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NUMERICAL STUDY OF HYDROMAGNETIC EFFECTS ON FLOW PAST AN OSCILLATING SEMI-INFINITE ISOTHERMAL VERTICAL PLATE WITH UNIFORM MASS DIFFUSION IN THE PRESENCE OF THERMAL RADIATION

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ABSTRACT: Magnetohydrodynmics effects on unsteady flow past an oscillating semi-infinite isothermal vertical plate with uniform mass diffusion in the presence of thermal radiation have been studied. The fluid considered is a gray, absorbing-emitting radiation but non-scattering medium. The effect of velocity and temperature for different parameters like magnetic field, thermal radiation, Schmidt number, thermal Grashof number and mass Grashof number are studied. It is observed that the velocity decreases in the presence of thermal radiation or magnetic field parameter.
KEYWORDS: Magnetic field, radiation, isothermal, vertical plate, finite-difference

❖ INTRODUCTION

The effect of radiation on MHD flow, heat and mass transfer problems has become industrially more important. Many engineering processes occur at high temperatures, the knowledge of radiation heat transfer plays significant role in the design of equipment. Free convection flow involving heat transfer occurs frequently in an environment where differences between land and air temperatures can give rise to complicated flow patterns. The subject of magnetohydrodynamics has attracted the attention of a large number of scientists due to its diverse application. In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere, etc. In engineering, it finds its application in MHD pumps, MHD bearing, etc. The study of effects of magnetic field on free convection flow is important in liquid metals, electrolytes, and ionized gasses. At the high temperature attained in some engineering devices, gas, for example, can be ionized and so becomes an electrical conductor.

England and Emery (1969) have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Soundalgekar and Takhar (1993) have considered the radiative free convective flow of an optically thin gray-gas past a semi-infinite vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar (1996). In all above studies, the stationary vertical plate is considered. Raptis and Perdikis (1999) have studied the effects of thermal radiation and free convection flow past a moving infinite vertical plate. Again, Raptis and Perdikis (2003) studied thermal radiation effects on moving infinite vertical plate in the presence of mass diffusion. Radiation effects on moving infinite vertical plate with variable temperature were studied by Muthucumaraswamy and Ganesan (2003). The dimensionless governing equations were solved by the Laplace transform technique.

Soundalgekar et al (1979) presented an exact analysis of MHD Stokes problem (also Rayleigh's problem) for the flow of an electrically conducting, incompressible, viscous fluid past an impulsively started vertical plate, under the action of transversely applied magnetic field. It observed that reverse flow occurs when the plate was being heated by the free convection currents. An exact solution to stokes problem for an infinite vertical plate whose temperature varies linearly with time has been found on taking into account the transversely magnetic field and incompressible, electrically conducting fluid was studied by Soundalgekar et al. (1981). The boundary-layer flow development of an electrically conducting fluid in the asymmetric stagnation point region of a two-dimensional body over a continuously stretching surface was analyzed by Kumari and Nath (1999).

The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was solved by Soundalgekar (1979). The effect on the flow past a vertical oscillating plate due to a combination of concentration and temperature differences was studied extensively by Soundalgekar and Akolkar (!983). The effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux has been studied by Soundalgekar et al. (1994).

Analytical or numerical work on transient natural convection along an oscillating isothermal vertical plate under the combined buoyancy effects of heat and mass diffusion in the presence of thermal radiation has not received attention of any researcher. Hence, the present study is to investigate the MHD flow past an oscillating semi-infinite isothermal vertical plate with uniform mass diffusion in the presence thermal radiation by an implicit finite-difference scheme of Crank-Nicolson type.

❖ MATHEMATICAL FORMULATION

A transient, laminar, unsteady natural convection flow of a viscous incompressible fluid past an oscillating semi-infinite isothermal vertical plate in the presence of thermal radiation has been considered. It is assumed that the concentration *C'* of the diffusing species in the binary mixture is very in comparison to the other chemical species which are present. Here, the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate starts oscillating in its own plane with frequency ω' against gravitational field and the temperature of the plate and the concentration level near the plate are also raised to T_w and C_w'. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium and the viscous dissipation is assumed to be negligible. Then under the above assumptions, the governing boundary layer equations of mass, momentum and concentration for free convective flow with usual Boussinesq's approximation are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t'} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2{}_0}{\rho}u \tag{2}$$

$$\rho C_p\left(\frac{\partial T}{\partial t'} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{3}$$

$$\frac{\partial C'}{\partial t'} + u\frac{\partial C'}{\partial x} + v\frac{\partial C'}{\partial y} = D\frac{\partial^2 C'}{\partial y^2} \tag{4}$$

The initial and boundary conditions are:

$$\begin{array}{l} t' \le 0: u = 0,\ \ u = 0,\ \ T = T_\infty,\ C = C_\infty \\ t' > 0: U = \cos\omega t,\ \ v = 0,\ T = T_w,\ C = C_W \text{ at } Y = 0 \\ U = 0,\quad T = T_\infty,\quad C = C_\infty \quad \text{at} \quad X = 0 \\ U \to 0,\ \ T \to T_\infty,\ \ C \to C_\infty \quad \text{as} \quad Y \to \infty \end{array} \tag{5}$$

For the case of an optically thin gray gas the local radiant absorption is expressed by:

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma\left(T_\infty^4 - T^4\right) \tag{6}$$

We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

By using equations (6) and (7), equation (3) reduces to

$$\rho C_p\frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} + 16a^*\sigma T_\infty^3(T_\infty - T) \tag{8}$$

On introducing the following non-dimensional quantities

$$U = \frac{u}{u_0},\ V = \frac{v}{u_0},\ t = \frac{t'u_0^2}{\nu},\qquad Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \tag{9}$$

$$X = \frac{xu_0}{\nu}Y = \frac{yu_0}{\nu},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\quad Gc = \frac{\nu g\beta^*(C'_w - C'_\infty)}{u_0^3},\ \omega = \frac{\omega'\nu}{u_0^2},$$

$$Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},$$

$$Gc = \frac{\nu g\beta^*(C'_w - C'_\infty)}{u_0^3},\ \omega = \frac{\omega'\nu}{u_0^2},$$

Equations (1) to (4) are reduced to the following non-dimensional form

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{10}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=Gr\,\theta+Gc\,C+\frac{\partial^2 U}{\partial Y^2}-MU \tag{11}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2}-\frac{R}{\Pr}T \tag{12}$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{13}$$

The corresponding initial and boundary conditions in non-dimensional quantities are

$$\begin{aligned} &t \le 0 : U = 0,\ V = 0,\ T = 0,\ C = 0 \\ &t > 0 : U = \cos\omega t,\ V = 0,\ T = 1,\ C = 1 \text{ at } Y = 0 \\ &U = 0,\ T = 0,\ C = 0 \quad \text{at} \quad X = 0 \\ &U \to 0,\ T \to 0,\ C \to 0 \quad \text{as} \quad Y \to \infty \end{aligned} \tag{14}$$

❖ NUMERICAL TECHNIQUE

In order to solve the unsteady, non-linear coupled equations (10) to (13) under the conditions (14), an implicit finite difference scheme of Crank- Nicolson type has been employed. The finite difference equations corresponding to equations (10) to (13) are as follows.

$$\frac{\left[U_{i,j}^{n+1}-U_{i-1,j}^{n+1}+U_{i,j}^{n}-U_{i-1,j}^{n}+U_{i,j-1}^{n+1}-U_{i-1,j-1}^{n+1}+U_{i,j-1}^{n}-U_{i-1,j-1}^{n}\right]}{4\Delta X}+\frac{\left[V_{i,j}^{n+1}-V_{i,j-1}^{n+1}+V_{i,j}^{n}-V_{i,j-1}^{n}\right]}{2\Delta Y}=0 \tag{15}$$

$$\begin{aligned} &\frac{\left[U_{i,j}^{n+1}-U_{i,j}^{n}\right]}{\Delta t}+U_{i,j}^{n}\frac{\left[U_{i,j}^{n+1}-U_{i-1,j}^{n+1}+U_{i,j}^{n}-U_{i-1,j}^{n}\right]}{2\Delta X}+V_{i,j}^{n}\frac{\left[U_{i,j+1}^{n+1}-U_{i,j-1}^{n+1}+U_{i,j+1}^{n}-U_{i,j-1}^{n}\right]}{4\Delta Y} \\ &=\frac{Gr}{2}\left[T_{i,j}^{n+1}+T_{i,j}^{n}\right]+\frac{Gc}{2}\left[C_{i,j}^{n+1}+C_{i,j}^{n}\right]-\frac{M}{2}\left[U_{i,j}^{n+1}+U_{i,j}^{n}\right] \\ &+\frac{\left[U_{i,j-1}^{n+1}-2U_{i,j}^{n+1}+U_{i,j+1}^{n+1}+U_{i,j-1}^{n}-2U_{i,j}^{n}+U_{i,j+1}^{n}\right]}{2(\Delta Y)^2} \end{aligned} \tag{16}$$

$$\begin{aligned} &\frac{\left[T_{i,j}^{n+1}-T_{i,j}^{n}\right]}{\Delta t}+U_{i,j}^{n}\frac{\left[T_{i,j}^{n+1}-T_{i-1,j}^{n+1}+T_{i,j}^{n}-T_{i-1,j}^{n}\right]}{2\Delta X}+V_{i,j}^{n}\frac{\left[T_{i,j+1}^{n+1}-T_{i,j-1}^{n+1}+T_{i,j+1}^{n}-T_{i,j-1}^{n}\right]}{4\Delta Y} \\ &=\frac{1}{Pr}\frac{\left[T_{i,j-1}^{n+1}-2T_{i,j}^{n+1}+T_{i,j+1}^{n+1}+T_{i,j-1}^{n}-2T_{i,j}^{n}+T_{i,j+1}^{n}\right]}{2(\Delta Y)^2}-\frac{R(T_{i,j}^{n+1}+T_{i,j}^{n})}{2Pr} \end{aligned} \tag{17}$$

$$\begin{aligned} &\frac{\left[C_{i,j}^{n+1}-C_{i,j}^{n}\right]}{\Delta t}+U_{i,j}^{n}\frac{\left[C_{i,j}^{n+1}-C_{i-1,j}^{n+1}+C_{i,j}^{n}-C_{i-1,j}^{n}\right]}{2\Delta X}+V_{i,j}^{n}\frac{\left[C_{i,j+1}^{n+1}-C_{i,j-1}^{n+1}+C_{i,j+1}^{n}-C_{i,j-1}^{n}\right]}{4\Delta Y} \\ &=\frac{1}{Sc}\frac{\left[C_{i,j-1}^{n+1}-2C_{i,j}^{n+1}+C_{i,j+1}^{n+1}+C_{i,j-1}^{n}-2C_{i,j}^{n}+C_{i,j+1}^{n}\right]}{2(\Delta Y)^2} \end{aligned} \tag{18}$$

❖ RESULTS AND DISCUSSION

The numerical values of the velocity, temperature and concentation are computed for different parameters like magnetic field parameter, radiation parameter, Schmidt number, thermal Grashof number and mass Grashof number. The purpose of the calculations given here is to assess the effects of the parameters M, R, Gr, Gc and Sc upon the nature of the flow and transport. The value of Prandtl number Pr is chosen such that they represent air (Pr = 0.71) and the Schmidt number Sc = 0.6(Water vapour).

The steady-state velocity profiles for different phase angle are shown in figure 1. The velocity profiles presented are those at X = 1.0. It is observed that for different phase angle ($\omega t = 0, \pi/6, \pi/4, \pi/3, \pi/2$), the velocity decreases with increasing phase angle. Here ωt = 0 represents vertical plate and note that the velocity profile grows from U = 1 and $\omega t = \pi/2$ refers horizontal plate and the velocity profiles starting with U = 0. The numerical value satisfies with the prescribed boundary conditions. The time to reach steady-state strongly depends on phase angle.

In figure 2, the velocity profiles for different thermal Grashof number and mass Grashof number are shown graphically. This shows that the velocity increases with increasing thermal Grashof number or mass Grashof number. As thermal Grashof number or mass Grashof number increases, the buoyancy effect becomes more significant, as expected, it implies that, more fluid is entrained from the free stream due to the strong buoyancy effects as Gr or Gc increases.

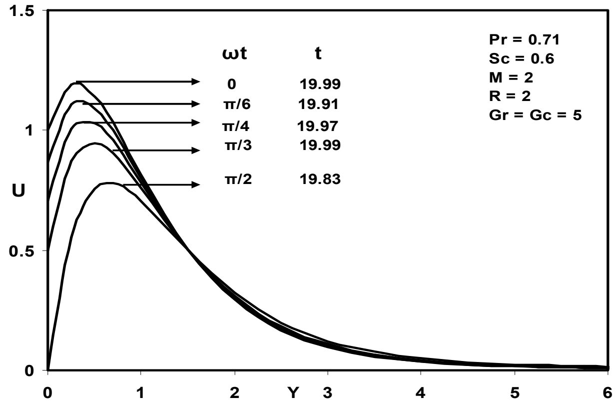

Fig. 1. Steady state profiles for different values of ωt

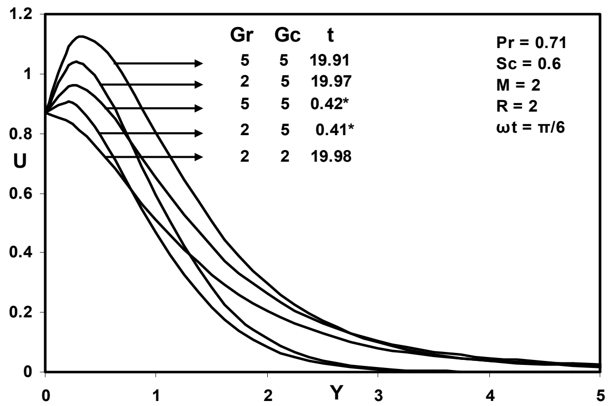

Fig. 2. Steady state velocity profiles for different values of Gr & Gc (*transient state)

The effect of velocity for different radiation parameter (R = 0.2, 2, 5, 10), and Sc = 0.6 are shown in figure 3. It is observed that the velocity increases with decreasing radiation parameter. This shows that velocity decreases in the presence of high thermal radiation. The steady-state velocity profiles for a different magnetic field parameter are shown in figure 4. It is observed that for (M = 2, 5, 10), Pr = 0.71 and Sc = 0.6, the velocity decreases in the presences of the magnetic field. This shows that the increase in the magnetic field parameter leads to a fall in the velocity. This agrees with expectations, since the magnetic field exerts a retarding force on the free convective flow.

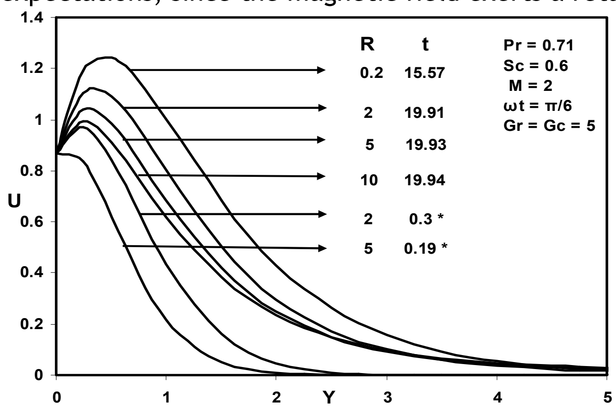

Fig. 3. Steady state profiles for different values of R (*transient state)

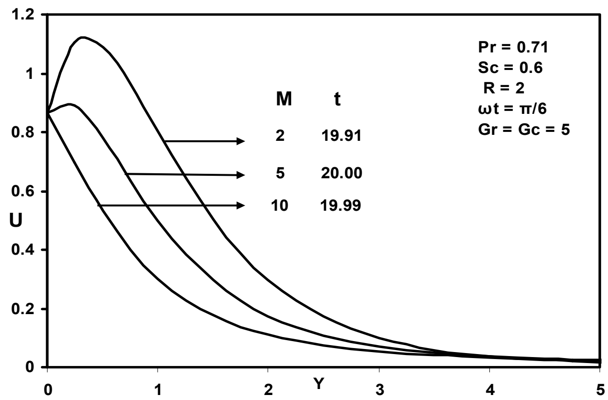

Fig. 4. Velocity profiles for different values of M

The transient and steady-state velocity profiles for different Schmidt number are shown in figure 5. It is observed that the velocity decreases with increasing Schmidt number and the steady-state value increases with increasing Schmidt number. The velocity boundary layer seems to grow in the direction of motion of the plate. It is observed that near the leading edge of a semi-infinite vertical plate moving in a fluid, the boundary layer develops along the direction of the plate.

The transient temperature profiles for different values of the thermal radiation parameter are shown in figure 6. It is observed that the temperature increases with decreasing R. This shows that the buoyancy effect on the temperature distribution is very significant in air (Pr = 0.71). It is known that the radiation parameter and Prandtl number plays an important role in flow phenomena because, it is a measure of the relative magnitude of viscous boundary layer thickness to the thermal boundary layer thickness.

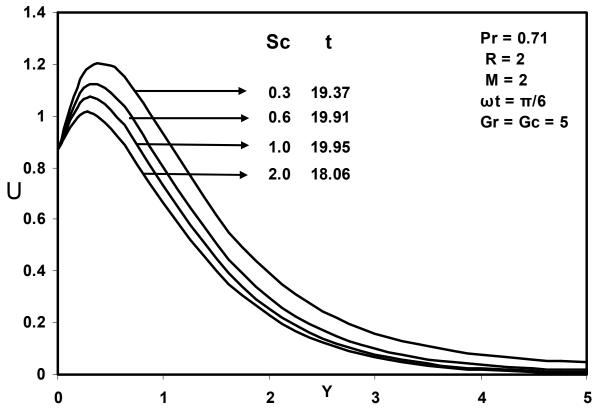

Fig. 5. Velocity profiles for different values of Sc

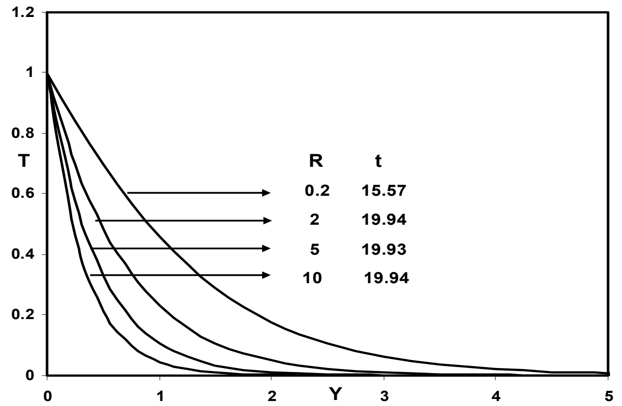

Fig.6.Temperature profiles for different values of R

Knowing the velocity and temperature field, it is customary to study the skin-friction and the Nusselt number. The local as well as average values of skin-friction and Nusselt number in dimensionless form are as follows. The derivatives involved in the Equations (19) - (24) are evaluated using five-point approximation formula and then the integrals are evaluated using Newton-Cotes closed integration formula.

$$\tau_X = -\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{19}$$

$$\bar{\tau} = -\int_0^1 \left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX \tag{20}$$

$$Nu_X = \frac{-X\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}} \tag{21}$$

$$\overline{Nu} = -\int_0^1 \left[\frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}}\right] dX \tag{22}$$

$$Sh_X = -X\left(\frac{\partial C}{\partial Y}\right)_{Y=0} \tag{23}$$

$$\overline{Sh} = -\int_0^1 \left(\frac{\partial C}{\partial Y}\right)_{Y=0} dX \tag{24}$$

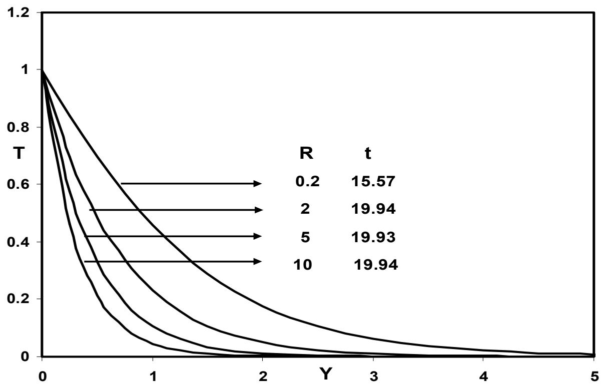

Fig.7. Concentration profiles for different values of Sc

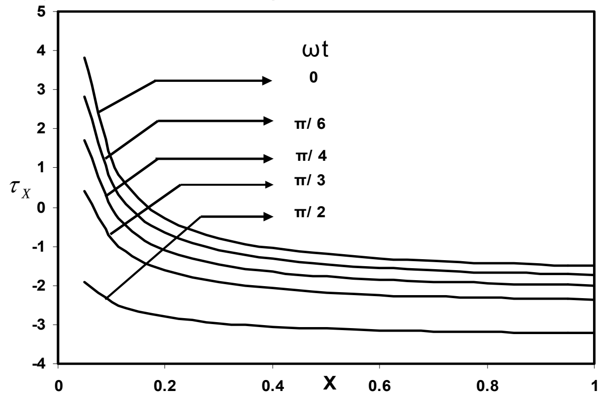

Fig. 8. Local skin friction

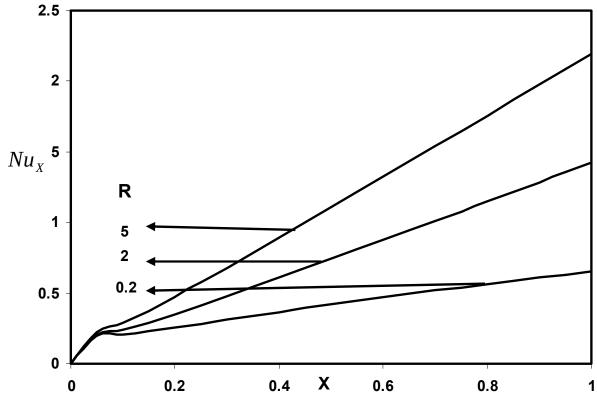

Fig 9. Local Nusselt number

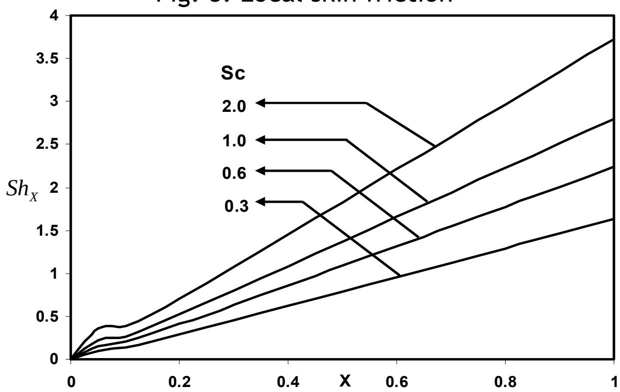

Fig. 10. Local Sherwood number

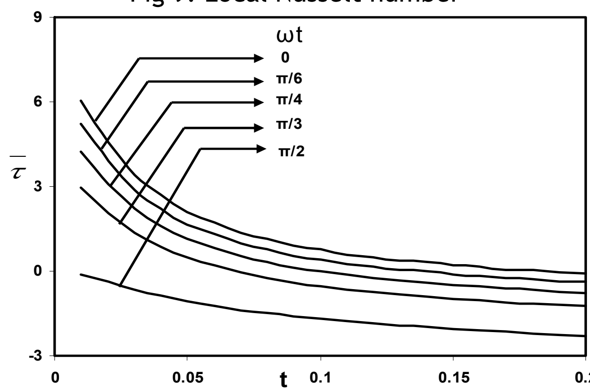

Fig. 11. Average skin friction

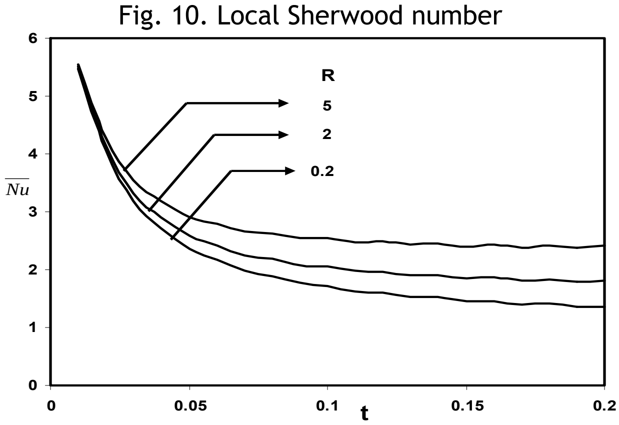

Fig. 12. Average Nusselt number

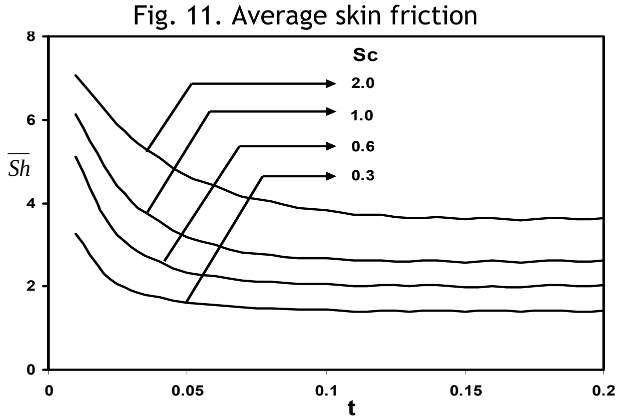

Fig. 13. Average Shearwood number

The local skin-friction, Nusselt number and Sherwood number are plotted in figures 8, 9 and 10 respectively. Local skin-friction values for different phase angle are evaluated from equation (19) and plotted in figure 8 as a function of the axial coordinate. The local wall shear stress increases with decreasing phase angle. The trend shows that the wall shear stress is more in the case of vertical plate than horizontal plate. The value of the skin-friction becomes negative, which implies, that after some time there occurs a reverse type of flow near the oscillating plate. Physically this is also true as the motion of the fluid is due to plate moving in the vertical direction against the gravitational field.

The local Nusselt number for different thermal radiation parameter is presented in figure 9 as a function of the axial co-ordinate. The trend shows that the Nusselt number increase with increasing values of the thermal radiation parameter. It is clear that the rate of heat transfer is more in the presence of thermal radiation. The local Sherwood number for different values of the Schmidt number are shown in figure 10. As expected, the rate of mass transfer increases with increasing values of the Schmidt number. This trend is just reversed as compared to the concentration field for different Schmidt number given in figure 7.

The average values of the skin-friction, Nusselt number and Sherwood number are shown in figures 11, 12 and 13 respectively. The effects of the different phase angle on the average values of the skin-friction are shown in figure 11. The average skin-friction decreases with decreasing with increasing values of the phase angle. Figure 12 illustrates the average Nusselt number increases with increasing radiation parameter. From figure 13, it is observed that the average Sherwood number increases with increasing values of the Schmidt number.

❖ CONCLUSION

Finite difference study has been carried out for unsteady hydromagnetic flow past an oscillating semi-infinite isothermal vertical plate in the presence of thermal radiation. The effect of velocity, temperature and concentration for different parameter are studied. The local as well as average skin-friction, Nusselt number and Sherwood number are shown graphically. It is observed that the contribution of mass diffusion to the buoyancy force increases the maximum velocity significantly. It is also observed that the velocity decreases in the presence of thermal radiation. The study shows that the number of time steps to reach steady-state depends strongly on the radiation parameter and magnetic field parameter.

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