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THE DYNAMIC MODEL OF THE CAPACITOR-RUN TWO-PHASE INDUCTION MOTOR - A VARIATIONAL APPROACH

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ABSTRACT: The dynamic models of the three-phase electric machines are obtained within the classical approach with the direct- and quadrature-axis theory via two transformations of the dynamic set of equations, i.e. the Park and Clarke transformations. Several assumptions such as no magnetic saturation, no space-harmonics are used for simplification purposes. The variational method also called the Euler-Lagrange method is another approach for modeling the dynamic behaviour of the electric machines that relates to the physical energy of the drive. The Euler-Lagrange models are more suitable than the two-axis theory models when magnetic saturation or/and space harmonics are to be taken into account. In this paper, an analysis of the coarse start-up of a capacitor-run two-phase induction motor based on the variational approach is presented. The core of this approach is the Lagrangian of the system i.e. a real function describing the dynamic behaviour of the system, [1], [2]. The basics of this approach and its main characteristics are discussed into the first chapter. In the second chapter of the paper an expression for the Lagrangian for the capacitor-run two-phase induction motor is introduced. In the third chapter, the evolution with time of the values of the Lagrangian's components is detailed. The discussion is based on a set of experimental data and a dedicated software application. The results of the analysis show that the variation approach may provide not only basic information such as the values of the induced currents, but can also give additional information about the stability of the drive. The conclusions and further developments are presented in the last chapter of the paper.
KEYWORDS: three-phase electric machines, variation approach

❖ INTRODUCTION

The dynamic models of systems are representations such as functions, sets of differential equations and so that allow estimations on the outputs based on input measurements. There are two basic ways to determine the dynamic models of a given system (1) either using explanatory theories or (2) with input and output measurements and system identification algorithms.

In the basic approach, the dynamic models of the electric drives are obtained with the direct and quadrature-axis theory tailored to the specific class of the electric machine and power converter. The implementation of the direct and quadrature-axis theory provide models that allow estimating system's response in the time domain, [3]. The estimate's consistency is affected by the accuracy of the measurements and the consistency of the parameters' estimates. With the space vector definition, the time domain model of the machine may be transformed into the complex representation. The complex representation of the electric machine model provides the easiest way to transform the dynamic model from one reference coordinate system to another. In addition, in the complex representation the command of the three-phase inverters can be handled in the most appropriate manner.

With the notations generally accepted in the literature, index 1 for the stator and index 2 for the rotor, the dynamic set of equations in complex representation is as follows, [3].

$$\underline{u}_1 = R_1 \cdot \underline{i}_1 + \frac{d\underline{\Psi}_1}{dt} \quad \text{voltage equation - stator,} \tag{1}$$

$$0 = R_2 \cdot \underline{i}_2 + \frac{d\underline{\Psi}_2}{dt} + j \cdot \underline{\Psi}_2 \cdot \frac{d\gamma}{dt} \quad \text{voltage equation - rotor,} \tag{2}$$

$$\underline{\Psi}_1 = L_1 \cdot \underline{i}_1 + L_h \cdot \underline{i}_2 \quad \text{flux linkages equation - stator,} \tag{3}$$

$$\underline{\Psi}_2 = L_1 \cdot \underline{i}_1 + L_h \cdot \underline{i}_2 \quad \text{flux linkages equation - rotor,} \tag{4}$$

$$M_{el} = \frac{3}{2} \cdot p \cdot \mathrm{Im}\left(\underline{\Psi}_2 \cdot \underline{i}_2^*\right) = \frac{J}{p} \cdot \frac{d\gamma}{dt} + M_W \quad \text{torque equation.} \tag{5}$$

Another approach issued from the quantum electro-dynamic theory relates to the Lagrangian function of the system represented as a function of two sets of generalized coordinates [1] as follows.

- the first is a set of complex numbers $q^c = \left(q_1, \ldots q_{2\cdot n^c}\right)$ with $2 \cdot n^c = \overline{0, n}$ and (6)

- the second, a set of real numbers $q^r = \left(q_{2\cdot n^c}, \ldots q_n\right)$ with $n^r = n - n^c$. (7)

The Lagrangian is a real-value and analytic function of complex variables, $\mathsf{L}\left(q^c, q^{c*}, q^r, \dot{q}^c, \dot{q}^{c*}, \dot{q}^r\right)$ related to the dynamic set of equations by the Euler-Lagrange equations on the following form.

$$\frac{d}{dt}\left(\frac{\partial \mathsf{L}}{\partial t}\right) = \frac{\partial \mathsf{L}}{\partial q_k} + S_k, \quad k = \overline{0, n} \tag{8}$$

where the S_k -terms correspond to the non-conservative energy exchanges with the environment.

For the electrical drives, the dynamic set of equations, in variation form is as follows, [2].

$$\frac{d}{dt}\left(\frac{\partial \mathsf{L}}{\partial \dot{\gamma}}\right) - \frac{\partial \mathsf{L}}{\partial \gamma} = -M_W \quad \text{torque equation,} \tag{9}$$

$$2 \cdot \frac{d}{dt}\left(\frac{\partial \mathsf{L}}{\partial \underline{i}_1^*}\right) - \frac{\partial \mathsf{L}}{\partial \underline{i}_1} = \underline{u}_1 - R_1 \cdot \underline{i}_1 \quad \text{voltage equation - stator} \tag{10}$$

$$2 \cdot \frac{d}{dt}\left(\frac{\partial \mathsf{L}}{\partial \underline{i}_2^*}\right) - \frac{\partial \mathsf{L}}{\partial \underline{i}_2} = -R_2 \cdot \underline{i}_2 \quad \text{voltage equation - rotor} \tag{11}$$

METHODOLOGY

The stator of the capacitor-run, two-phase induction machine has two distributed windings, the main and the auxiliary windings. The axes of these windings are located at 90 electrical degrees with respect to the air gap of the machine. The numbers of turns of these windings usually differ. The main winding is supplied directly from the grid whereas the auxiliary winding is supplied through a capacitor from the grid to produce the quadrature current component. The rotor is symmetrical and similar to the rotor of the squirrel-cage three-phase induction machine. Due to these features, the stator produces an elliptic magnetic motion field into the air gap, [4].

The dynamic set of equations in complex representation with the stator asymmetry taken into account will be as follows.

$$\underline{u}_{S1} = R_{1m} \cdot \underline{i}_{S1} + \frac{d\underline{\Psi}_{S1}}{dt} - R_{1a} \cdot \underline{i}_{S1}^* \quad \text{voltage equation - stator} \tag{12}$$

$$0 = R_2 \cdot \underline{i}_{R1} + \frac{d\underline{\Psi}_{R1}}{dt} + j \cdot \underline{\Psi}_{R1} \cdot \frac{d\gamma}{dt} \quad \text{voltage equation - rotor} \tag{13}$$

$$\underline{\Psi}_{S1} = L_{hm} \cdot (1 + \sigma_1) \cdot \underline{i}_{S1} + L_{hm} \cdot \underline{i}_{R1} - L_{ha} \cdot \underline{i}_{S1}^* - L_{ha} \cdot \underline{i}_{R1}^{'*} \quad \text{flux linkage equation - stator} \tag{14}$$

$$\underline{\Psi}_{R1}^{'} = L_{hm} \cdot (1 + \sigma_2) \cdot \underline{i}_{R1}^{'} + L_{hm} \cdot \underline{i}_{S1} - L_{ha} \cdot \underline{i}_{S1}^* - L_{ha} \cdot \underline{i}_{R1}^* \quad \text{flux linkage equation - rotor} \tag{15}$$

$$M_{el} = p \cdot \mathrm{Im}\left(L_{hm} \cdot \underline{i}_{S1} \cdot \underline{i}_{R1}^* - L_{ha} \cdot \underline{i}_{S1}^* \cdot \underline{i}_{R1}^* - L_{ha} \cdot \left(\underline{i}_{R1}^*\right)^2\right) \quad \text{torque equation} \tag{16}$$

The magnetizing inductances over the two axes of magnetic asymmetry of the stator are given by the following expressions.

$$L_{hm} = \frac{L_{h\alpha} + L_{h\beta}}{2} \text{ , and } L_{ha} = \frac{L_{h\alpha} - L_{h\beta}}{2} \text{ respectively.} \tag{17}$$

We introduce the Lagrangian of the run-capacitor induction machine as follows.

$$\begin{aligned}
&\mathsf{L}\left(\gamma, \dot{\gamma}, \underline{i}_{R2}, \underline{i}_{R2}^*, \underline{i}_{S1}, \underline{i}_{S1}^*\right) = \mathsf{L}_{mec} + \mathsf{L}_{mag} \\
&= \frac{J}{2} \cdot \dot{\gamma}^2 + \frac{L_{hm}}{2} \cdot \left|\underline{i}_{S1} + \underline{i}_{R2} \cdot e^{j\cdot\gamma}\right|^2 - \frac{1}{4} \cdot L_{ha} \cdot \left[\left(\underline{i}_{S1} + \underline{i}_{R2} \cdot e^{j\cdot\gamma}\right)^2 + \left(\underline{i}_{S1}^* + \underline{i}_{R2}^* \cdot e^{-j\cdot\gamma}\right)^2\right] + \\
&+ \frac{L_{\sigma1}}{2} \cdot \left|\underline{i}_{S1}\right|^2 + \frac{L_{\sigma1}}{2} \cdot \left|\underline{i}_{R2}\right|^2
\end{aligned} \tag{18}$$

With the magnetic fluxes given by the expressions:

$$\underline{\Psi}_{S1} = 2 \cdot \frac{\partial L}{\partial \underline{i}_{S1}^*} \tag{19}$$

$$\underline{\Psi}_{R1} = 2 \cdot \frac{\partial L}{\partial \underline{i}_{R1}^*} \text{ .} \tag{20}$$

As seen from the notation (18), the magnetic Lagrangian has four components. An evaluation of the variation of these components at the coarse start-up of the capacitor-run induction machine is subsequently analysed.

❖ DISSCUSIONS/RESULTS/ANALYSES

The motor under the investigation was of MSP311 type. The nominal parameters of the motor are given in Table 1. The electrical and magnetic parameters of the motor had been earlier determined by direct measurements for the stator windings or had been computed through the FEM method for inductances, [5]. The results are presented in Table 2.

Table 1: The Nominal Parameters of the MSP311 Motor

Denomination	Rated supply voltage	Rated frequency	Rated angular speed	Number of pair poles	Capacitance
Units	[V]	[Hz]	[rpm]	[-]	[μF]
Value	220	50	2820/420	1/6	14

Table 2. The Electrical and Mechanical Parameters of the MSP311 Motor

Denomination	Stator resistance d/q axis	Stator self-in-ductance d/q axis	Mutual inductance d/q axis	Rotor resistance	Rotor self-in-ductance
Units	[Ω]	[H]	[H]	[Ω]	[H]
Value	20,8/57,5	0,358/0,665	0,275/0,504	17,0	0,523

The values were defined as follows:

- excitation values: $\underline{u}_{S1}$, M_W
- state values: $\underline{\Psi}_{S1}$, $\underline{\Psi}_{R1}$, ω

The supply voltage components were acquired on a dedicated test-band with a computer-aided measurement system. The state values and the Lagrangian were determinate through a dedicated Matlab application. The dynamic model used to estimate the rotor currents and fluxes was the flux model of the machine issued from the equations set (1) to (5).

The supply voltages dependencies with time and the components of the flux linkages within the air gap of the machine are presented in Figure 1 and Figure 2.

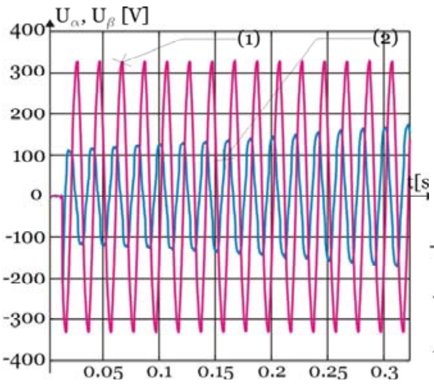

Figure 1: the input supply voltages at the motor terminals; (1) – the main winding supply voltage and (2) – the auxiliary winding supply voltage.

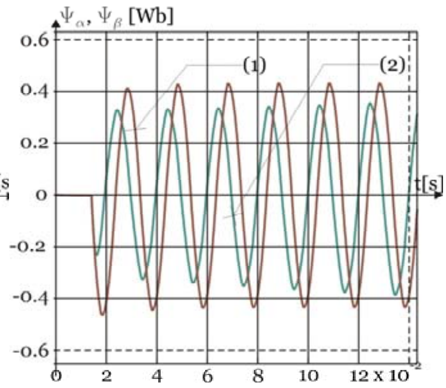

Figure 2: the components of the flux linkages within the air gap; (1) – the direct component and (2) – the quadrature component.

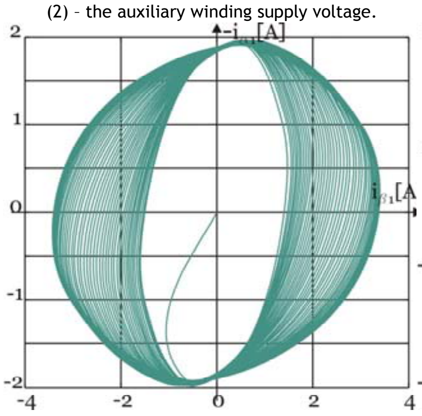

Figure 3: The space vector of the stator currents at coarse, no-load start-up.

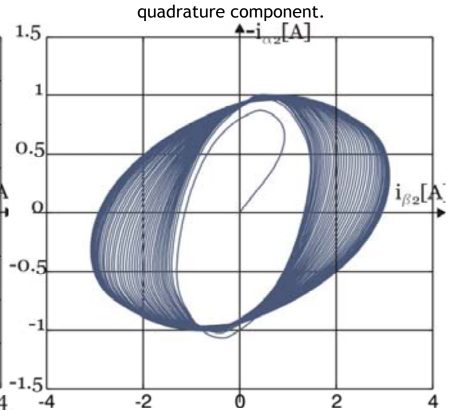

Figure 4: the estimated space vector of the rotor currents at coarse, no-load start-up.

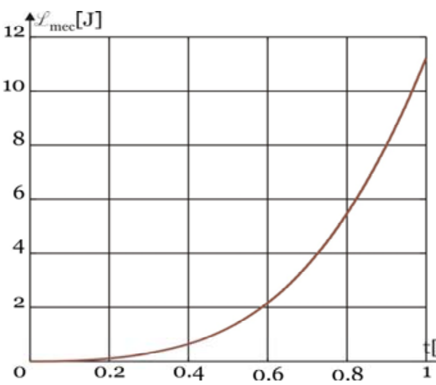

Figure 5: the mechanical component of the Lagrangian at coarse start-up.

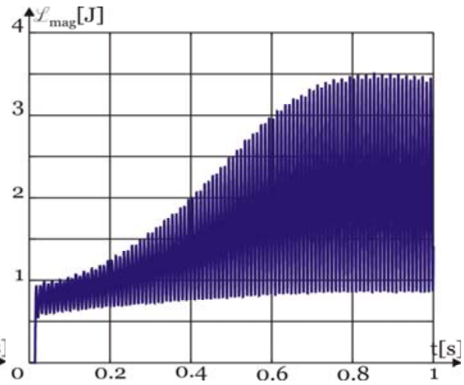

Figure 6: the magnetic Lagrangian at coarse start-up.

The space vectors of the stator and rotor currents are presented in Figures 3 and 4, respectively. As shown in Figure 1, the auxiliary voltage increased to the steady state magnitude with a time constant of about 0.6 seconds. The magnetic motion field (mmf) into the air gap of the machine was highly elliptic during the start-up.

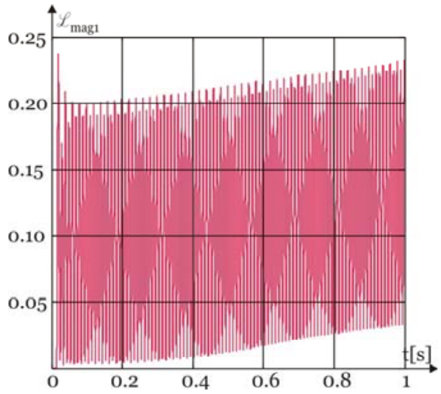

Figure 7: the symmetrical magnetizing component of the Lagrangian at coarse start-up.

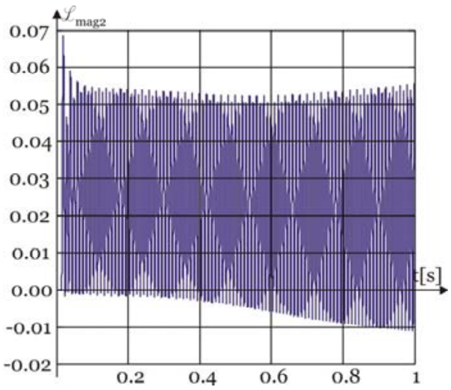

Figure 8: the asymmetrical magnetizing component of the Lagrangian at coarse start-up.

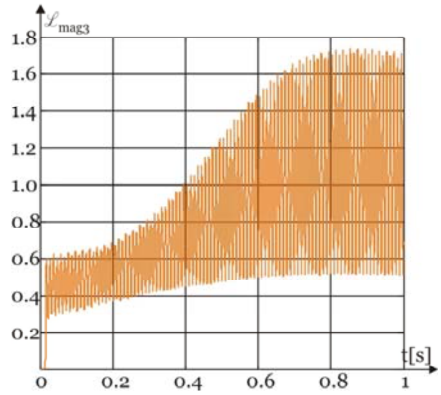

Figure 9: the symmetrical leakage magnetic component of the Lagrangian at coarse start-up

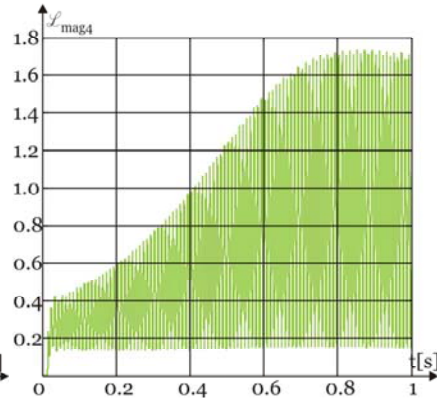

Figure 10: the asymmetrical leakage magnetic component of the Lagrangian at coarse start-up

In the steady-state operation the mmf became almost circular, Figures 3 and 4. However, in direct operation of the machine, the shape of the mmf is dependent on the load. Therefore, in direct operation, the efficiency of the machine cannot be optimized. To optimize the efficiency, an electronically controlled voltage supply must be added to the drive.

In figures 5 and 6 the mechanical and the magnetic components of the Lagrangian are depicted. The mechanical Lagrangian represents the kinetic energy of the drive. The experiment was performed at no-load operation of the drive therefore the angular speed increased proportional with time. The magnetic Lagrangian takes into account the total amount of the magnetic energy into the machine. As seen from the plot, at the beginning of the process the magnetic circuitry of the machine was not magnetized. Therefore the start-up took some time until reaching the steady-state operation. This phenomenon is similar with the coarse start-up of the DC-shunt machine. The components of the magnetic Lagrangian are presented in Figures 7 to 10. As seen from the plots, the magnitude of the magnetizing components of the Lagrangian both symmetrical and asymmetrical parts are almost unchanged during the start-up process. However, the two magnitudes of the leakage components of the Lagrangian increase during the process. The maximum magnitude is achieved near the synchronous angular speed before the steady-state regime of the drive. In addition, the leakage components of the Lagrangian are much greater in comparison with the previous two components of the Lagrangian.

❖ CONCLUSIONS/FURTHER PROPOSALS

In this paper, the variation of the Lagrangian of a capacitor-run two-phase induction motor during the coarse start-up of the drive has been investigated. The investigation proved an interesting similarity between the coarse start-up of the capacitor-run two-phase induction motor and the coarse start-up of the DC-shunt machine. As for the DC-shunt machine, the capacitor-run two-phase induction motor is not suitable for fast tracking servo-drives due to its large time constants.

The analysis also proved that the magnetic component of the Lagrangian has four components. The information regarding the dynamics of the start-up is mainly contained into the two leakage components of the Lagrangian. Because the Lagrangian and its components may not be measured through direct transducers, in the on-line applications, a digital signal processor should be used. Further developments consist in the use of the drive's Lagrangian to produce the command law for the drive system.

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