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MHD EFFECTS ON OSCILLATING VERTICAL PLATE IN THE PRESENCE OF CHEMICAL REACTION OF FIRST ORDER

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ABSTRACT
First order chemical reaction on unsteady free convective flow of a viscous incompressible flow past an infinite vertical oscillating plate with variable temperature and mass diffusion in the presence of magnetic field. The plate temperature as well as wall concentration are raised linearly with time. An exact solution to the dimensionless governing equations has been obtained by the Laplace transform method, when the plate is oscillating harmonically in its own plane. The effects of velocity, and concentration field are studied for different parameters like magnetic field parameter, phase angle, chemical reaction parameter, thermal Grashof number, mass Grashof number and time are studied. It is observed that the velocity increases with decreasing phase angle ωt or magnetic field parameter.

Keywords:
chemical reaction, oscillating, vertical plate, heat and mass , magnetic field.

1. INTRODUCTION

MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion of earths core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. In the field of power generation, MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants.

The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar *et al* [6]. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar *et al* [8]. The dimensionless governing equations were solved using Laplace transform technique.

Chambre and Young [1] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das *et al* [2] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das *et al* [3]. The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level.

The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was solved by Soundalgekar[4] . The effect on the flow past a vertical oscillating plate due to a combination of concentration and temperature differences was studied extensively by Soundalgekar and Akolkar[5]. The effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux has been studied by Soundalgekar et al.[7].

However the simultaneous heat and mass transfer effects on infinite oscillating vertical plate in the presence of chemical reaction and MHD is not studied in the literature. It is proposed to study chemical reaction effects on unsteady flow past infinite vertical oscillating plate with variable temperature and mass diffusion, in the presence of external magnetic field. The dimensionless governing equations are tackled using the Laplace transform technique. The solutions are in terms of exponential and complementary error function.

2. MATHEMATICAL FORMULATION

Hydromagnetic effects on infinite vertical oscillating plate with variable temperature and uniform mass diffusion in the presence of chemical reaction of first order. Here, the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate starts oscillating in its own plane with frequency ω'. The temperature of the plate as well as the wall concentration are raised linearly with respect to time The plate is also subjected to a uniform magnetic field of strength B_0. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} \tag{2}$$

In most cases of chemical reactions, the rate of reaction depends on the concentration of the species itself. A reaction is said to be of the order n, if the reaction rate if proportional to the n^{th} power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself.

With the following initial and boundary conditions:

$$\begin{array}{llll} t' \le 0: & u = 0, & T = T_\infty, & C' = C'_\infty \quad \text{for all } y \\ t' > 0: & u = u_0 \cos\omega' t', & T = T_\infty + (T_w - T_\infty)\,A\,t', & C' = C'_\infty + (C'_w - C'_\infty)\,A\,t' \quad \text{at } y = 0 \\ & u = 0, & T \to T_\infty, & C' \to C'_\infty \quad \text{as } y \to \infty \end{array} \tag{4}$$

where $A = \frac{u_0^2}{\nu}$.

On introducing the following non-dimensional quantities:

$$\begin{gathered} U = \frac{u}{u_0},\ t = \frac{t' u_0^2}{\nu},\ Y = \frac{y u_0}{\nu},\ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \\ Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ Gc = \frac{\nu g\beta^*(C'_w - C'_\infty)}{u_0^3},\ \omega = \frac{\omega'\nu}{u_0^2}, \\ Pr = \frac{\mu C_p}{k},\ Sc = \frac{\nu}{D},\ M = \frac{\sigma B_0^2 \nu}{\rho u_0^2},\ K = \frac{\nu K_1}{u_0^2} \end{gathered} \tag{5}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} - M\,U \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - K\,C \tag{8}$$

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{lllll} & U = 0, & \theta = 0, & C = 0, & \text{for all } Y, t \le 0 \\ t > 0: & U = \cos\omega t, & \theta = t, & C = t, & \text{at } Y = 0 \\ & U = 0, & \theta \to 0, & C \to 0 & \text{as } Y \to \infty \end{array} \tag{9}$$

The solutions are obtained for hydromagnetic flow field in the presence of first order chemical reaction. The equations (6) to (8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = t\left[(1+2\eta^2 \Pr)\,\mathrm{erfc}(\eta\sqrt{\Pr}) - \frac{2}{\sqrt{\pi}}\eta\sqrt{\Pr}\exp(-\eta^2\Pr)\right] \tag{10}$$

$$C = \frac{t}{2}\left[\exp(2\eta\sqrt{KtSc})\,\mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{Kt})\right]$$
$$-\frac{\eta\sqrt{Sct}}{2\sqrt{K}}\left[\exp(-2\eta\sqrt{KtSc})\,\mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{Kt}) - \exp(2\eta\sqrt{KtSc})\,\mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{Kt})\right] \tag{11}$$

$$U = \frac{\exp(i\omega t)}{4}\left[\exp(2\eta\sqrt{(M+i\omega)t})\mathrm{erfc}(\eta+\sqrt{(M+i\omega)t}) + \exp(-2\eta\sqrt{(M+i\omega)t})\,\mathrm{erfc}(\eta-\sqrt{(M+i\omega)t})\right]$$
$$+\frac{\exp(-i\omega t)}{4}\left[\exp(2\eta\sqrt{(M-i\omega)t})\,\mathrm{erfc}(\eta+\sqrt{(M-i\omega)t}) + \exp(-2\eta\sqrt{(M-i\omega)t})\mathrm{erfc}(\eta-\sqrt{(M-i\omega)t})\right]$$
$$+[c(1+at)+d(1+bt)]\left[\exp(2\eta\sqrt{Mt})\mathrm{erfc}(\eta+\sqrt{Mt}) + \exp(-2\eta\sqrt{Mt})\mathrm{erfc}(\eta-\sqrt{Mt})\right] \tag{12}$$
$$-2c\,\mathrm{erfc}\left(\eta\sqrt{pr}\right)$$
$$-c\exp(at)\left[\exp(2\eta\sqrt{(M+a)t})\mathrm{erfc}(\eta+\sqrt{(M+a)t}) + \exp(-2\eta\sqrt{(M+a)t}\;\mathrm{erfc}(\eta-\sqrt{(M+a)t})\right]$$

where, $a = \frac{M}{\Pr-1}, b = \frac{M-K\,Sc}{Sc-1}, c = \frac{Gr}{2a^2(1-\Pr)}$ and $d = \frac{Gc}{2b^2(1-Sc)}$.

where $\eta = \frac{Y}{2\sqrt{t}}$ and erfc is called complementary error function.

3. DISCUSSION OF RESULTS

The numerical values of the velocity and concentration are computed for different parameters like Magnetic field parameter, chemical reaction parameter, Schmidt number, thermal Grashof number, mass Grashof number and time. The purpose of the calculations given here is to assess the effects of the parameters $\omega t, M, K, Gr, Gc$ and Sc upon the nature of the flow and transport. The solutions are in terms of exponential and complementary error function.

Figure 1 demonstrates the effect of the concentration profiles for different values of the chemical reaction parameter ($K = 0.2, 2, 5, 10$), $Sc = 0.6$ and time $t = 0.2$. It is observed that the concentration increases with decreasing chemical reaction parameter.

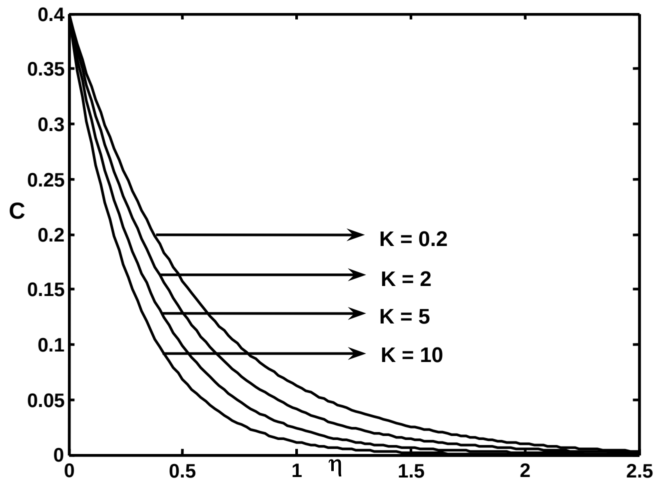

Figure 1. Concentration profiles for different values K

The velocity profiles for different phase angles ($\omega t = 0, \pi/6, \pi/3, \pi/2$), M=2,K=2

$Gr = Gc = 5, Sc = 0.6, \Pr = 7.0$ and $t = 0.2$ are shown in figure 2. It is observed that the velocity increases with decreasing phase angle ωt. It is interesting to note that at $\omega t = 0$, the plate is considered to be vertical and the velocity profile developed from $U = 1$. At $\omega t = \pi/2$, the plate is considered to be horizontal and the velocity profiles developed from the origin.

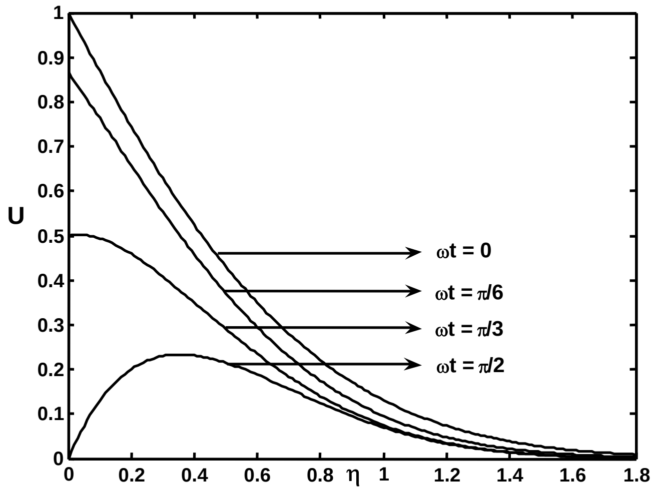

Figure 2. Velocity profiles for different values of ωτ

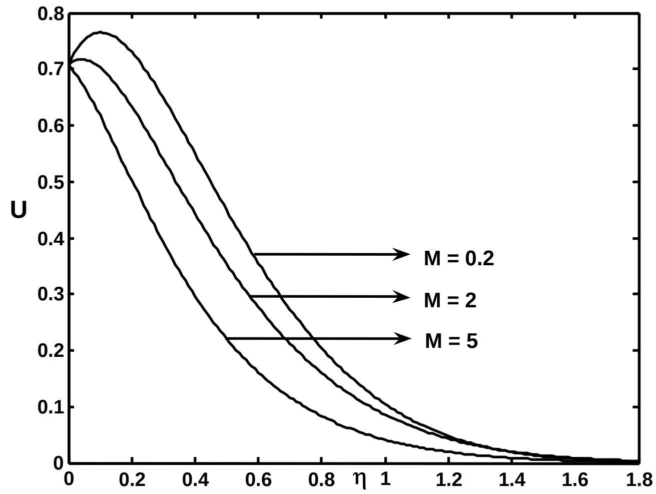

Figure 3. Velocity profiles for different values of M

Figure 3. demonstrates the effects of the magnetic field parameter on the velocity when $(M = 0.2, 2, 5)$, $\omega t = \pi/4, Gr = Gc = 5, Sc = 0.6, K = 8, Pr = 7.0$ and $t = 0.6$. It is observed that the velocity increases with decreasing magnetic field parameter.

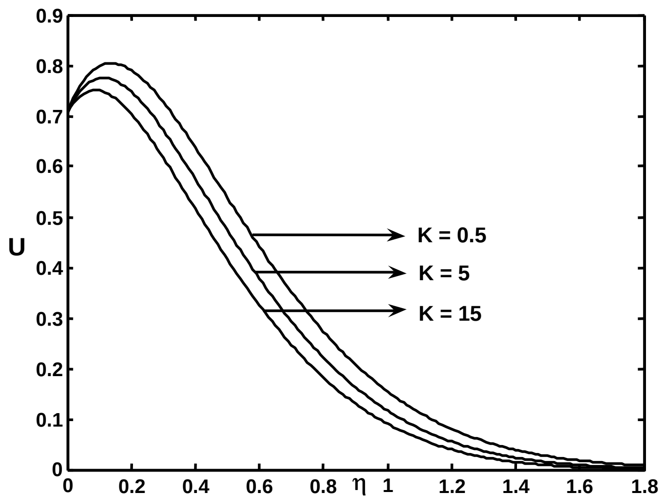

Figure 4. Velocity profiles for different values of K

Figure 4 illustrates the effect of the velocity for different values of the reaction parameter $(K = 0.5, 5, 15)$, $\omega t = \pi/4$, $M = 0.2$, $Gr = Gc = 5$, $Sc = 0.6$, $Pr = 7.0$ and $t = 0.6$. The trend shows that the velocity increases with decreasing chemical reaction parameter. The velocity profiles for different thermal Grashof number $(Gr = 5, 10)$, mass Grashof number $(Gc = 5, 10)$, $\omega t = \pi/4$, $K = 8$, $M = 2$, $Sc = 0.6$, $Pr = 7.0$ and time $t = 0.2$ are shown in Figure 5. It is clear that the velocity increases with increasing thermal Grashof number or mass Grashof number.

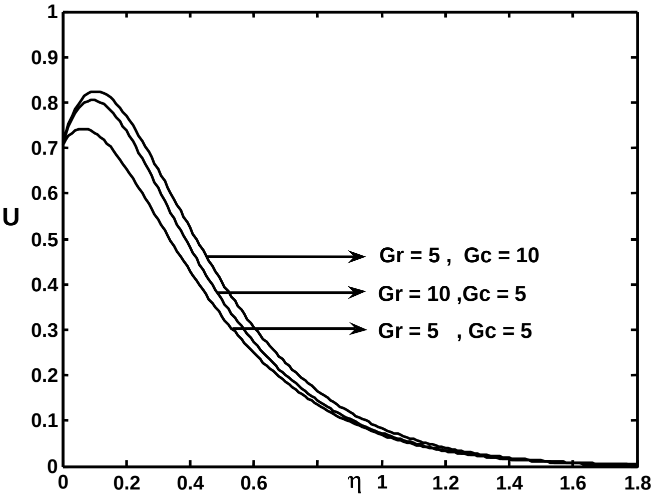

Figure 5. Velocity profiles for different values of Gr and Gc

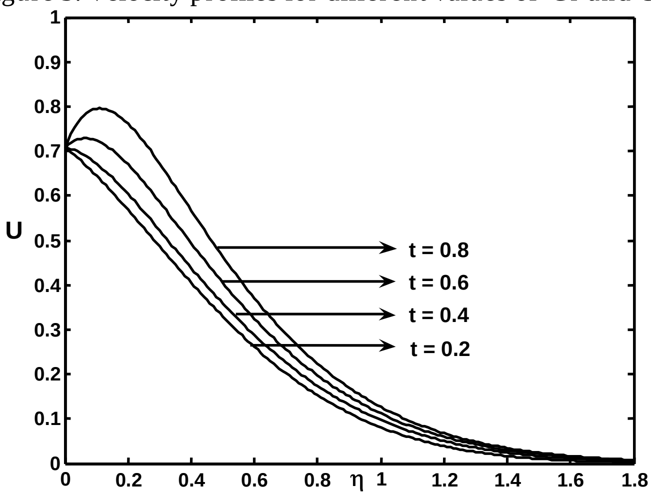

Figure 6. Velocity profiles for different values of t

The effect of velocity profiles for different time $(t = 0.2, 0.4, 0.6, 0.8)$, $M = 2$, $K = 2$ $\omega t = \pi/4$, $Gr = Gc = 5$, $Pr = 7.0$, $Sc = 0.6$ are shown in Figure 6. In this case, the velocity increases gradually with respect to time t.

4. CONCLUSION

Hydromagnetic flow past an oscillating infinite vertical plate with variable temperature and mass diffusion, in the presence of chemical reaction of first order studied. The dimensionless equations are solved using Laplace transform technique. The effect of velocity, temperature and concentration for different parameters like ωt, M, K, Gr, Gc and t are studied. The study concludes that the velocity increases with decreasing phase angle ωt, magnetic field parameter M and chemical reaction parameter K. The trend is just reversed with respect to time t. As expected, the plate concentration increases with decreasing chemical reaction parameter K.

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