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FIRST ORDER CHEMICAL REACTION ON UNSTEADY FLOW PAST AN ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION

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ABSTRACT: An exact solution of unsteady flow of a viscous incompressible fluid past a uniformly accelerated infinite vertical plate with variable temperature and mass diffusion in the presence of homogeneous chemical reaction of first-order. The plate temperature is raised linearly with respect to time and the concentration level near the plate is also raised linearly with time. The dimensionless governing equations are solved using the Laplace transform method. The effects of velocity, temperature and concentration are studied for different physical parameters like chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number and time. It is observed that the velocity increases with decreasing chemical reaction parameter.
KEYWORDS: accelerated, vertical plate, heat and mass transfer, chemical reaction

❖ INTRODUCTION

In many chemical engineering processes, there is the chemical reaction between a foreign mass and the fluid. These processes take place in numerous industrial applications such as manufacturing of ceramics, wire drawing, food processing and polymer production.

Chambre and Young (1958) have analyzed a first order chemical reaction in the neighborhood of a stationary horizontal plate. Ramanamurthy and Govindarao (1971) have made an analysis of flow around a cylindrical catalyst pellet with first order chemical reaction. Apelblat (1980) studied analytical solution for mass transfer with a chemical reaction of the first order. Das et al.(1994) have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Gupta et al (1979) studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis(1981) extended the above problem to include mass transfer effects subjected to variable suction or injection. Mass transfer effects on flow past an uniformly accelerated vertical plate was studied by Soundalgekar (1982). Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh (1983). Basant Kumar Jha and Ravindra Prasad(1990) analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo(1986).

It is proposed to study the effects of the homogeneous chemical reaction of first order on unsteady flow past a uniformly accelerated infinite vertical plate with variable temperature and mass diffusion. The dimensionless governing equations are solved using the Laplace transform technique. The solutions are in terms of exponential and complementary error function.

❖ GOVERNING EQUATIONS

First order chemical reaction effects on unsteady flow of a viscous incompressible fluid past an accelerated infinite vertical plate with variable temperature and mass diffusion is studied. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, the plate and the fluid are of the same temperature T_∞ and concentration C'_∞. At time t' > 0, the plate is accelerated with a velocity $u = \frac{u_0^3}{v} t'$ in its own plane and the temperature of the plate is raised linearly with respect to time and the concentration level near the plate is also raised linearly with time. Then by usual Boussinesqs' approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\rho C_P \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \quad (2) \qquad \frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - K_l C' \quad (3)$$

With the following initial and boundary conditions: $u = 0, \quad T = T_\infty,\ C' = C'_\infty$ for all $y, t' \le 0$

$$t' > 0: \ u = \frac{u_0^3}{\upsilon} t', \quad T = T_\infty + (T_w - T_\infty) A t', \quad C' = C_\infty + (C_w - C_\infty) A t', \ \text{at } y = 0 \quad (4)$$

$$u \to 0, \quad T \to T_\infty,\ C' \to C'_\infty \ \text{as } y \to \infty$$

where $A = \frac{u_0^2}{\nu}$.

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0},\ t = \frac{u_0^2 t'}{\nu},\ Y = \frac{y u_0}{\nu},\ K = \frac{K_l \nu}{u_0^2},\ \Pr = \frac{\mu C_P}{k},\ Sc = \frac{\nu}{D}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},\ Gr = \frac{g\nu\beta(T_w - T_\infty)}{u_0^3},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ Gc = \frac{g\nu\beta^*(C'_w - C'_\infty)}{u_0^3} \quad (5)$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} \quad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial Y^2} \quad (7) \qquad \frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - KC \quad (8)$$

The negative sign of K in the last term of the equation (8) indicates that the chemical reaction takes place from higher level of concentration to lower level of concentration.

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{llllll} & U = 0, & \theta = 0, & C = 0 & \text{for all} & y, t \le 0 \\ t > 0: & U = t, & \theta = t, & C = t & \text{at} & y = 0 \\ & U = 0, & \theta \to 0, & C \to 0 & \text{as} & y \to \infty \end{array} \quad (9)$$

❖ SOLUTION PROCEDURE

The solutions are obtained for hydrodynamic flow field in the presence of first order chemical reaction. The dimensionless governing equations (6) to (8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = t\left[(1 + 2\eta^2 \Pr)\, erfc(\eta\sqrt{\Pr}) - 2\eta\sqrt{\frac{\Pr}{\pi}} \exp(-\eta^2 \Pr)\right] \quad (10)$$

$$\begin{aligned} C = {} & \frac{t}{2}\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \\ & - \frac{\eta\sqrt{Sct}}{2\sqrt{K}}\left[\exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt}) - \exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt})\right] \end{aligned} \quad (11)$$

$$\begin{aligned} U = {} & (1 + 2cb)\, t\left[(1 + 2\eta^2)\, erfc(\eta) - \frac{2\eta}{\sqrt{\pi}} \exp(-\eta^2)\right] + 2b\, erfc(\eta) \\ & - \frac{at^2}{6}[(3 + 12\eta^2 + 4\eta^4)\, erfc(\eta) - \frac{\eta}{\sqrt{\pi}}(10 + 4\eta^2)\exp(-\eta^2) \\ & - (3 + 12\eta^2 \Pr + 4\eta^4 \Pr^2)\, erfc(\eta\sqrt{\Pr}) + \frac{\eta\sqrt{\Pr}}{\sqrt{\pi}}(10 + 4\eta^2 \Pr)\exp(-\eta^2 \Pr)] \\ & - b\exp(ct)\left[\exp(2\eta\sqrt{ct})\, erfc(\eta + \sqrt{ct}) + \exp(-2\eta\sqrt{ct})\, erfc(\eta - \sqrt{ct})\right] \\ & - b\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \\ & + b\exp(ct)[\exp(-2\eta\sqrt{Sc(K + c)t})\, erfc(\eta\sqrt{Sc} - \sqrt{(K + c)t}) \\ & + \exp(2\eta\sqrt{Sc(K + c)t})\, erfc(\eta\sqrt{Sc} + \sqrt{(K + c)t})] \\ & - bct\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \\ & + bc\,\eta\sqrt{\frac{tSc}{K}}\left[\exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt}) - \exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt})\right]. \end{aligned} \quad (12)$$

where, $a = \dfrac{Gr}{1-Pr}, b = \dfrac{Gc}{2c^2(1-Sc)}, c = \dfrac{K\,Sc}{1-Sc}$ and $\eta = \dfrac{Y}{2\sqrt{t}}$.

❖ RESULTS AND DISCUSSION

For physical interpretation of the problem, numerical calculations are carried out for different physical parameters Gr, Gc, Sc, Pr, K and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number Pr are chosen such that they represent air(Pr = 0.71) and water(Pr = 7.0). The numerical values of the velocity, temperature and concentration fields are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, chemical reaction parameter, Schmidt number and time are studied graphically.

Figure 1 demonstrates the effect velocity fields for different thermal Grashof number (Gr = 2,5), mass Grashof number (Gc = 2,5,10), K = 0.2, Pr = 0.71 and t = 0.2. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number. Figure 2 represents the effect of velocity profiles for different Schmidt number (Sc = 0.16, 0.3, 0.6), Gr = 5, Gc = 5, K = 0.2, Pr = 0.71 and t = 0.2. The trend shows that the velocity increases with decreasing Schmidt number. It is observed that the relative variation of the velocity with the magnitude of the Schmidt number.

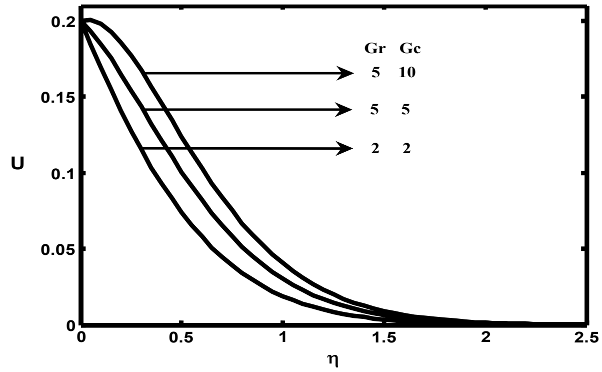

Figure 1. Velocity profiles for different Gr, Gc

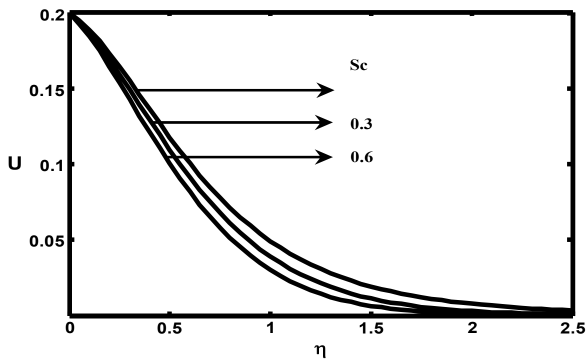

Figure 2. Velocity profiles for different Sc

The velocity profiles for different (t = 0.2, 0.4, 0.6), K = 0.2, Gr = Gc = 5, Pr = 0.71 are studied and presented in figure 3. It is observed that the velocity increases with increasing values of the time t. Figure 4 illustrates the effect of velocity for different values of the chemical reaction parameter (K = 0.2, 15), Gr = 5, Gc = 5, Pr = 0.71 and t = 0.2. This shows that the increase in the chemical reaction parameter leads to a fall in the velocity.

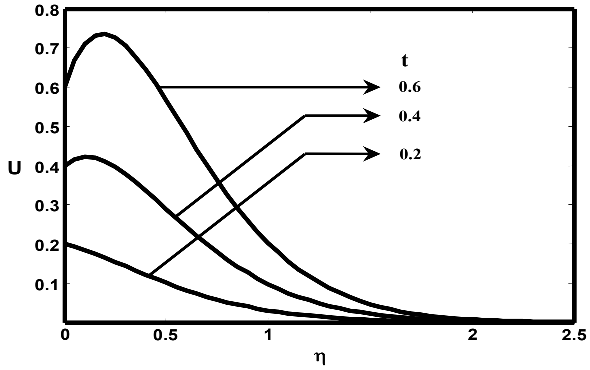

Figure 3. Velocity profiles for different t

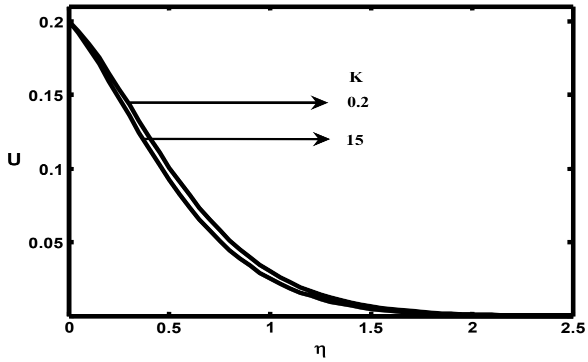

Figure 4. Velocity profiles for different K

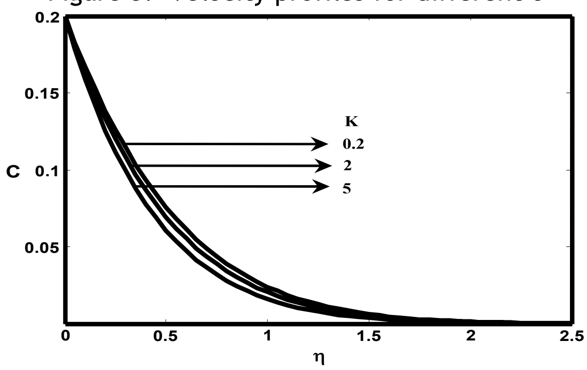

Figure 5. Concentration profiles for different K

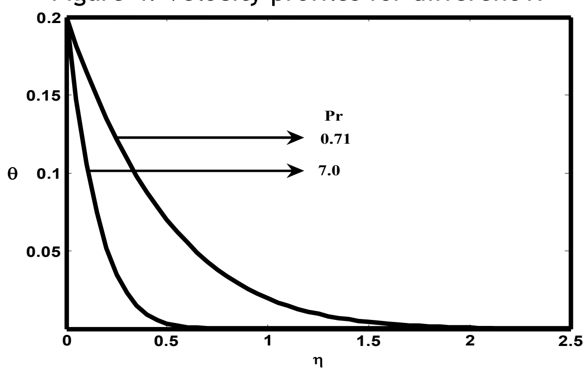

Figure 6. Temperature Profiles for different Pr

The effect of concentration profiles for different values of the chemical reaction parameter (K = 0.2, 2, 5) and time t = 0.2 are presented in figure 5. The effect of the chemical reaction parameter is dominant in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the chemical reaction parameter. It is observed that the concentration increases with decreasing chemical reaction parameter.

The temperature profiles for air (Pr = 0.71) and water (Pr = 7.0) are studied at time t = 0.2in figure 6. It is observed that the heat transfer is more in air than in water. It is clear that there is a sudden drop in temperature in water compared to that in air.

❖ CONCLUSIONS

An exact solution of unsteady flow past a uniformly accelerated infinite vertical plate with variable temperature and mass diffusion, in the presence of homogeneous chemical reaction of first order has been studied. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different physical parameters like thermal Grashof number, mass Grashof number and t are studied graphically. It is observed that the velocity increases with increasing values of Gr, Gc and t. But the trend is just reversed with respect to the chemical reaction parameter and Schmidt number.

❖ NOMENCLATURE

A - constant
C' - species concentration in the fluid
C - dimensionless concentration
C_p - specific heat at constant pressure
C'_w - concentration of the plate
C'_∞ concentration of the fluid far away form the plate
D - mass diffusion coefficient
Gr - mass Grashof number
Gc - thermal Grashof number
g - accelerrated due to gravity
k - thermal conductivity
K_l - chemical reaction parameter
K - dimensionless chemical reaction parameter
Pr- Prandtl number
Sc- Schmidt number
T - temperature of the fluid near the plate
T_w - concentration of the plate
T_∞ - concentration of the fluid far away form the plate
t' - time s
t - dimensionless time
u - velocity of the fluid in the x-direction
u_0 - velocity of the plate
U - dimensionless velocity
x - spatial coordinate along the plate
y' - coordinate axis normal to the plate m
y - dimensionless coordinate axis normal to the plate
β - volumetric coefficient of thermal expansion
β^* - volumetric coefficient of expansion with concentration
μ - cofficient of viscosity
υ - kinematic viscosity
ρ - density of the fluid
θ - dimmensionless temperature
η - similarity parameter
erfc - complementary error function

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