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PREDICTIVE CONTROL OF SERVO – PNEUMATIC SYSTEM

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ABSTRACT: This article describes an adaptive control of pneumatic system with the predictive controller that is based on minimizing of the quadratic criterion. As the predictive controller was designed in the state space it is necessary to observe the progress of the state vector by the observer. The design of the observer is also included into this text. At the adaptive control an online identification of the system is an important part and so there is an individual chapter devoted to the proper choice of the model and proper algorithm of recursive identification. The progress of the control of pneumatic system with the designed predictive controller is discussed and shown in the final part of the article.
KEYWORDS: pneumatic system; observer; controller; state – space; MPC

❖ INTRODUCTION

Pneumatic servomechanism is highly astatic and nonlinear system of which nonlinearity is caused mainly by the compressibility of the air and by the friction. During the work of pneumatic system the air is compressed and it changes its temperature and density. To obtain the transfer function of the system that describes the actual state of the system best, the system is identificated by the recursive least - squares method (LSM). The designed predictive controller minimizes the quadratic criterion and works with the state vector, therefore the estimator of reduced order was designed for the estimation of state vector. The estimator was designed from the model structure which is used for current identification.

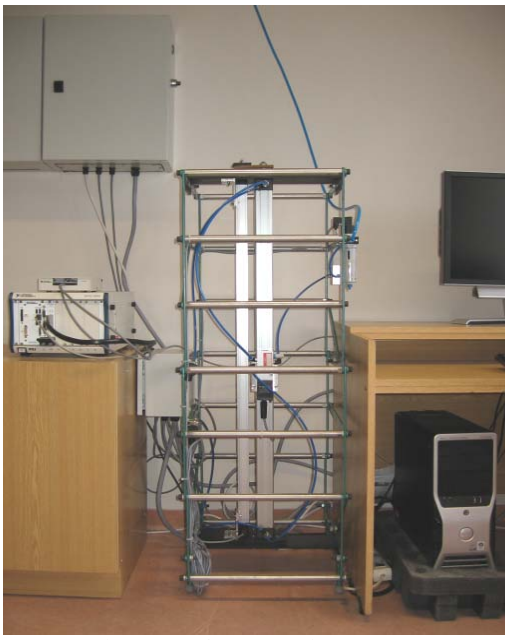

Figure 1: Pneumatic system

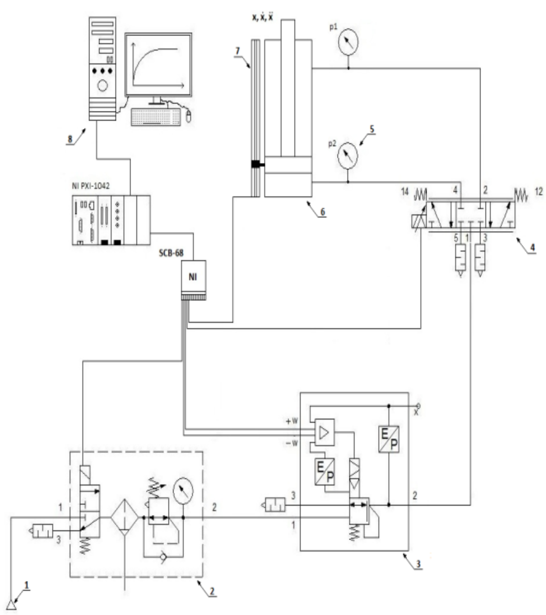

Figure 2: Scheme of pneumatic system

(Legend: 1 – compressor, 2 – reduction valve, 3 – proportional reduction valve, 4 – proportional mass valve, 5 – pressure sensors, 6 - cylinder with one - sided piston rod, 7 - linear potentiometric sensor ,6 – control system)

The pneumatic system requires a fast response of the controller and so it is necessary to ensure sufficiently small period of sampling. The design of predictive controller and the pneumatic system are described in several following paragraphs.

❖ PNEUMATIC SYSTEM

The air to the pneumatic system is brought from the compressor on the input of reduction valve that enables switching on and off the air supply to the system. As the pressure of compressed air in the supply pipe fluctuates the pressure of the brought air to the system has to be kept on constant level by means of proportional reduction valve Festo VPPM-6L-L-1-618-0L10H-A4P-S1C1. After setting the pressure of the air on required value the air is brought to the input of proportional mass valve Festo MPYE-5-1/8-HF-010-B. The pressure in the working chambers of pneumatic cylinder with one-sided piston with the 960 mm stroke is set by means of the movement of slide valve. The movement of the piston of pneumatic cylinder is scanned by the linear potentiometric sensor and the pressure in working chambers of the cylinder is measured out by two DMP-331 pressure sensors. The regulation of pneumatic system is ensured by NI PXI-1042Q control system with PXI-6259 DAQ board. Pneumatic system is represented in Figure 1 and 2.

❖ ONLINE IDENTIFICATION OF PNEUMATICS SYSTEM

For the online identification of the pneumatic system it is necessary at first to determine the structure of the transfer function of the system. This transfer function, that expresses the dependence of the piston position on the control voltage on the servo - valve, was gained by means of the excitation of the system by the pseudorandom signal and the subsequent offline identification that was made by means of quadratic criterion (Eq.1). The offline identification of the system was performed with various structures of the transform function when the most accurate approximation was gained with the transfer function form defined by the Eq.2.

$$J_2 = J_x = \int_0^T [y(t) - y_m(t)]^2 = \sum_{i=0}^{N} [y(i) - y_m(i)]^2 \tag{1}$$

$$G(s) = \frac{b_1 s + b_0}{a_3 s^3 + a_2 s^2 + a_1 s} \tag{2}$$

where *x* is the vector of transfer function parametres, $y_m(t)$ is the response of the identificated system to the excitation signal, $y(t)$ is the response of the system model to the excitation signal, a_4 - a_1 are the coeficients of the nominator of the transfer function, b_1 - b_0 are the coeficients of the numerator of the transfer function.

The transfer function $G(s)$ (Eq.2) was converted from the continuous time to the discrete time and the structure of recursive models intended for the online identification of the pneumatic system was chosen on the basis of the discrete transfer function. Two recursive models from the above mentioned were chosen, these are the ARX and ARMAX models of which difference equations are

$$(1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}) y(k) = (b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}) u(k) + e(k) \tag{3}$$

$$(1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}) y(k) = (b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}) u(k) + (1 + c_1 z^{-1} + c_2 z^{-2} + c_3 z^{-3}) e(k) \tag{4}$$

At the recursive identification of the system by the ARX model LSM with the adaptive directional forgetting [2] was used and at the usage of ARMAX model the RPEM method [5] was used. From the point of view of accuracy both the methods are equivalent but the RPEM method is more stable than the LSM with the adaptive directional forgetting. So the ARMAX model in form [3,3,3,1], was chosen for the description of the pneumatic piston position. The calculation of ARMAX model parameters is done by RPEM algorithm of which scheme is given by the following equations

$$\hat{\theta} = \hat{\theta}(k-1) + L(k)\varepsilon(k) \tag{5}$$

$$\varepsilon(k) = y(k) - \hat{y}(k) \tag{6}$$

$$L(k) = \frac{P(k-1)\psi(k)}{\lambda(k) + \psi(k)^T P(k-1)\psi(k)} \tag{7}$$

$$P(k) = \frac{1}{\lambda(k)} \left[\frac{P(k-1)\psi(k)\psi(k)^T P(k-1)}{\lambda(k) + \psi(k)^T P(k-1)\psi(k)} \right] \tag{8}$$

where $P(k)$ is covariance matrix of the estimated parameters, $\psi(k)$ is the gradient vector of $\hat{y}(k)$

$$\psi^T(k) = \frac{\partial \hat{y}(k)}{\partial \theta(k)} = \left[\frac{\partial \hat{y}(k)}{\partial a_1},\dots,\frac{\partial \hat{y}(k)}{\partial a_{na}},\frac{\partial \hat{y}(k)}{\partial b_1},\dots,\frac{\partial \hat{y}(k)}{\partial b_{nb}},\frac{\partial \hat{y}(k)}{\partial c_1},\dots,\frac{\partial \hat{y}(k)}{\partial c_{nc}} \right] \tag{10}$$

and $\hat{y}(k)$ is the estimate $y(k)$

$$\hat{y}(k) = \varphi^T(k)\theta(k-1) \tag{11}$$

where $\varphi^T(k)$ is the data vector constructed from the past input-output data *u(k)* and *y(k)*

$$\varphi^T(k) = \left[-y(k-1),\dots,-y(k-na),u(k-1),\dots,u(k-nb),\varepsilon(k-1),\dots,\varepsilon(k-nc)\right] \tag{12}$$

and $\theta(k)$ is the parametric vector with model parameters

$$\hat{\theta}^T(k) = \left[a_1,\dots,a_{na},b_1,\dots,b_{nb},c_1,\dots,c_{nc}\right].$$

Predictive control of the pneumatic system was designed in the state space and so it is necessary to form the state matrixes - *M,N,C,D* from the ARMAX model structure. The normal form of observability that reconstructs the response of pneumatic system $y_m(k)$ was chosen for the description of the system in the state space. And the reduced order observer, that is briefly described in the following chapter, was designed for observe of state vector.

❖ REDUCED ORDER OBSERVER

The state vector consists of three variables where the first one is the measured response of the pneumatic system $y_m(k)$. The response of this system (position of the piston rod of the pneumatic cylinder) is measured by the linear displacement encoder and so it is not necessary to observe the variable $x_1(k)$ of the state vector. The two rest variables $x_2(k)$, $x_3(k)$ of the state vector are observed. Not a full - state observer but the reduced order observer is used for the observation of the state vector [1]. The reduced order observer was designed on the basis of equations

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} y_m(k+1) \\ x_{E1}(k+1) \\ x_{E2}(k+1) \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} y_m(k) \\ x_{E1}(k) \\ x_{E2}(k) \end{bmatrix} + \begin{bmatrix} N_{11} \\ N_{21} \\ N_{31} \end{bmatrix} u(k) \tag{14}$$

$$y(k) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \end{bmatrix} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} + Du(k) \tag{15}$$

During the project of the observer it is necessary to select the eigenvalues so that the error of the observe converges to zero. The dynamic qualities of the error of the observe are given by the eigenvalues $\lambda_{1,2}$, that's why the eigenvalues of the designed observer are zero. The observer was designed on the basis of equations 16, 17, 18, 19, where $x_E(k)$ is the estimate $\hat{x}_E(k)$.

$$\hat{\hat{x}}_E(k+1) = M_E\hat{\hat{x}}_E(k) + H_E y_m(k) + N_E u(k) \tag{16}$$

$$x_E(k) = \hat{\hat{x}}_E(k) + Qy_m(k) \tag{17}$$

$$H_E = H - QM_{11} - QGQ + FG \tag{18}$$

$$N_E = K - QN_{11} \tag{19}$$

where

$$H_E = \begin{bmatrix} M_{21} \\ M_{31} \end{bmatrix}, K = \begin{bmatrix} N_{21} \\ N_{31} \end{bmatrix}, F = \begin{bmatrix} M_{22} & M_{23} \\ M_{32} & M_{33} \end{bmatrix}, G = \begin{bmatrix} M_{12} & M_{13} \end{bmatrix} \tag{20, 21, 22, 23}$$

❖ MODEL PREDICTIVE CONTROL

Model Predictive Controller is a controller that is nowadays often used in practise for the control of various technological processes. This controller combines elements of feedback regulation from measured outputs together with the elements of straightforward regulation from the predictions of mathematical models obtained during the online identification.

Pneumatic system is described in state space by equations

$$x(k+1) = Mx(k) + Nu(k) \quad (24)$$

$$y(k) = Cx(k) + Du(k) \quad (25)$$

where *x(k)* is the state vector, *u(k)* is the input vector, *y(k)* is the output vector, *M* is the state matrix, *N* is the input matrix, *C* is the output matrix, *D* is the feedforward matrix.

The future process of system response is defined from predictive equation

$$\hat{y} = Hx(k) + Gu = V + Gu \quad (26)$$

where *x(k)* is state vector (Eq.24), $\hat{y}$ is the prediction of output

$$\hat{y} = [\hat{y}(k+1) \quad \hat{y}(k+2) \quad \hat{y}(k+3) \quad . \quad . \quad . \quad \hat{y}(k+N)]^T \quad (27)$$

u is the vector of the demanded control actions

$$u = [u(k+1) \quad u(k+2) \quad u(k+3) \quad . \quad . \quad . \quad u(k+N)]^T \quad (28)$$

and

$$H = \begin{bmatrix} CM \\ CM^2 \\ CM^3 \\ . \\ . \\ . \\ CM^N \end{bmatrix}, G = \begin{bmatrix} CN & 0 & . & . & . & . & 0 \\ CMN & CN & . & . & . & . & . \\ CM^2N & CMN & CN & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ CM^{(N-1)}N & CM^{(N-2)}N & . & . & . & . & CN \end{bmatrix}. \quad (29, 30)$$

Predictive controller is based on minimalize a certain quadratic criterion in which the predicted outpu $\hat{y}$ (Eq.27) is included, the criterion in form by Eq.31 was chosen for the control of pneumatic system [4]:

$$J = \sum_{i=N_1}^{N_2} [\hat{y}(k+i) - w(k+i)]^T Q[\hat{y}(k+i) - w(k+i)] + \sum_{i=1}^{Nu} u^T(k+i-1)Pu(k+i-1) \quad (31)$$

where N_2 is the horizon of prediction, N_1 is the horizon of initial nonsensitivity, *Nu* is the horizon of controlling, $u(k+i-1)$ is the vector of control actions, $\hat{y}(k+i)$ is the vector of predicted outputs, $w(k+i)$ is the vector of desired values, *Q* and *P* are the correction matrixes.

The vector of demanded control actions *u* (Eq.28) is obtained by the substitution of Eq.26 into the quadratic criterion of Eq. 31 in the matrix form and then by its minimalization

$$J = \mathrm{E}\{(\hat{y} - w)^T Q(\hat{y} - w) + u^T Pu\} = \mathrm{E}\{(V + Gu - w)^T Q(V + Gu - w) + u^T Pu\} \Rightarrow \min \quad (32)$$

$$\frac{\partial J}{\partial u^T} = (G^T QG + P)u + G^T Q(V - w) = 0 \quad (33)$$

thus vector of control actions *u* is given

$$u = (G^T QG + P)^{-1} + G^T Q(w - V) \quad (34)$$

where w is the vector of desired values

$$w = [w(k+1) \quad w(k+2) \quad w(k+3) \quad . \quad . \quad . \quad w(k+N)]^T. \quad (35)$$

The control sequence of control actions *u* (Eq.28) is counted out in each sampling period according by Eq.(34). The horizon of initial nonsensitivity N_1 and the horizon of control *Nu* in criterion by Eq.(31) are usually chosen of the same length as the horizon of prediction N_2.

During the regulation of pneumatic system the “*receding horizont*” method is applied. This method uses only the first control action from the whole vector of control actions and it makes the calculation of the whole vector of the control actions in the following sampling period again. The required position at servomechanisms is usually set in the shape of ramp and this is why the self generator of position was designed and this generates the process of the ramp *N* steps forward.

The algorithm of predictive control of pneumatic system was designed in LabView programme and was tested on the NI PXI-1042Q real-time system. The initial parameters of the estimate of the ARMAX model were set on the zero value and the sampling rate was set 0,001s. So the new estimates of parameters of recursive model, the value of state vector and the value of control actions of predictive controller are calculated in each sampling rate. The progress of control of the pneumatic piston with predictive controller position is shown in Figure 3 and the progress of control action is in Figure 4.

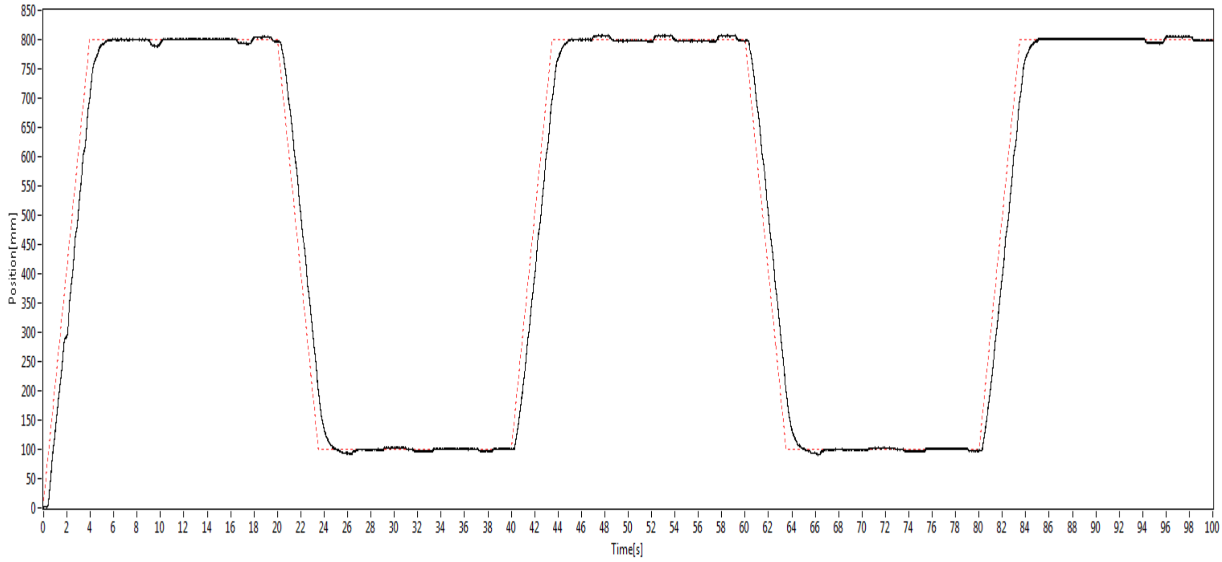

Figure 3: Position of the pneumatic piston

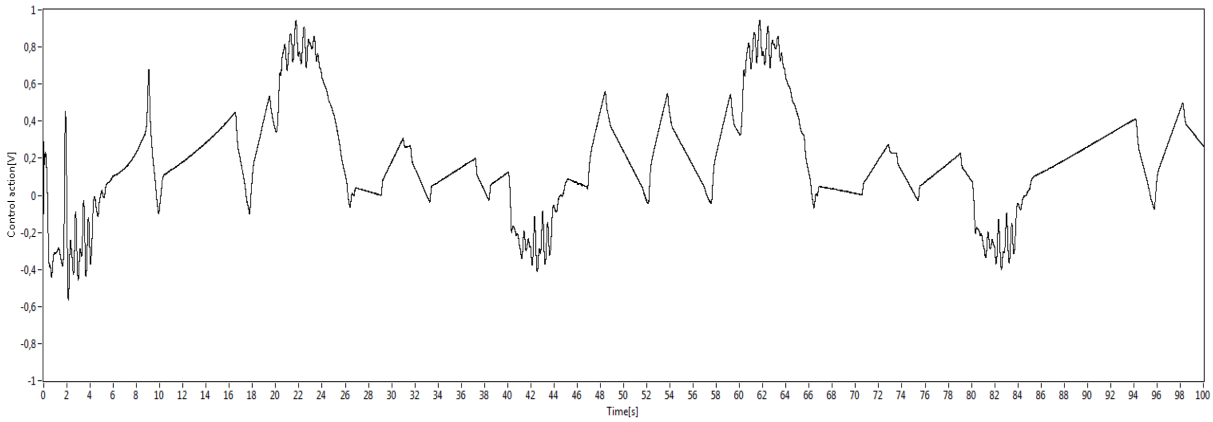

Figure 4: Control action of the predictive controller

❖ RESULTS & CONCLUSION

At the process of control of the pneumatic system it is possible to gain an high-quality control process with the predictive controller and this is observable from Figure 3. Generally, the control of pneumatic systems needs to chose really small sampling rates so that the controller would be able to react to the fast changing dynamic of the system.

The designed algorithm of online identification and the control action of predictive controller calculation contains a great amount of mathematical operations and if the requirement of really small sampling rate should be met, it is necessary to place high demands to the control system and this also restricts the usage of the controller. By using the predictive controller we did not remove the problem concerning the auto-oscilation of the piston around the demanded position.

The deadband was installed to supress the auto-oscilation. In this deadband the system control switched off if the piston reached the demanded position. Unfortunately, even the instalation of the deadband did not remove the auto-oscilation, for its removal it is necessary to reduce the influence of passive resistances.

❖ ACKNOWLEDGEMENTS

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