

RADIATION AND CHEMICAL REACTION EFFECTS ON FLOW PAST A VERTICAL PLATE WITH RAMPED WALL TEMPERATURE

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ABSTRACT

An analysis is performed to study the effects of thermal radiation and chemical reaction on the unsteady free convection and mass transform flow of a viscous incompressible fluid past an infinite vertical plate containing a ramped type temperature profile with respect to time. The fluid considered here is a gray, absorbing/emitting radiation but a non-scattering medium. It is assumed that there exists a homogeneous first order chemical reaction between the fluid and species concentration. The dimensionless governing coupled linear partial differential equations are solved using the Laplace transform technique. The influence of the various parameters, entering into the problem, on the velocity field and temperature field is extensively discussed with the help of graphs.

Keywords: Free convection, mass diffusion, thermal radiation, chemical reaction, ramped temperature, Laplace transform technique.

1. PRRODUCTION

Free convection flows past a vertical surface or plate were studied extensively in the literature due to its applications in engineering and environmental processes. Several investigations were performed using both analytical and numerical methods under different thermal conditions which are continuous and well-defined at the wall. Practical problems often involve wall conditions that are nonuniform or arbitrary. To understand such problems, it is useful to investigate problems subject to step change in wall temperature. Keeping this in view, Schetz [1] made an attempt to develop an approximate analytical model for free convection flow from a vertical plate with discontinuous wall temperature conditions. Several investigations were continued on this problem using an experimental technique [2], numerical methods [3], and by using series expansions [4, 5]. Lee and Yovanovich [6] presented a new analytical model for the laminar natural convection from a vertical plate with step change in wall temperature. The validity and accuracy of the model is demonstrated by comparing with the existing results. Chandran et al. [7] have presented an analytical solution to the unsteady natural convection flow of an incompressible viscous fluid near a vertical plate with ramped wall temperature and they have compared the results with constant temperature. Saha et al. [8] investigated the natural convection boundary layer adjacent to an inclined semi-infinite flat plate subjected to ramp heating. The flow development from the start-up to an eventual steady state has been described based on scaling analysis and verified by numerical simulations. Narahari and Anwar Beg [9] studied the effects of mass transfer and free convection currents on the flow past an infinite vertical plate with ramped wall temperature. Recently Singh and Singh [10] analyzed transient MHD free convection flow near a semi-infinite vertical wall having ramped temperature.

Free convection flows occur not only due temperature difference, but also due to concentration difference or the combination of these two. The study of combined heat and mass transfer play an important role in the design of chemical processing equipment, nuclear reactors, formation and dispersion of fog etc. The effect of presence of foreign mass on the free convection flow past a semi-infinite vertical plate was first studied by Gebhart and Pera [11]. Soundalgekar [12] has studied mass transfer effects on flow past an impulsively started infinite isothermal vertical plate. Dass et al. [13] considered the mass transfer effects on flow past an impulsively started infinite isothermal vertical plate with constant mass flux. Muthucumaraswamy et al. [14] presented an exact solution to the problem of flow past an impulsively started infinite vertical plate in the presence of uniform heat and mass flux at the plate using Laplace transform technique.

Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion devices for



aircraft, missiles, satellites and space vehicles are examples of such engineering applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. England and Emery [15] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Soundalgekar and Takhar [16] have considered the radiative free convective flow of an optically thin gray-gas past a semi-infinite vertical plate. Radiation effects on mixed convection along an isothermal vertical plate were studied by Hossain and Takhar [17]. Raptis and Perdikis [18] studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Dass et al.[19] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace transform technique. Muthucumaraswamy and Janakiraman [20] studied MHD and Radiation effects on moving isothermal vertical plate with variable mass diffusion. Rajesh and Varma [21] studied Radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. Recently Rajesh and Varma [22] analyzed radiation effects on MHD flow through a porous medium with variable temperature or variable mass diffusion.

The study of heat and mass transfer problems with chemical reaction are of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. A few representative fields of interest in which combined heat and mass transfer along with chemical reaction play an important role in chemical process industries such as food processing and polymer production. Chambre and Young [23] have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Dass et al. [24] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Dass et al. [25]. The dimensionless governing equations were solved by the usual Laplace Transform technique. Recently Rajesh and Varma [26] presented chemical reaction effects on free convection flow past an exponentially accelerated vertical plate

However, the effects of thermal radiation and chemical reaction on free convection and mass transform flow past an infinite vertical plate subject to discontinuous or non-uniform wall temperature conditions have not been studied in the literature. Hence it is now proposed to study the effects of thermal radiation and chemical reaction on the free convection and mass transform flow of an incompressible viscous fluid past an infinite vertical plate subject to ramped wall temperature. Exact solutions to the non dimensional coupled linear partial differential equations are derived by the Laplace transform method.

2. MATHEMATICAL ANALYSIS

The unsteady free convection and mass transform flow of a viscous incompressible fluid past an infinite vertical plate has been considered. The fluid considered here is a gray, absorbing/emitting radiation but a non-scattering medium. The flow is assumed to be in x'-direction which is taken along the plate in the vertically upward direction, and y'-axis is taken normal to the plate. Initially, for time $t' \leq 0$, both the fluid and the plate are assumed to be at the same temperature T'_{∞} and concentration C'_{∞} . At time t' > 0, the temperature of the plate is raised or lowered to $T'_{\infty} + (T'_w - T'_{\infty})\frac{t'}{t_0}$ when $t' \leq t_0$, and thereafter, for $t' > t_0$, is maintained at the constant

temperature T'_w and the concentration level at the plate is raised to C'_w . It is assumed that there exists a homogeneous first order chemical reaction between the fluid and species concentration. Applying the Boussinesq approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = g\beta \left(T' - T'_{\infty}\right) + g\beta^* \left(C' - C'_{\infty}\right) + v\frac{\partial^2 u'}{\partial {y'}^2}$$
(1)

$$\rho C_{p} \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^{2} T'}{\partial v'^{2}} - \frac{\partial q_{r}}{\partial v'}$$
(2)

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial {y'}^2} - K_r \left(C' - C'_{\infty} \right)$$
(3)

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With the initial and boundary conditions

$$u' = 0, \quad T' = T'_{\infty}, \quad C' = C'_{\infty} \quad \text{for all } y' \ge 0 \quad \text{and} \quad t' \le 0$$

$$u' = 0 \quad \text{at} \quad y' = 0 \quad \text{for} \quad t' > 0$$

$$T' = T'_{\infty} + \left(T'_{w} - T'_{\infty}\right) \frac{t'}{t_{0}} \quad \text{at} \quad y' = 0 \quad \text{for} \quad 0 < t' \le t_{0}$$

$$T' = T'_{w} \quad \text{at} \quad y' = 0 \quad \text{for} \quad t' > t_{0}$$

$$C' = C'_{w} \quad \text{at} \quad y' = 0 \quad \text{for} \quad t' > 0$$

$$f' = 0, \quad T' \to T'_{\infty}, \quad C' \to C'_{\infty}, \quad \text{as} \quad y' \to \infty \quad \text{for} \quad t' > 0$$
(4)

u' = 0, $T' \to T'_{\infty}$, $C' \to C'_{\infty}$, as $y' \to \infty$ for t' > 0The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a^* \sigma \left(T_{\infty}'^4 - T'^4 \right)$$
(5)

It is assumed that the temperature differences with in the flow are sufficiently small that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_{α} and neglecting the higher order terms, thus

$$T'^{4} \cong 4T_{\infty}'^{3}T' - 3T_{\infty}'^{4}$$
(6)

By using equations (5) and (6), equation (2) reduces to

$$\rho C_{p} \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^{2} T'}{\partial {y'}^{2}} + 16a^{*} \sigma T_{\infty}'^{3} \left(T_{\infty}' - T' \right)$$
⁽⁷⁾

On introducing the following non-dimensional quantities:

$$y = \frac{y'}{\sqrt{vt_0}}, t = \frac{t'}{t_0}, u = \frac{u'}{G_r} \sqrt{\frac{t_0}{v}}, \theta = \frac{T' - T'_{\infty}}{T'_w - T'_{\infty}}, P_r = \frac{\rho v C_p}{\kappa}, G_r = \frac{g\beta (T'_w - T'_{\infty})t_0^{\frac{3}{2}}}{\sqrt{v}},$$

$$\frac{16a^* \sigma T'_{\infty}^3 v t_0}{\sqrt{v}}, C = \frac{C' - C'_{\infty}}{\sigma t_0}, S_r = \frac{v}{r}, G_m = \frac{g\beta^* (C'_w - C'_{\infty})t_0^{\frac{3}{2}}}{\sqrt{v}}, N = \frac{G_m}{\sigma t_0}, K = K_r t_0, \quad (8)$$

$$R = \frac{16a^* \sigma T_{\infty}'^3 v t_0}{\kappa}, C = \frac{C' - C_{\infty}'}{C_w' - C_{\infty}'}, S_c = \frac{v}{D}, G_m = \frac{g\beta^* (C_w' - C_{\infty}')t_0^2}{\sqrt{v}}, N = \frac{G_m}{G_r}, K = K_r t_0,$$
(8)

in equations (1) - (3), leads to

$$\frac{\partial u}{\partial t} = \theta + NC + \frac{\partial^2 u}{\partial y^2}$$
(9)

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} - \frac{R}{P_r} \theta$$
(10)

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} - KC \tag{11}$$

According to the above non- dimensionalisation process, the characteristic time t_0 can be defined as

$$t_{0} = \frac{\nu^{\frac{1}{3}}}{\left[g\beta(T'_{w} - T'_{\infty})\right]^{\frac{2}{3}}}$$
(12)

The initial and boundary conditions given by equation (4) now become u = 0, $\theta = 0$, C = 0 for all $y \ge 0$ and $t \le 0$

$$u = 0 \text{ at } y = 0 \text{ for } t > 0; \ \theta = t \text{ at } y = 0 \text{ for } 0 < t \le 1$$

$$\theta = 1 \text{ at } y = 0 \text{ for } t > 1; \ C = 1 \text{ at } y = 0 \text{ for } t > 0$$

$$u \to 0, \ \theta \to 0, \ C \to 0 \text{ as } y \to \infty \text{ for } t > 0$$
(13)

All the physical parameters are defined in the nomenclature. The dimensionless governing equations (9) and (11), subject to the boundary conditions (13), are solved by the usual Laplace transform technique and the solutions are derived as follows.









$$+\left[\exp\left(y\sqrt{R}\right)erfc\left(\frac{y\sqrt{P_r}}{2\sqrt{t-1}}+\sqrt{\frac{R(t-1)}{P_r}}\right)\right]\left[-\frac{H(t-1)}{2d}+\frac{b(t-1)H(t-1)}{2d}+\frac{byP_rH(t-1)}{4d\sqrt{R}}\right]$$
$$-\frac{N}{2e}\left[\exp\left(-y\sqrt{KS_c}\right)erfc\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}-\sqrt{Kt}\right)+\exp\left(y\sqrt{KS_c}\right)erfc\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}+\sqrt{Kt}\right)\right]$$
$$+\frac{N\exp\left(-ft\right)}{2e}\left[\exp\left(-y\sqrt{(K-f)S_c}\right)erfc\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}-\sqrt{(K-f)t}\right)\right]$$
$$+\exp\left(y\sqrt{(K-f)S_c}\right)erfc\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}+\sqrt{(K-f)t}\right)\right]$$
(16)

Where $b = \frac{R}{P_r - 1}$, d = bR, $e = KS_c$, $f = \frac{KS_c}{S_c - 1}$ and H(t - 1) is the unit step function

3. RESULTS AND DISCUSSION

In order to get physical insight into the problem, the numerical values of the velocity and temperature are computed for different values of the physical parameters such as K(Chemical reaction parameter), R(Radiation parameter), N(Buoyancy ratio parameter), Sc(Schmidt number) and t(time). The Buoyancy ratio parameter, N, represents the ratio between mass and thermal buoyancy forces. When N=0, there is no mass transfer and the buoyancy force is due to the thermal diffusion only. N>0 implies that mass buoyancy force acts in the same direction of thermal buoyancy force i.e., the buoyancy- assisted case, while N<0 means the mass buoyancy force acts in the opposite direction i.e., the buoyancy-opposed. The value of the prandtl number is chosen as Pr=0.71, which represents air.

The velocity profiles for different values of K (Chemical reaction parameter) are presented in figures (1) and (2) at t=0.5 and t=1.3 in the presence of both aiding (N>0) and opposing flows (N<0) respectively. It is observed that the velocity decreases with an increase in K in the presence of aiding flows (N>0) whereas it increases in the presence of opposing flows (N<0). It is also observed that the velocity increases with y near the plate and becomes maximum and then decreases away from the plate and finally takes asymptotic value for all values of K.



Figures (3) and (4) illustrate the influences of R (Radiation parameter) on the velocity field at t=0.5 and t=1.3 for the cases of both aiding (N>0) and opposing flows (N<0) respectively. The velocity is found to decrease with an increase in R in the presence of both aiding and opposing flows. It is also found that the velocity increases with y near the plate and becomes maximum and then decreases away from the plate and finally takes asymptotic value for all values of R.

Figures (5) and (6) display the effects of N (Buoyancy ratio parameter) on the velocity field at t=0.5 and t=1.3 for the cases of both aiding (N>0) and opposing flows (N<0) respectively. It is found that the velocity increases with an increase in N in the presence of aiding flows (N>0) whereas it decreases in the presence of opposing flows (N<0).













In figures (7) and (8) the velocity profiles are shown for different values of Sc (Schmidt number) at t=0.5 and t=1.3 for the cases of both aiding (N>0) and opposing flows (N<0) respectively. It is observed that the velocity decreases with increasing Schmidt number in the presence of aiding flows (N>0). But the reverse effect is observed in the presence of opposing flows (N<0).

Figures (9) and (10) represent the velocity profiles for different values of t (time) for both aiding (N>0) and opposing flows (N<0) respectively. The velocity is observed to increase with an increase in t for both aiding and opposing flows. It is also observed that the velocity increases with y near the plate and becomes maximum and then decreases away from the plate and finally takes asymptotic value for all values of t. More over the points of maxima on the curves get shifted to the right as t increases.

The temperature profiles for different values of R (Radiation parameter) and t (time) are presented in figures (11) and (12). It is observed that the temperature decreases with an increase in R (Radiation parameter) whereas it increases with an increase in t (time). It is also observed that the temperature is maximum near the plate and decreases away from the plate and finally takes asymptotic value for all values of R and t.

ACKNOWLEDGEMENT

I would like to acknowledge Dr. S. Vijaya kumar varma, Professor of Mathematics, S.V.University, Tirupati (A.P), India for fruitful discussion on the subject of this paper.

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