ANNALS OF FACULTY ENGINEERING HUNEDOARA
- INTERNATIONAL JOURNAL OF ENGINEERING
Tome IX (Year 2011). Fascicule 1. (ISSN 1584 - 2665)

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# APPLICATION OF THE QUEUING THEORY MODEL WITH STOCHASTIC CREATION OF SERVICING CHANNELS

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**ABSTRACT:** This study outlines and emphasises the fact that high downtime values at the exploitation stage and over-production of cars is the source of economic and especially environmental losses, which could be minimised by a more efficient use of the said products. By moment observation and subsequent analyses it was possible to identify the structure of the need of car usage at a standard residential area, which became the basis for constructing a logistic model of products treatment with the criterion function of minimising the downtimes of a selected machine product – a car. This study describes the products treatment model the specific feature of which is the stochastic creation of servicing channels, and is a contribution to the development of the queuing theory.
**KEYWORDS:** queuing theory, moment observation, downtime, environment

## INTRODUCTION

The maturity of the society can presently be assessed not only from a material point of view, but also from the point of view of care for the environment and efforts made to reduce the impacts on the burden placed on the environment [1]. Rational use of products is one of the activities where the environmental effects achieved without any material and energy inputs is such as in the case of few remedial measures to protect the environment [1].

Based on moment observation and subsequent analysis, the structure of the need of car usage in a standard residential area (Figure 1) was identified, upon which the logistic model of products treatment was constructed with a criterion function of minimising the downtimes of the selected machine product - the car. Selected input data: number of cars at a fully occupied parking lot: 53 cars, average number of cars at the parking lot: 39.2821 cars, average downtime rate: 74.1172%.

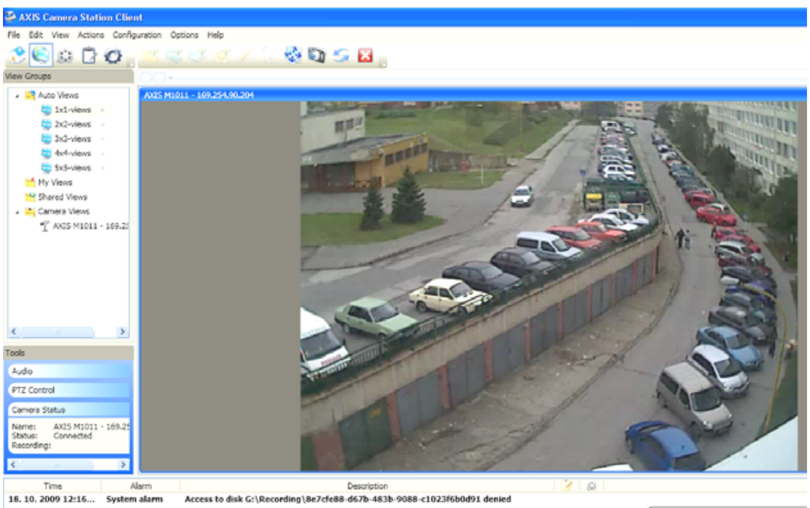
Figure 1 – A Record from Moment Observation by IP Camera

The current percentage value of the cars downtime rate was determined at 74.12%. Can our society afford such an uneconomical use of a product? In what part of production or in what other phase of product life cycle could we afford so high downtime values? And until when is this situation sustainable? Moreover, the raising number of cars on roads results in ever higher downtime values of these products. It is necessary to introduce system measures that would enable a more economical use of the ever increasing number of cars and, at the end, reduce over-production of these products.

The analysis of the numbers of parking places of cars and statistical observation of the features of the need for cars manifested the problem of describing the probability conditions at the parking lot, the need for parking places and the overall use of cars. The modelling of these conditions led to the conclusion that the Kendall classification of models lacks in its description the condition where the number of servicing channels changes, and it counts with a fixed number of servicing channels only, which is either given or is the subject of calculation for the set input requests.

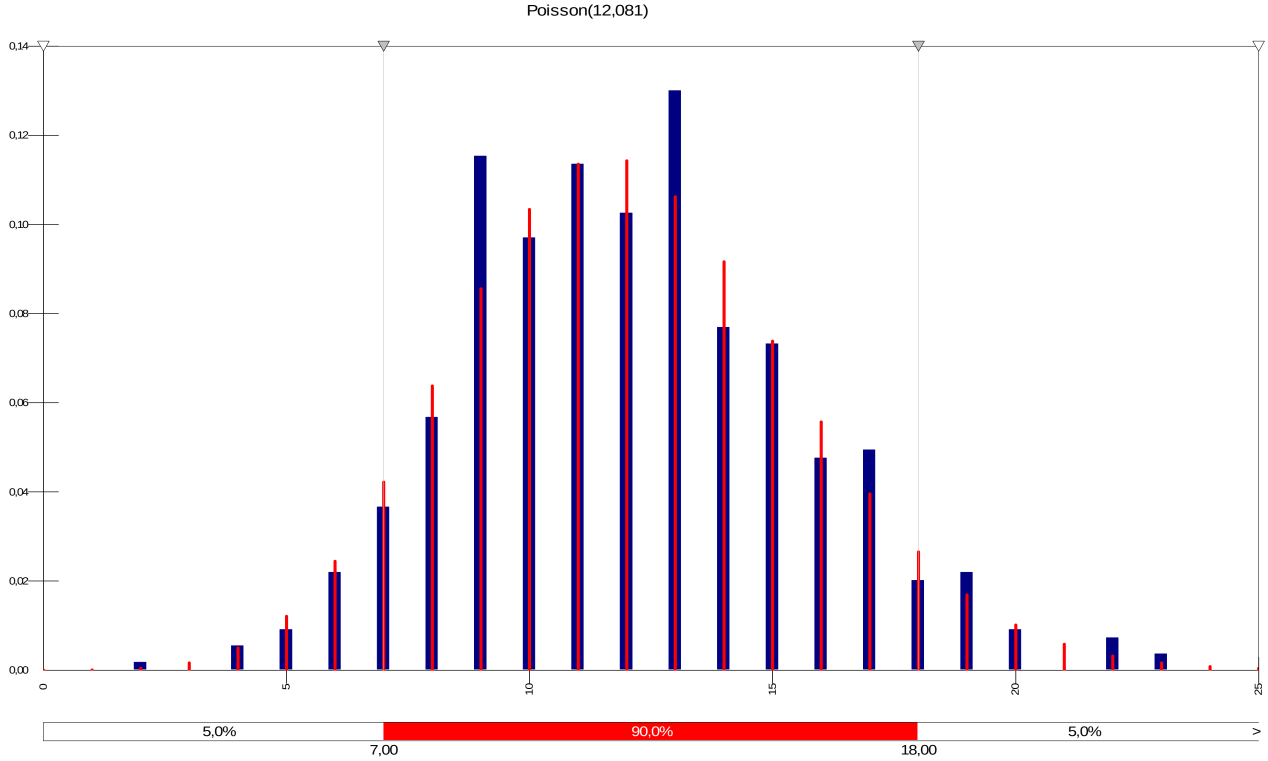


Figure 2 **–** Graphical Depiction of the Approximation of Quantities Distribution
Poisson Distribution Indicator: DEPARTURES

The description and analysis of the distribution of quantities at which car departures and arrivals occur in the observed parking lot resulted in the finding that their clearly stochastic nature in the creation of servicing requests can be described as the Poisson process, which was used for the capacity calculations of the needs for cars.

## ❖ NEW OPERATIONAL LOGISTICAL MODEL OF PRODUCTS TREATMENT. THE MODEL DEFINITION ACCORDING TO KENDALL CLASSIFICATION

An analysis of motor cars usage demonstrated their enormously low use. It is evident that this situation is unbearable and that it is necessary to consider a new way of using cars, especially in city transport, where there is an absence of parking places, and the abundance of cars causes traffic problems.

The way of using cars can be characterised by a model that has not been described by the Kendall classification [1] and outlines a method of a more efficient use of cars.

We will model the state of arrivals and departures of cars at a parking lot, as shown in Figure 3, supposing that the arriving (parked) cars are available for use by other customers, and therefore form servicing channels. The creation of these channels is stochastic with respect to the course of observations. The leaving cars then represent the accepted servicing requests, and also have a stochastic nature.

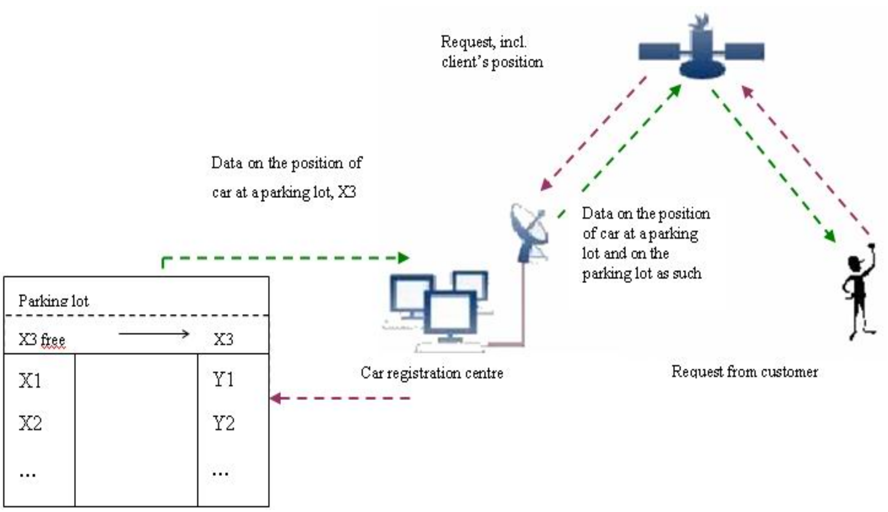


Figure 3 **–** Simplified Principle of the Proposed System

Figure 4 show that the cars arriving at the parking lot form a random flow, which follows the Poisson distribution with $\lambda_B$ parameters according to measurements and recent theoretical findings. Paradoxically, the arriving cars do not constitute a servicing request, but a request for creating new servicing channels. The leaving cars then characterise a similar stochastic Poissonian flow of servicing requests with $\lambda_A$ parameter, i. e. customers with available stand-by cars.

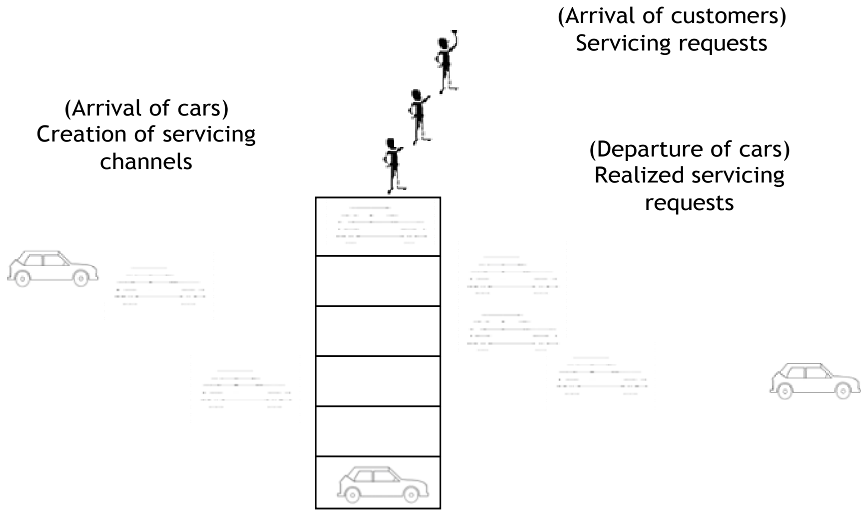


Figure 4 - Graphical Depiction of the New Model

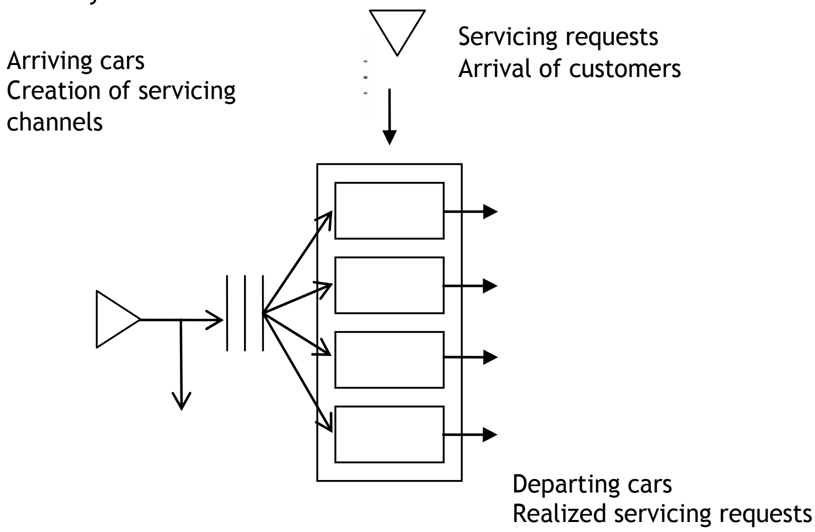


Figure 5 - New Model Scheme

The task of the model is to find out under what circumstances the servicing requests (transportation) of customers will be realised and what will be the probability that servicing is not possible at a time interval (0, t).

If the model description is based on the Kendall classification, which has six symbols:

a ... type of input ,
b ... type of servicing,
c ... number of channels,
d ... waiting discipline,
e ... limitation of the queue length,
f ... limitation of the entry flow.

Classification symbols:

D … deterministic input or servicing.
P … Poisson distribution.
M … Exponential distribution.
Ek … Erlang distribution of the k series.
U … Uniform distribution.
G … General distribution.

## ❖ DESCRIPTION OF THE NEW MODEL ACCORDING TO KENDALL CLASSIFICATION

The type of inputs is stochastic; while observing the quantity of the creation of servicing requests (arrival of customers, i. e. departing cars observed) it can be stated that it constitutes the Poisson distribution of quantities. The type of servicing is deterministic and constant (the average time of getting in the car is considered only). The number of servicing channels is stochastic (Poisson distribution, too) and is characterised by arrivals of cars. The waiting discipline is FIFO (first in, first out). The limitation of the queue length is constant, and is determined by the number of places in the parking lot. Limitation of the entry flow – number of cars in the catchment area. This means that if:

a … type of input, P (Poisson distribution),
b … type of servicing, D (deterministic),
c … number of channels, P (Poisson distribution),
d … waiting discipline distribution, F (FIFO),
e … limitation of the queue length, K (constant, number of places at the parking lot),
f … limitation of the entry flow, K (size of population, number of cars in the town).

than the following model characteristics can be attributed to the given model:

$$M = P \,/\, D \,/\, P \,/\, F \,/\, K \,/\, K \qquad (1)$$

The new model proposed in THO is characterised by a change in its classification structure; it has not been reported yet in the Kendall classification of models, and is exceptional, in particular, for the stochastic creation of servicing channels.

The task of the model is to characterise the need for cars under the condition of not losing the comfort of municipal car transport.

Servicing requests:

Event A (car departures measured), A indices

The number of servicing requests arising at a certain time interval (0, t) is a random variable with Poisson distribution and $\lambda_A$ parameter, where $\lambda_A > 0$:

$$p(x) = \frac{\lambda_A{}^x}{x!}.e^{-\lambda_A} \quad (2)$$

Probability of exactly "k" servicing requests input at a time interval (0, t):

$$P_A(K_t = k) = \frac{(\lambda_A\, t)^k}{k!} e^{-\lambda_A\, t} \quad (k = 0, 1, 2, \dots) \quad (3)$$

The mean value and variance of the number of requests created at a time interval (0, t):

$E(K_t) = \lambda_A \qquad D(K_t) = \lambda_A$.

$E(K_t) = \lambda_A$, which is input intensity (average number of requests entering the system at a unit of time).

Creation of servicing channels:

Event B (car arrivals measured), B indices

The number of servicing channels created at a time interval (0, t) is a random variable with Poisson distribution and $\lambda_B$ parameter, where $\lambda_B > 0$:

$$p(x) = \frac{\lambda_B{}^x}{x!}.e^{-\lambda_B} \quad (4)$$

Probability of creating exactly "k" servicing channels at a time interval (0, t):

$$P_B(K_t = k) = \frac{(\lambda_B\, t)^k}{k!} e^{-\lambda_B\, t} \quad (k = 0, 1, 2, \dots) \quad (5)$$

The mean value and variance of the number of created servicing channels at a time interval (0, t):

$E(K_t) = \lambda_B \qquad D(K_t) = \lambda_B$.

$E(K_t) = \lambda_B$, which is intensity of creating the number of channels (average number of new servicing channels at a unit of time).

Creation of downtimes:

Event C (measured number of cars at the parking lot – downtimes) C indices

The number of cars at various moments is a random variable with Poisson distribution and $\lambda_C$ parameter, where $\lambda_C > 0$:

$$p(x) = P_C(X = k) = \frac{\lambda_C{}^x}{x!}.e^{-\lambda_C} \quad (6)$$

Mean value and variance: $E(K_t) = \lambda_C = \overline{x}$.

$E(K_t) = \lambda_C$, which is average downtime intensity (average number of cars at a parking lot during the observed period).

The time between the departures of two consecutive requests realized is a random variable with exponential distribution and $\lambda_D$ parameter (time between departures of two cars).

The time between arrivals of two consecutive creation of servicing channels is a random variable with exponential distribution and $\lambda_E$ parameter (time between arrivals of cars).

If we assume that the parked cars can be used by the next customer, it is evident that this model can be managed as a system of stocks management in real time with the probability function and possibly with the intelligent mode

## ❖ MODEL BEHAVIOUR ANALYSIS

An analysis of cars movement at a representative parking lot shows that the creation of servicing requests (departures of cars observed) follows the Poisson distribution with parameters:

$\lambda_A$ = 12.0806, t = 1 hour for (k = 0, 1, 2, ...). The creation of servicing channels (arrivals of cars observed) follows the Poisson distribution with parameters:

$\lambda_B$ = 12,1502, t = 1 hour for (k = 0, 1, 2, ...) whereas

$$P_A(K_t = k) = \frac{(\lambda_A\, t)^k}{k!} e^{-\lambda_A\, t} \quad (k = 0, 1, 2, \dots)$$

$$P_B(K_t = k) = \frac{(\lambda_B\, t)^k}{k!} e^{-\lambda_B\, t} \quad (k = 0, 1, 2, \dots)$$

Let us assume that the downtimes, i. e. the observed number of cars at a parking lot, which is also indicated by the results of analyses, is a random variable with Poisson distribution of quantities and parameter $\lambda_c = 39{,}2821$.

Table 1 – Probability of creating “k” requests for servicing, or “k” servicing channels

| k | $P_A(k)$ | $P_B(k)$ | $\sum_k^{25} P_B(k)$ | | $P(A/B_{\min.})$ | $P_C(x{=}k)$ | $\sum_k^{53} Pc(x=k)$ |
|---|---|---|---|---|---|---|---|
| | Departures | Arrivals | Min. „k“ arriving | | | Downtimes | Min. „k“ standing |
| 0 | | | | | | 8.71009E-18 | 0.985154 |
| 1 | | 6.42E-05 | 1.00E+00 | | | 3.4215E-16 | 0.985154 |
| 2 | 0.000414 | 0.00039 | 0.999561 | > | 4.14E-04 | 6.72018E-15 | 0.985154 |
| 3 | 0.001666 | 0.001581 | 0.999171 | > | 1.67E-03 | 8.79941E-14 | 0.985154 |
| 4 | 0.00503 | 0.004801 | 0.99759 | > | 5.04E-03 | 8.64147E-13 | 0.985154 |
| 5 | 0.012154 | 0.011667 | 0.992789 | > | 1.22E-02 | 6.7891E-12 | 0.985154 |
| 6 | 0.024472 | 0.023627 | 0.981122 | > | 2.49E-02 | 4.44483E-11 | 0.985154 |
| 7 | 0.042233 | 0.04101 | 0.957495 | > | 4.41E-02 | 2.49431E-10 | 0.985154 |
| 8 | 0.063775 | 0.062285 | 0.916485 | > | 6.96E-02 | 1.22477E-09 | 0.985154 |
| 9 | 0.085604 | 0.084086 | 0.8542 | > | 1.00E-01 | 5.34573E-09 | 0.985154 |
| 10 | 0.103415 | 0.102166 | 0.770114 | > | 1.34E-01 | 2.09991E-08 | 0.985154 |
| 11 | 0.113574 | 0.112849 | 0.667948 | > | 1.70E-01 | 7.49898E-08 | 0.985154 |
| 12 | 0.114337 | 0.114261 | 0.555099 | > | 2.06E-01 | 2.45479E-07 | 0.985154 |
| 13 | **0.106251** | 0.106792 | 0.440838 | > | 2.41E-01 | 7.41764E-07 | 0.985154 |
| 14 | **0.091684** | 0.092682 | 0.334046 | > | 2.74E-01 | 2.08129E-06 | 0.985154 |
| 15 | **0.07384** | 0.075074 | 0.241364 | < | 3.06E-01 | 5.45048E-06 | 0.985151 |
| 16 | **0.055752** | 0.05701 | 0.16629 | < | 3.35E-01 | 1.33816E-05 | 0.985146 |
| 17 | 0.039619 | 0.040746 | 0.10928 | < | 3.63E-01 | 3.09211E-05 | 0.985132 |
| 18 | **0.02659** | 0.027504 | 0.068534 | < | 3.88E-01 | 6.74801E-05 | 0.985101 |
| 19 | **0.016906** | 0.017588 | 0.04103 | < | 4.12E-01 | 0.000139514 | 0.985034 |
| 20 | **0.010212** | 0.010685 | 0.023442 | < | 4.36E-01 | 0.000274019 | 0.984894 |
| 21 | **0.005875** | 0.006182 | 0.012757 | < | 4.61E-01 | 0.000512573 | 0.984620 |
| 22 | **0.003226** | 0.003414 | 0.006575 | < | 4.91E-01 | 0.000915223 | 0.984108 |
| 23 | **0.001694** | 0.001804 | 0.003161 | < | 5.36E-01 | 0.001563124 | 0.983193 |
| 24 | | 0.000913 | 0.001357 | < | 4.14E-04 | 0.002558446 | 0.981629 |
| 25 | | 0.000444 | 0.000444 | < | 1.67E-03 | 0.004020040 | 0.979071 |
| 26 | $\sum_{k=26}^{53} Pc(x=k) = 0{,}975051$ | | | | | 0.006073667 | 0.975051 |
| . | | | | | | 0.008836522 | 0.968977 |
| . | | | | | | 0.012397026 | 0.960141 |
| . | | | | | | … | … |
| 53 | | | | | | 0.006329771 | 0.006330 |

(Remark to Table 1: The capacity of the parking lot is maximum 53 cars. The arrival quantities attained values from 1 to 25. The departure quantities spanned from 2 to 23.)

The probability that the exactly “k” cars would arrive at the parking lot at the assumed k interval is $P_B(k)$. The probability the at least “k” cars would arrive at the parking lot is, according to [1], the sum of probabilities:

$$\sum_k^{25} P_B(k) = P_B(k) + P_B(k+1) + P_B(k+2) + \dots + P_B(k_{\max.}) \tag{7}$$

Symbol P(A) will now represent the probability that exactly “k” cars would leave the at the observed interval, P(B) as the probability that exactly “k” cars would arrive, P($B_{\min}$.) as the probability that at least “k” cars would arrive at the parking lot, and P(C) as the probability that there are exactly “k” cars at the observed moment. Based on the conditional probability theory according to [1] it is than possible to determine the probability that exactly “k” customers would be served:

$$P(A/C) = \frac{\mathrm{P}(A \cap C)}{\mathrm{P}(C)} \tag{8}$$

whereas events A and C are, with the present way of treating cars, independent from each other and, according to [1], the following applies for events independent from each other: $P(A \cap C) = P(A).(C)$. With the present way of treating cars, events A and B are also independent from each other – everyone has its own car. The following applies to the new model:

$$P(A / B_{min.}) = \frac{P(A \cap B_{min.})}{P(B_{min.})} \tag{9}$$

Exactly "k" cars can leave the parking lot only if at least "k" cars arrive at the parking lot. An analysis of interdependence of the events A and B $_{min.}$ in the case of the proposed model led to the conclusion that $P(A \cap B_{min.}) = P(A)$, that is:

$$P(A / B_{min.}) = \frac{P(A)}{P(B_{min.})} \tag{10}$$

Let us return to the original symbols:

$$P_{A/B}(k) = \frac{P_A(k)}{\sum_{k}^{25} P_B(k)} \tag{11}$$

Table 1 clearly indicates that to satisfy the servicing requests under the proposed model, the number of cars showing a change in the quantity of probabilities in relation to (11) is sufficient, which means that with just 15 cars it will be possible to ensure the comfort of servicing, along with attaining considerable economic and environmental savings.

This model can be considered as a basis for further considerations about the new system of treating cars, described in this study more in detail. This model is also a contribution to the development of the queuing theory, a specific feature of which is the stochastic creation of servicing channels.

An analysis of the need and use of cars at a representative parking constituting a segment of a densely populated agglomeration lot was performed. It is evident that the individual parking localities correspond to each other and create the room for further considerations.

## ❖ CONCLUSION

The results described above show that the present way of using cars is largely uneconomic, and it requires innovative thinking of people and in municipal car traffic system and car industry, and use of modern communication systems potential for its optimization [4].

By applying moment observation, the current percentage value of car downtimes was determined – 74.12%. Based on the queuing theory principles it can be said that the downtime rate of cars can be reduced up to 28.30% by a more effective use of cars.

Perhaps the mostly discussed issue concerning the introduction of the proposed model is the issue of cars ownership, its representative purpose and a sort of natural opposition of conservative-thinking car users in relation to the introduction of the described model. This prejudiced perception of the new model can be avoided by an objective assessment on the basis of exact methods of assessing the benefits and losses of the current condition compared to the proposed condition, such as the CBA analysis [5]. It would be rather unfair to assess these conditions in a manner other than the measurement of preferences. The main objective of this study is to point out the need to introduce a new, more effective system of cars treatment, which will not only be economic, but also environmentally sustainable, as the idea of common usage of cars is technically viable at present, and it is necessary to ensure its general application.

This study was prepared under VEGA 1/0052/08 "System Approach to Rationalisation of Work Processes in Manufacturing Enterprises" and VEGA 1/0679/08 "Integrated System for Innovative Design, Planning, Organisation and Management of Production".

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