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OPTIMIZATION CONSIDERATION ON A PARACHUTE PROBLEM

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ABSTRACT: The heavy non-permanent fluid flow facing a turbine bucket along its symmetry axis (which is also the "parachute problem"), in the presence of gravity is considered. Our purpose is to evaluate the drag on the concave profile in view of some optimization considerations.
KEYWORDS: vertical wind turbines with buckets; parachute type profile; mathematical model of Helmholtz type; B.V.P. of mixed (Volterra) type; optimization considerations
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❖ INTRODUCTION

The analysis of the problem of a heavy non-permanent fluid flow facing a bucket along its symmetry axis, in the presence of gravity seems to be important within the mathematical models for vertical wind turbines with buckets. This problem is also "the parachute problem" which allows us to get information on the total drag in view of some optimization process.

Problems of this type were in the attention of many scientists [1], [2], [4], [5] who also have studied the optimization inverse joint problem, i.e., to determine the parachute shape which assures a maximum drag. But all these authors did take into consideration only the steady case in the absence of gravity. More recently it has been developed a mathematical model of Helmholtz type for the unsteady plane flow of an inviscid incompressible fluid, in the presence of gravity, past a curvilinear obstacle (bucket or parachute type profile) with its concave side towards the "attack" stream [7], [8]. In this approach under some certain hypotheses an analytical approximate solution of the problem based upon a solution of the Bernoulli integral considered on a jet (separation) line is firstly obtained, which allows us the effective determination of these unknown jet lines.

❖ THE PARACHUTE PROBLEM

It is shown that besides the initial stagnation (attack) point *B* (the middle of the symmetrical concave contours *CC'*), there is a second "stagnation point" *D*, placed downstream on the symmetry vertical axis *Ox*, which will "close" the backward obstacle cavity zone, our configuration belonging to the case when the separation lines are crossing. Then the effective construction of the complex potential *f* of the whole flow is made in some appropriate auxiliary planes. Precisely one considers firstly the image of our physical flow domain of the plane (*z*) onto the superior half-plane of the plane $f=\varphi+i\psi$. There by using a conformal mapping of Joukovski type, this domain of the plane (*f*) is represented on a unit superior half-disk, centered at the origin, of a new plane (τ), so that the jet lines of the physical plane correspond to its semi-circumferences.

Finally the solving of our initial problem comes to a B.V.P. of mixed (Volterra) type which can be reduced, in a classical way [2] to a Dirichlet problem for another half-plane [2].

Our purpose is to evaluate the drag on the concave profile in view of some optimization considerations.

The total pressure of the stream, of mass density ρ and complex potential *f*(*z*), on our curvilinear obstacle *CC'* of perimeter *L* is

$$\int_C^B (p-p_0)ds=(p_{s^0}-p_0)\frac{L}{2}-\rho a\frac{L}{2}-\rho g\int_0^{x_C} x\sqrt{1+y'(x)^2}\,dx-\frac{1}{2}\int_0^{x_C}\left|\frac{df}{dz}\right|^2 ds\,,$$

where p_0 is the exterior (atmospheric) constant pressure, p_{s^0} is the pressure at the stagnation point *B*, *g* is the gravity acceleration while $y=y(x)$, $x\in(0,x_C)$ is the equation of the half-contour *BC* in the physical plane corresponding to the abscissas *O* and x_C.

Concerning the last term, by using the results of [7], [8], it comes to

$$\int_0^1[(\tau+1)^\alpha(b-\tau)^{1/2}+(\tau+1)^{\alpha+\varepsilon}](1-\frac{1}{\tau^2})e^{\frac{1}{\pi}\sqrt{\frac{(1+\tau)^2}{2\tau}[\frac{(1+\tau)^2}{2\tau}-1]}\int_b^1\frac{y'(x(\xi))}{\sqrt{\frac{(1+\xi)^2}{2\xi}[\frac{(1+\xi)^2}{2\xi}-2]}}\frac{d\xi}{\frac{M}{2\xi}(1+\xi)^2-\xi}}$$

where α and ε are positive less than unity exponents, observing the restriction $0<\alpha+\varepsilon<1$, while $a,b>0$ and $M<0$ are constants precised by some regularity requirements for our solution [7], [8].

In fact, in order to optimize the drag, we have to find that shape such that the last integrals should be maximized.

❖ OPTIMIZATION CONSIDERATIONS

Problem A:

Let $J(y')=\int_{x_C}^{0} x\sqrt{1+y'(x)^2}\,dx$ be a functional defined on the differentiable functions $y'<0$, $x\in(x_C,0)$. It is obvious that $\max J(y')$, subject to $y'\leq 0$ for all $x\in[x_C,0]$ is $y'(x)=0$. Indeed since x is negative then by increasing x (i.e., decreasing of $|x|$) one leads to the decreasing of $\sqrt{1+y'(x)^2}$, hence to increase of $x\sqrt{1+y'(x)^2}$.

Actually some additional constrains are needed in order to have nontrivial solutions.

Problem B:

The last term of the total pressure evaluation implies a functional *J* of the following structure

$$J(y')=\int_0^1 f(\tau)e^{kg(\tau)F(y')}d\tau\,,$$

where $F(y')=\int_b^1 y'(x(\xi))l(\xi)d\xi$, with corresponding expressions for the functions *f*, *g*, *l* and for the constant *k*.

By considering the change of variable $\eta=x(\xi)$, $F(y')$ could be represented as $F(y')=\int_0^{x_C} y'(\eta)h(\eta)d\eta$, where *h* is a certain negative function $h=l(x^{-1}(\eta))$ (with *l* negative in our case).

Because *F* is a linear functional its gradient ∇F satisfies the equality $\nabla F=F$, that is $\nabla F(y')(w)=F(w)$ for all y' and *w*. Now the functional *J* can be written under the form $J(y')=G(F(y'))$ where *G* is a function of real variable defined by $G(v)=\int_0^1 f(\tau)e^{kg(\tau)v}d\tau$. Obviously $\nabla G(v)=G'(v)=\int_0^1 f(\tau)kg(\tau)e^{kg(\tau)v}d\tau$.

Using the chain rule theorem we have

$$\nabla J(y')(w)=\nabla G_{F(y')}(\nabla F(y')(w))=G'(F(y'))F(w)=F(w)\int_0^1 f(\tau)kg(\tau)e^{kg(\tau)F(y')}d\tau\,.$$

Assume that the function y' is given and we are considering $\nabla J(y')(w)$ as a function of *w*.

Let $\gamma(y')=\int_0^1 f(\tau)kg(\tau)e^{kg(\tau)F(y')}d\tau$, so we have

$$\nabla J(y')(w)=F(w)\cdot\gamma(y')=\gamma(y')\cdot\int_0^{x_C} w(\eta)h(\eta)d\eta\,.$$

Let $\bar{y}'$ be a maximum. Assume that $\bar{y}'$ is strictly negative or is equal to zero only on a set with Lebesque measure equals to zero. Then the necessary condition for maximum over a set of functions *w* such that $y'+\lambda w<0$ (for a constant $\lambda>0$) is that $\nabla J(y')(w)\geq 0$ [6], [9]. But in our case this can be done if and only if $\int_0^{x_C} w(\eta)h(\eta)d\eta=0$.

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