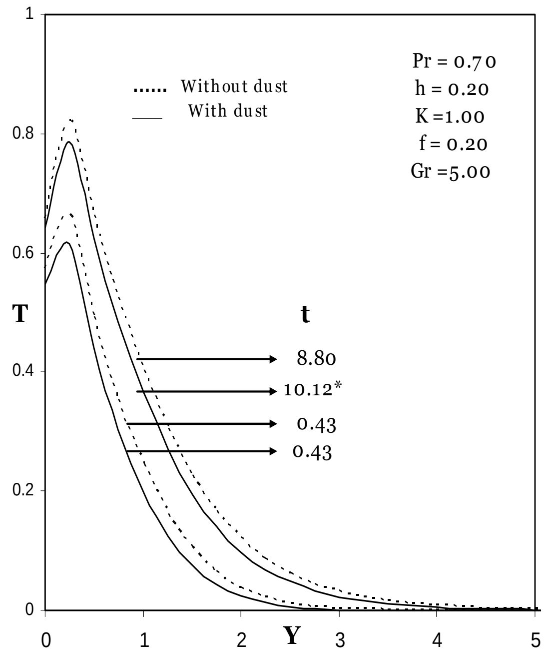

Figure 2. Transient temperature profiles at X = 1.0 (*-steady state)

The temperature profiles for the fluid with dust particles are shown in Figure 2. It is observed that the temperature increases with time. Here time taken to reach the steady state increases since these these dust particles carry away heat and the fluid gets cooled.

The effects of the parameters Gr, f on the dusty gas velocity are shown in Figure 3. An increase of f (mass concentration of the dust) leads to a fall in the gas velocity because these dust particles oppose the motion of the gas. It is observed that time taken to reach the steady state increases with f. Also the dusty gas velocity increases as Gr increases in irrespective of time. The transient dust-particles velocity are shown in Figure 4. This figure shows the effect of f and Gr in dust particles velocity(U_p). According to the numerical results, the velocity of the dust particles decreases with increasing mass concentration of the dust. This is quite nature. The time taken to reach the steady state increases for higher values of f. It is also observed that the velocity of dust particles increases with an increasing value of κ. Dust- particles velocity increases with increasing values of Gr.

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Temperature profiles are presented in Figure 5. for different values of Gr. It is observed that the temperature decreases as Gr increases.

Knowing the velocity and temperature field, it is customary to study the skin-friction and the Nusselt number. The local values of skin-friction and Nusselt number in dimensionless form are as follows:

$$\tau_X = -\left(\frac{\partial U}{\partial Y}\right)_{Y=0}, \quad Nu_X = -X\left[\frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}}\right] \tag{11}$$

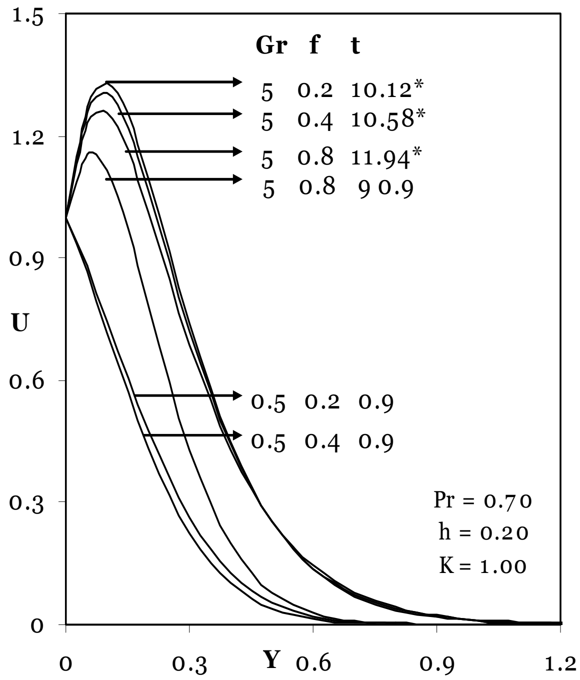

Figure 3. Transient dusty-gas velocity profiles at X=1.0 for different Gr, f (*-steady

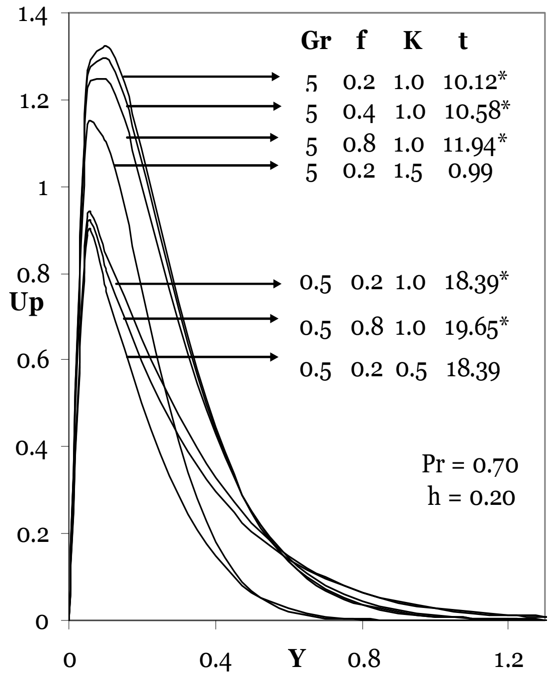

Figure 4. Transient dust particles velocity profiles at X =1.0 for different f, k (*-steady state)

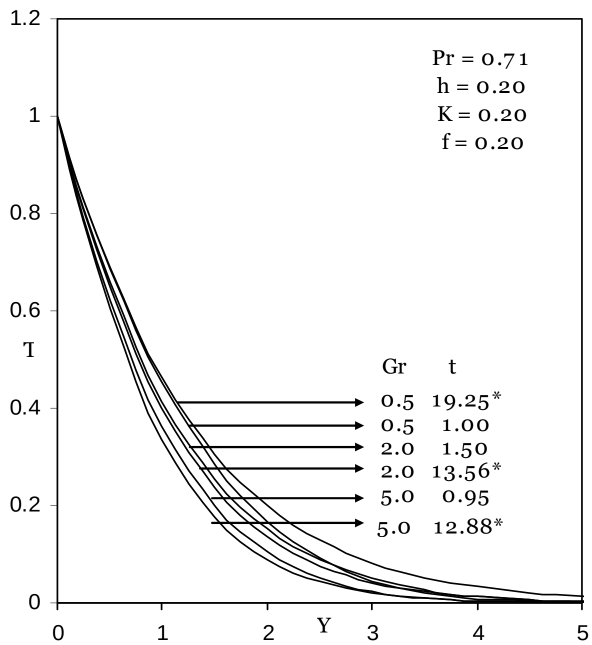

Figure 5. Temperature profiles for different values of Gr (*-steady state)

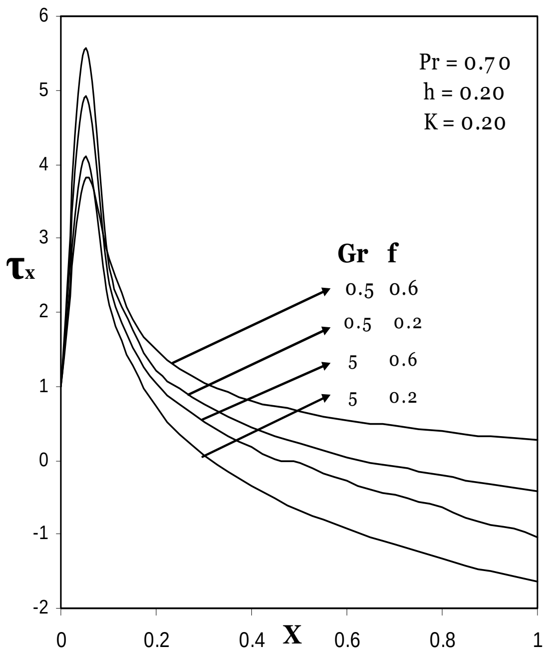

Figure 6. Local skin-friction

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The derivatives involved in equations (11) are evaluated by using a five-point approximation formula and then the integrals are evaluated by Newton-Cotes closed integration formula. The effects of f and Gr on local skin-friction are shown in Figure 6. The local wall shear stress increases as f increases. It is observed that local skin friction decreases as Gr increases.

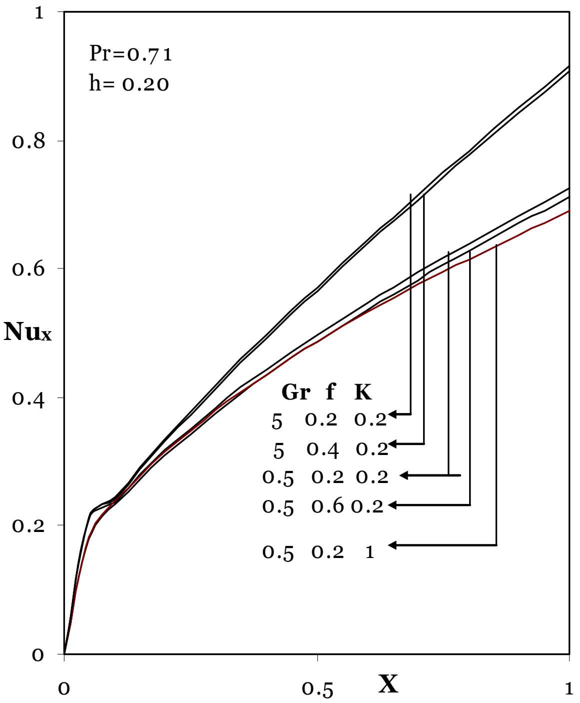

Figure 7. Local Nusselt number

Local Nusselt number for different values of f, κ and Gr are shown in Figure 7. It increases as X increases. It is observed that local Nusselt number increases by decreasing value of f and κ and it is increases as increasing value of Gr.

5. CONCLUSION

Finite difference study has been carried out for the dusty gas flow past a semi-infinite isothermal vertical plate with heat flux. The dimensionless governing equations are solved by an implicit finite difference scheme of Crank-Nicolson method. Conclusions of the study are as follows:

1. The velocity of the dusty gas increases with time. Time taken to reach the steady state increases due to the presence of dust particles in the fluid,. i.e., the velocity of the gas decreases since these dust particles oppose the motion of the gas.
2. The temperature increases with time. The time taken to reach the steady state increase since these dust particles carry away heat and the fluid gets cooled.
3. The velocity of the dusty gas decreases with increasing mass concentration of the dust because these dust particles oppose the motion of the gas. Also the dusty gas velocity increases as Gr increases in irrespective of time.
4. The velocity of the dust particles decreases with increasing mass concentration of the dust. It is also observed that the velocity of dust particles increases with an increasing value of κ. Dust-particles velocity increases with increasing values of Gr.
5. It is observed that the temperature decreases as Gr increases.
6. It is observed that local skin friction decreases as κ increases and local Nusselt number increases by decreasing value of f and increasing value of Gr.

NOMENCLATURE

C_p - specific heat of the fluid at constant pressure
C_{pL} - specific heat of the particle at constant pressure
d - radius of the spherical particle
f - mass concentration of dust
Gr- Grashof number
h - non dimensional dust parameter
κ - thermal conductivity
K_1 - Stokes resistance coefficient
K - nondimensional dust parameter
M - mass of a dust particle
N_o - number density of small dust particle
Pr - Prandtl number
T' - temperature
t - dimensionless time
u,v - velocity components of fluid in x,y directions respectively
u_p,v_p - velocity components of dust particles in x,y-directions respectively
U,V - dimensionless velocity components of fluid in X,Y directions respectively
U_p,V_p - dimensionless velocity components of dust particles in X,Y- directions
x - spatial coordinate along the plate
X- dimensionless spatial coordinate along the plate
y - spatial coordinate normal to the plate
Y- dimensionless spatial coordinate normal to the plate

Greek symbols

α - thermal diffusivity
β - coefficient of volume expansion
λ - nondimensional parameter [λ =Pr(1+h)]
μ - coefficient of viscosity
ν - kinematic viscosity
ρ - density
τ - average skin-friction
τ_x - local skin-friction

Subscript

w - conditions at the wall
∞ - conditions in the free stream

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