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CAD GENERATING OF 3D SUPERSHAPES IN DIFFERENT COORDINATE SYSTEMS

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ABSTRACT:
This paper presents a CAD study for generating of 3D supershapes in different coordinate systems, based on descriptive geometry and computer programming. The Maple programme helps in obtaining conclusions refering to shape and profile for different 3D supershapes, but also facilitate the design of new sculptures. Results from this study are applied in geometric constructions and computer aided design used in sculpture design.
KEYWORDS: *Sculpture design, 3D supershapes, CAD*

1. THEORETICAL CONSIDERATIONS

In modern art, mathematics and CAD constitute a powerfull tool in creation, edit and visualisation in interactive ways of new and fascinating artistical forms.

The supershapes, has been proposed by by Johan Gielis in 1997 [1], to describe numerous complex curves and shapes that are found in nature. Supershapes have been widely studied in different fields such as biological structures, computer graphics, computer vision and sculpture design [2, 3, 4].

For the sake of simplicity, geometric objects are represented in cartesian (rectangular) coordinate system.

In polar coordinates, the radius of a supershape is defined as:

$$r(\phi)=\frac{1}{\left[\left(\left|\frac{1}{a}\cos\left(\frac{m}{4}\phi\right)\right|\right)^{n2}+\left(\left|\frac{1}{b}\sin\left(\frac{m}{4}\phi\right)\right|\right)^{n3}\right]^{1/n1}},\ \text{with } n_i\in R^+, m\in R^+, a\in R_0^+, b\in R_0^+. \quad (1)$$

where a and b are positive real numbers but not zero; m and n_i are positive real numbers.

Parameters a and b control the scale, m represents the number of symmetry axes, and coefficients $n1$, $n2$ and $n3$ control the shape. For $n2 \neq n3$, asymmetrical shapes can be obtained.

Extension to 3D is performed by the spherical product of two 2D supershapes.

The 3D parametric formula of supershapes, in polar coordinates, can be written as:

$$\begin{cases} x(\theta,\phi)=r_1(\theta)\cos(\theta)r_2(\phi)\cos(\phi) \\ y(\theta,\phi)=r_1(\theta)\sin(\theta)r_2(\phi)\cos(\phi); \quad -\pi/2\le\phi\le\pi/2; \quad -\pi\le\theta\le\pi \\ z(\theta,\phi)=r_2(\phi)\sin(\theta) \end{cases} \quad (2)$$

where radius r of a supershape, in polar coordinates.

The implicit function $F(x, y, z)$ can be written as:

$$F(x,y,z)=1-\frac{x^2+y^2+r_1^{\,2}z^2}{r_1^{\,2}r_2^{\,2}}. \quad (3)$$

2. COMPUTER PROGRAMME

Using the Maple programming language next procedure was written for generation of 3D supershapes, in different coordinate systems [5].

Programme:

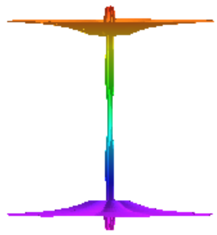
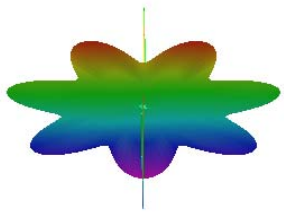
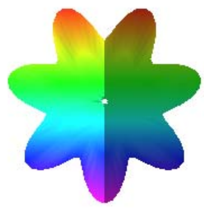
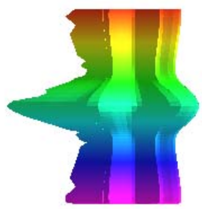
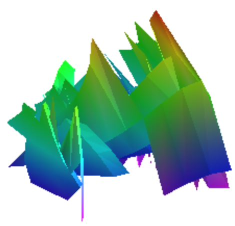
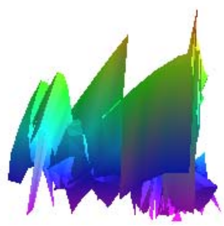
```
> restart:
> with(plots):
> C:= [bipolarcylindrical, bispherical, casscylindrical, confocalellip, conical, cylindrical, ellcylindrical, hypercylindrical, invcasscylindrical, invellcylindrical, invprospheroidal, logcoshcylindrical, logcylindrical, maxwellcylindrical, oblatespheroidal, paraboloidal, rectangular, rosecylindrical, sixsphere, spherical]:
> a := 1: b := 1: m := 7: n1 := 0.2: n2 := 1.7: n3 := 1.7: coordinate_system := C[25]: nops(C):
> r1 := evalf((abs(1/a*cos(m*theta/4))^n2+abs(1/b*sin(m*theta/4))^n3)^(-1/n1)):
> r2 := evalf((abs(1/a*cos(m*phi/4))^n2+abs(1/b*sin(m*phi/4))^n3)^(-1/ n1)):
> x := r1*cos(theta)*r2*cos(phi):
> y := r1*sin(theta)*r2*cos(phi):
> z := r2*sin(phi):
> Supershape:=plot3d([x,y,z], theta=-180*Pi/180..180*Pi/180, phi=-90*Pi/ 180..90*Pi/180, style=patchnogrid, grid=[100,100], lightmodel=light4, shading =zhue, transparency=1, coords=coordinate_system): display(Supershape);
```

3. GRAPHICAL REPRESENTATIONS IN DIFFERENT COORDINATE SYSTEMS

Let's consider the particular 3D supershape with next parameters:
$a = 1$, $b := 1$, $m = 7$, $n1 = 0.2$, $n2 = 1.7$, $n3 = 1.7$.

The graphical representations of this 3D supershape in different coordinate systems are given in Table 1.

Table 1. The graphical representations of this 3D supershape in different coordinate systems

No.	Type of coordinate system	Graphical representation in 3D	
		a) Axonometric representation	b) Representation on plane [XOZ]
1	Bipolarcylindrical		
2	Bispherical		
3	Casscylindrical		
4	Confocalellip		

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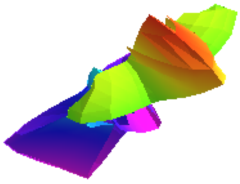
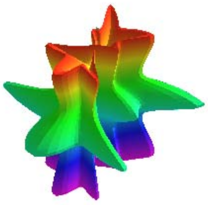
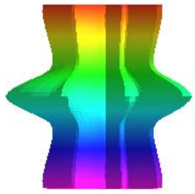
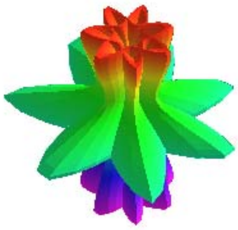
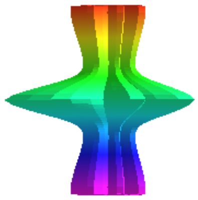
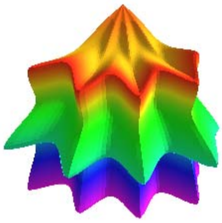
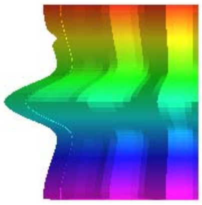
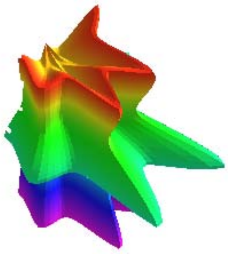
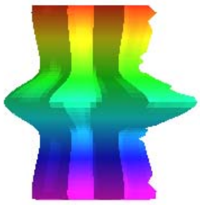
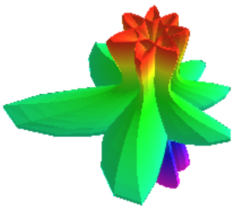
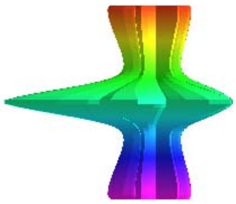
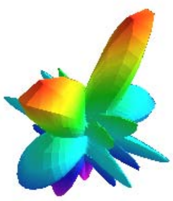
5	Conical		
6	Cylindrical		
7	Ellcylindrical		
8	Hypercylindrical		
9	Invcasscylindrical		
10	Invellcylindrical		
11	Invprospheroidal		

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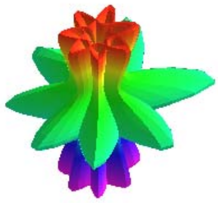
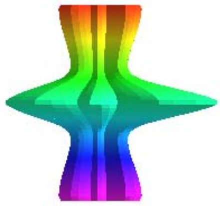
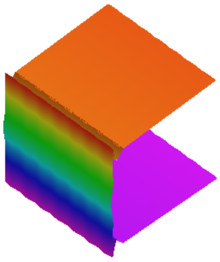
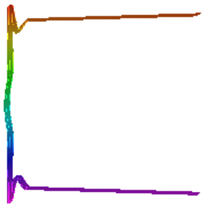
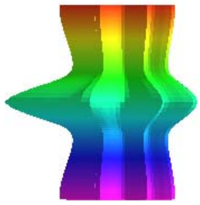
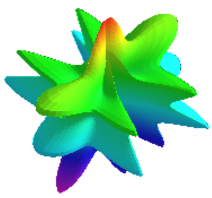
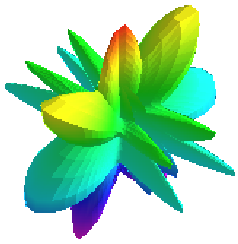
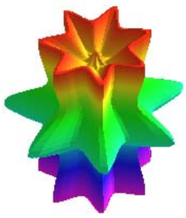
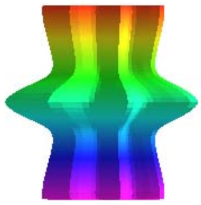
12	Logcoshcylindrical		
13	Logcylindrical		
14	Maxwellcylindrical		
15	Oblatespheroidal		
16	Paraboloidal		
17	Cartesian (rectangular)		
18	Rosecylindrical		

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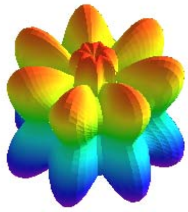
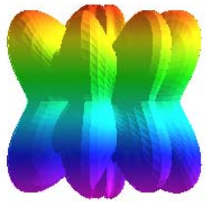
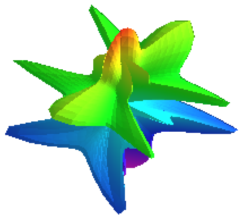
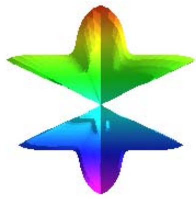
19	Sixsphere		
20	Spherical		

4. CONCLUSIONS

This paper investigates a CAD method for generation of 3D supershapes in different coordinate systems, based on descriptive geometry and computer programming. Using different coordinate systems in representation of the 3D supershapes interesting 3D shapes can be obtained.

The Maple programming language helps in obtaining conclusions refering to shape and profile of 3D supershapes, but also facilitate the new ideas in shape design.

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