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[1.]Adolfo SENATORE, [2.]Mario PISATURO

AN EXPERIMENTAL PROCEDURE FOR THE EVALUATION OF VIBRATION FREQUENCIES OF ROTATING STRUCTURAL ELEMENTS IN MECHANICAL AND AEROSPACE ENGINEERING

[1.] DEPARTMENT OF INDUSTRIAL ENGINEERING, UNIVERSITY OF SALERNO – FISCIANO (SA), ITALY

ABSTRACT: This paper describes an experimental procedure about out-of-plane vibrations in rotating uniform Euler-Bernoulli beams with double symmetry in the cross-section. Several theoretical studies have been directed to the evaluation of natural frequencies of rotating structural elements such as space frames, windmill rotors, aircraft propeller, etc. In this research, an experimental procedure capable to overcome the testing limits due to the inference of mechanical coupling with vibrations of hub and driving motor, the rotating transducers requirement, the problems of rubbing electrical contact is offered.
Finite element analysis and literature works provide the numerical reference for all the considered laoding and spinning cases.
KEYWORDS: beam vibration and mode shape, FE Analysis, rotating structural elements, natural frequencies

INTRODUCTION

This research may be seen as a part of the recent investigations addressed to the evaluation of natural frequencies of rotating structural elements such as space frames, windmill rotors, aircraft propellers, turbine blades, light spinning mechanical devices.

The most simplified representation of a rotating beam is a one-dimensional Euler-Bernoulli model. A uniform rotating beam of doubly symmetric cross-section is a special case; for his system no torsional motions appear: i.e., out-of-plane "flapping" vibration and in-plane "lead-lag" vibration are uncoupled. Owing to the stiffening effect of the centrifugal tension, one can expect the natural frequencies to increase with an increase of the speed of rotation and/or increase in the hub radius. In several publications a cantilever under centrifugal tension has been considered and approximate methods such as Rayleigh-Ritz, Galerkin, etc., have been used to derive natural frequencies. Fox and Burdess [1] used the Galerkin procedure to analyze the out-of-plane vibration. Hodges and Rutkowski [2] analyzed the problem by a finite element method of variable order; other results based on this approach are described by Udupa and Varadan [3]. Wright et al. [4] presented an analytical solution of the mode shape equation for beams attached to a rotating hub and with particular flexural rigidity/mass distribution.

Naguleswaran [5] presented the exact results achieved using a differential equation and its solution approach rather than the dynamic stiffness method. Storti and Aboelnaga [6] listed the class of beams which admit solutions in terms of hypergeometric functions. The results are presented in tabular form to preserve the accuracy to seven significant figures.

Banerjee [7] analysed the application of the derived dynamic stiffness matrix investigating the free-vibration characteristics of uniform and non-uniform (tapered) rotating beams with particular reference to the Wittrick-Williams algorithm [8-10]. In the paper of Banerjee [7] the mode shape equation is solved by the method of Frobenius and the general solution is expressed as the superposition of four linearly independent converging polynomials. In that paper, out-of-plane and in-plane vibration are considered for combinations of clamped, pinned and free end conditions. The frequency equations for six combinations of boundary conditions are presented in closed form, the roots obtained by a trial and error search followed by an iterative procedure based on linear interpolation. The first three natural frequencies are presented for out-of-plane vibrations for several combinations of offset and rotational speed parameters.

The present paper proposes an experimental procedure able to overcome the laboratory limits about the undesidered inference of mechanical coupling with vibrations of hub and driving motor, the rotating transducers requirement, problems of rubbing electrical contact of the spinning tests.

❖ LATERAL VIBRATION OF A ROTATING UNIFORM BEAM

An interesting case in the field of vibrations in continuous media is the rotating beam under the action of the centrifugal force field which is the cause of a tensile stiffening effect. The configuration clamped-free (C-F) is the idealisation of a number of dynamical systems as turbine shovel, ship helix, helicopter rotor.

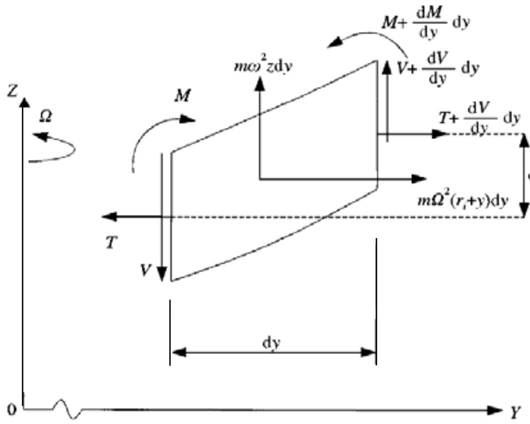

Figure 1: Forces acting on an incremental element: out-of-plane vibration [7].

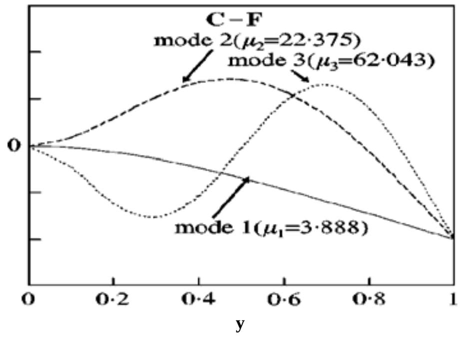

Figure 2: Mode shapes for clamped-free conditions and dimensionless frequency [7].

The theoretical approach to this phenomenon can be analysed with reference to the Fig. 1 and the force equilibrium while the beam exhibits flapping and lead-lag motion [5, 8]. The theoretical studies involved in the final comparisons consider this mode shapes as uncoupled. The beam stiffness increment appears in the out-of-plane vibrations and the displacements find place in the YZ plane as in the Fig. 1. Mode shapes for clamped-free conditions are given in Fig. 2.

❖ FINITE ELEMENT ANALYSIS

In this section a FE modal analysis to seek the natural frequencies of a rotating uniform beam is proposed. The FE model has been implemented by applying ANSYS Parametric Design Language (APDL). The beam is subjected to the action of centrifugal force field. Since a modal analysis provides only the natural frequencies and mode shapes of an unloaded structure, this study needs a previous static analysis to obtain the prestress results. In the FE model the effect of the centrifugal force has been applied on the beam by taking into account that the force varies linearly along the axial direction.

The beam is modelled through *beam3* ANSYS element type (see Ansys library [12] for details) with clamped-free end conditions and elastic linear material law. For the chosen element type this analysis needs only these informations: cross-section, area moment of inertia and the total beam lenght. The Block Lanczos algorithm, based on sparse matrix solver allows seeking the first three eigenfrequencies and it is as accurate as the subspace method but faster.

The following plots in Fig. 3 depict the results of the analysis above described:

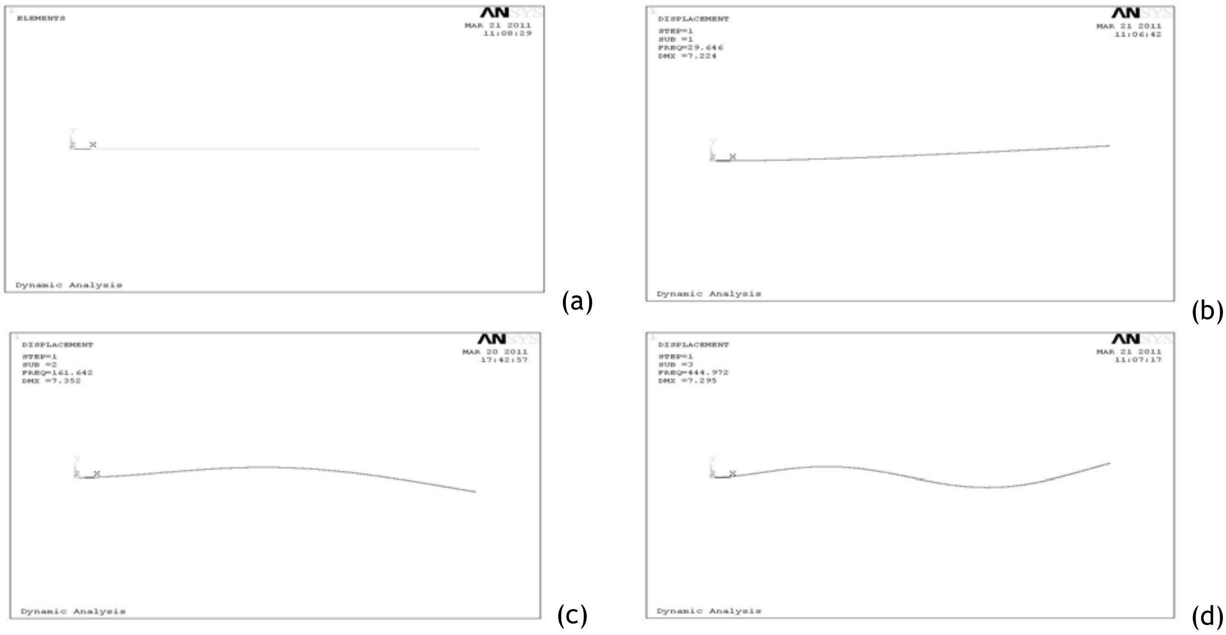

Figure 3: Undeformed and deformed structure: a) undeformed; b) first mode; c) second mode; d) third mode

The following table indicates the results of the FE analysis both for the distributed load and for the suspended weights configuration:

Table 1: FEA results: frequencies of the first three modes

Dimensional speed, Ω	Dimensionless speed, ν	Distributed load		FE Analisys results	
				Two weights system	Three weights system
44.9 rad/s	1	f_1	26.4 Hz	26.9 Hz	26.9 Hz
		f_2	158.6 Hz	159.6 Hz	159.3 Hz
		f_3	442.0 Hz	442.9 Hz	442.6 Hz
89.8 rad/s	2	f_1	29.6 Hz	32.1 Hz	31.4 Hz
		f_2	161.6 Hz	165.9 Hz	164.5 Hz
		f_3	445.0 Hz	448.5 Hz	447.6 Hz

❖ EXPERIMENTAL INVESTIGATION

As stated above, the present paper proposes an experimental procedure able to overcome the laboratory limits about tests on spinning light mechanical elements. A direct method to get assessments on the natural frequencies of a rotating uniform beam by means of a Laser Doppler Vibrometer as transducer to perform the test also on very light structures and its results are presented.

In order to reproduce the presence of the centrifugal force field the beam has been clamped on the top with the longitudinal Y-axes along the vertical while a system of two or three calibrated weights has been used as lumped parameter axial loads. All the tests have been performed on aluminium beams with sizes 310 mm, 30 mm and 3 mm, rectangular section.

By using the dimensionless variables, it’s possible calculating the rotational speed for comparison puroposes [5]. The weights to be suspended in order to complete the required setup are given by:

$$P_1 = \int_0^{\frac{L}{3}} m\Omega^2 y \, dy \tag{1}$$

$$P_2 = \int_{\frac{L}{3}}^{\frac{2}{3}L} m\Omega^2 y \, dy \tag{2}$$

$$P_3 = \int_{\frac{2}{3}L}^{L} m\Omega^2 y \, dy \tag{3}$$

The following table indicates the values of the suspended weights for both the configuration:

Table 2: weights values for the two simulated operating conditions (ν=1, ν=2)

Dimensionless rotational speed ν [5]	Two-weights system	Three-weights system
1	P_1 = 6.08 N P_2 = 18.25 N	P_1 = 2.70 N P_2 = 8.11 N P_3 = 13.52 N
2	P_1 = 24.32 N P_2 = 73.00 N	P_1 = 10.80 N P_2 = 32.44 N P_3 = 54.08 N

The suspended weights was attached to the beam by means of nylon wires 0.5 mm-diameter, with a good tension resistance and poor bending behaviour (Fig. 4); in this way, the lateral beam dynamics is not perturbed.

In fact, with this anchorage system the suspended weight behaves as a pendulum with up-end excited with an oscillatory motion with frequency higher than the natural one; in this case the vibration amplitude is very low [12].

The vibration measurements have been achieved by means of: Brüel & Kjær Pulse 3560 analyzer, front-end 2825 and Labshop software while the laser Doppler vibrometer was an Ometron 8329, according to the scheme in Fig. 4.

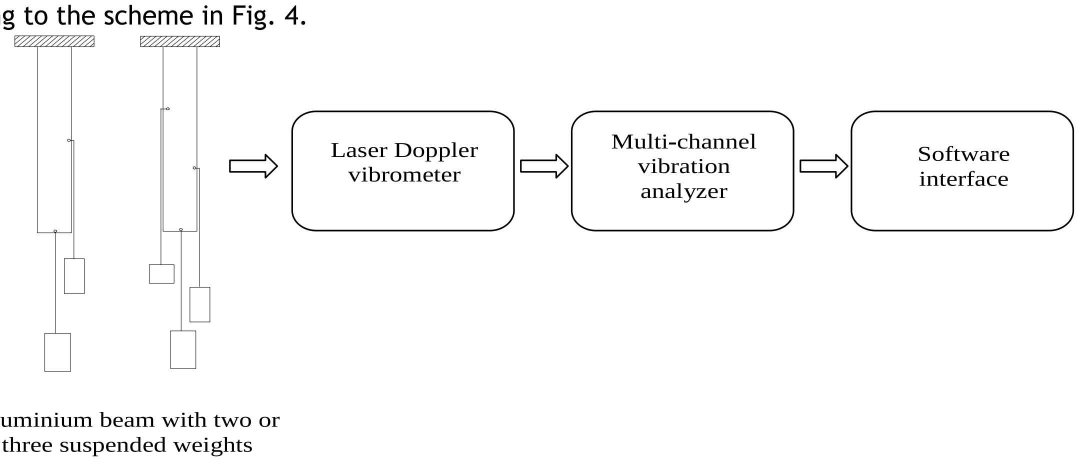

Figure 4: experimental setup and transducer/analyzer block diagram

❖ EXPERIMENTAL RESULTS

The Figures 5 and 6 show the spectra for dimensionless rotational speed ν=1 and ν=2, respectevely, in the test with three weights loading.

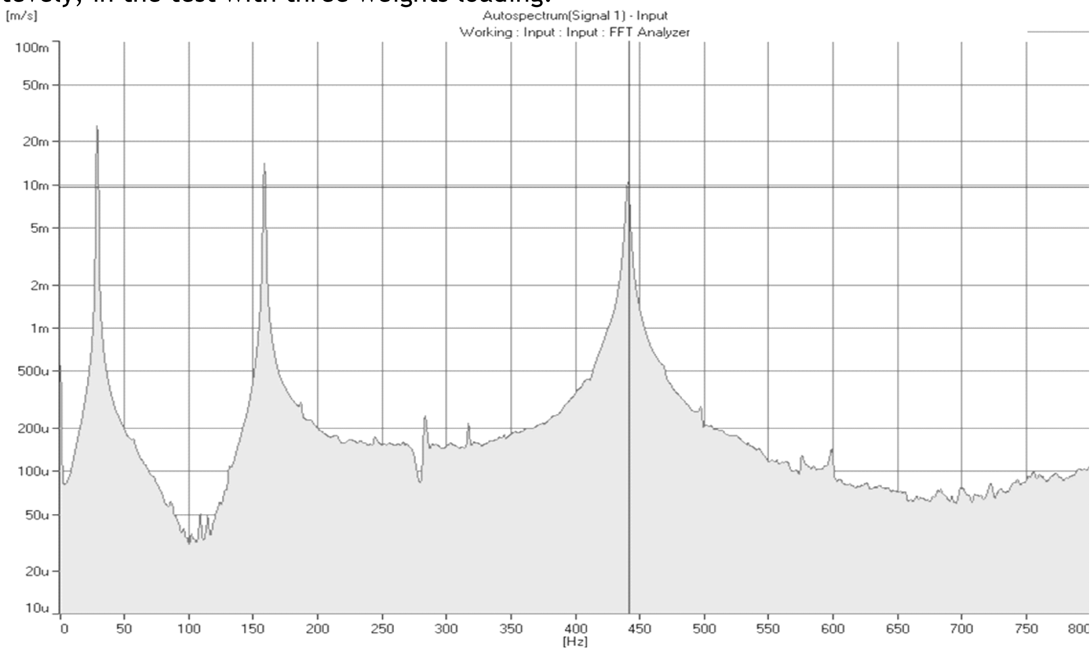

Figure 5: spectrum for dimensionless rotational speed ν=1 (three weights)

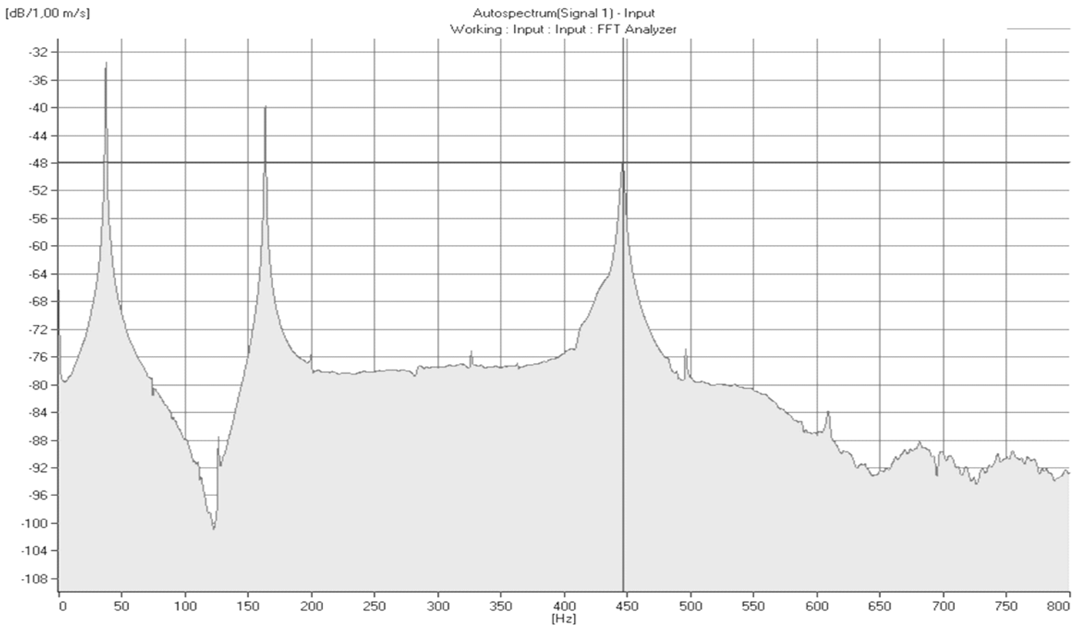

Figure 6: spectrum for dimensionless rotational speed ν=2 (three weights)

The following tables show the remarkable agreement among the theoretical experimental data about the second and third natural frequencies with the three weights system in the both operating condition (ν=1 and ν=2 simulated rotational speeds, Tab. 5) while the results exhibit a good agreement simulating the same speeds but with the simpler two weights loading system.

The difference between theoretical and experimental results for the simulation of both the dimensionless speed ν=1 and ν=2, are a fraction of the percentile, as well as in the case of unloaded beam (Tab. 4), for the second and third natural frequencies.

About the first natural frequency, the experimental results show higher values if compared with the theoretical ones because the lumped P_k loads on the tested beam allows a step representation of the axial force T, with two or three steps according to the number of weights, while the theoretical model takes into account the centrifugal force field with the exact linear dependence with the y co-ordinate. Therefore, the beam portion closer to the clamping end exhibits higher flexural rigidity than the modelled beam. This affects the first natural frequency for the greater amplitude and the flexural shape of the corresponding mode.

Table 4: Comparison between theoretical, numerical and experimental results (unloaded beam)

Dimensionless speed ν		Theoretical natural frequencies [5]	FE Analysis natural frequencies, present work	Experimental natural frequencies, present work
0	f_1	25.3 Hz	25.1	24.8 Hz
	f_2	157.0 Hz	157.5	155.5 Hz
	f_3	438.0 Hz	441.0	436.8 Hz

Table 5: Comparison between theoretical, numerical and experimental results (loaded beam)

Dimensionless speed ν	Dimensional speed Ω		Theoretical natural frequencies [5]	FE Analysis natural frequencies for distributed load, present work	Experimental natural frequencies Two weights system	Experimental natural frequencies Three weights system
1	44.9 rad/s	f_1	26.3 Hz	26.4 Hz	28.0 Hz	29.0 Hz
		f_2	158.5 Hz	158.6 Hz	158.0 Hz	158.5 Hz
		f_3	442.0 Hz	442.0 Hz	440.0 Hz	442.0 Hz
2	89.8 rad/s	f_1	29.6 Hz	29.6 Hz	37.0 Hz	34.5 Hz
		f_2	161.6 Hz	161.6 Hz	163.3 Hz	161.3 Hz
		f_3	445.0 Hz	445.0 Hz	446.0 Hz	445.0 Hz

❖ CONCLUDING REMARKS

The present paper proposes an experimental procedure able to simulate the tensile effect due to centrifugal force fields on several flexural systems as turbine shovel, ship helix, helicopter rotor, etc.

The aim is the overcoming of known limits about the inference drawback of mechanical coupling amon flexural element under test and the vibrations of hub and driving motor, the need of rotating transducers, the problems of rubbing electrical contact in spinning tests.

The outcomes of this paper encourage the development of similar tests for rotating machine elements, providing a reliable prediction of the stiffening effect due to the centrifugal force.

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