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APPROXIMATE PERFORMANCE ANALYSIS OF A CLOSED LINE WITH RANDOM FAILURES BY AGGREGATION OF SEGMENTS

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ABSTRACT: *Closed lines subject to random station failures have been a subject of analysis for a few decades. Analytical models especially for approximate analysis have proven to be effective. Discrete time modeling, typically Markov chains, is commonplace but the underlying state spaces grow very fast as the size approaches the real-life cases. We represent the line by a number of functional segments treated as open serial lines. Consequently a simpler closed line is formed with fewer stages. Key performance parameters from the open lines are reflected as additional failure modes from a first round of calculation. This is followed by solving the simplified line in a second round. The results show acceptable levels of accuracy can be achieved.*
KEYWORDS: *Closed line, performance analysis, discrete stochastic modeling, Markov Chains*

INTRODUCTION

We address closed production lines formed by randomly failing stations in series. It is quite common in industrial practice to assess such configurations by a mix of analysis techniques. This is mainly because of the difficulty to arrive at conclusive results in satisfactory detail by a single approach.

Automotive assembly lines have often been the subjects in these detailed performance assessment. Recent work on such studies in well known car makers are mentioned in the open literature [1, 2]. Analytical methods in performance evaluation have some advantages although they come at a cost. They provide more comprehensive and exploratory outcomes in a shorter time than experimental (i.e., simulation based) methods. However these often come with some loss of realism due to their structure and need for extended computational work especially in realistic sized cases.

In particular we take production lines for which discrete stochastic modeling is viable. We propose an aggregation method to improve the speed in conducting analytical performance assessment by reducing the problem size. We claim that the aggregation of stages on an open line segment by our approach into a single stage with an extra failure mode does not cause much loss in computed performance measures. Our suggestion is based on available methods and is inspired from the premises that starvation caused by upstream stages can be taken as an additional failure mode and that an open linear line can be collapsed to a single stage. We report comparisons of the approximate solutions with simulation conducted for different closed lines.

ANALYSIS OF PRODUCTION LINES WITH RANDOM FAILURES

A production line is a series of process stages performing successive tasks on parts in the same order. Process stages may be subject to random stoppages due to failures. Parts flow will be interrupted at such instances. Intermediate buffer storages are often inserted to maintain flow at least for a while until the broken stage gets repaired. A closed line with intermediate buffer storages is sketched in Figure 1. Every stage is numbered uniquely with an identically designated succeeding buffer. Material flows in the direction shown by the arrows. The flow is often in unit parts rather than batches to achieve higher line efficiency.

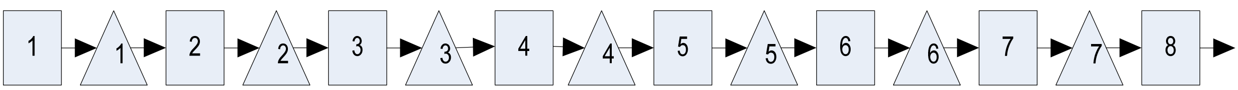

Figure 1- Schematic description of a product line

Besides the inserted buffers, line balance is sought to maintain a continuous flow as much as possible for acceptable line efficiency. Such product line arrangements are often found in assembly, in particular in its oldest industrial form, car body assembly.

In a product line with unit part flow and no failures, the rhythm (expressed in parts/unit time) of part completions is given by the slowest among the stages. The periodicity of part completions is the cycle time of the line. In a perfectly balanced line unit process times at all stages are identical and hence are equal to the cycle time. In lines with random failing stages in perfect balance, a measure of line efficiency, E, is defined

$$E = \frac{\text{Cycle Time}}{\text{Average Interdeparture Time}} \quad (1)$$

Thus E is the percentage of time any stage will be busy on the average rather than being down, starved or blocked.

Production lines may be open or closed. An open line is characterized by the input stream to its first stage being independent of the completion on its last stage. The schematic line in Figure 1 is open. In an open line the first stage can potentially receive a fresh part as soon as it completes processing on a part and, the last stage always has room to place a completed part.

On a closed line, however, the first stage is tied up to the completions at the last stage. This is caused by the presence of some sort of carriages (for instance, fixtures, bins, skids) carrying the parts and circulating on the line. The line has a finite number of these. Their count is comparatively less than the total buffer storage spaces. Figure 2 depicts such a closed line. Unless the last stage continues processing for a long enough time, the first stage is bound to starve. Stage 8 and Stage 1 operations are somehow related through the circulating character.

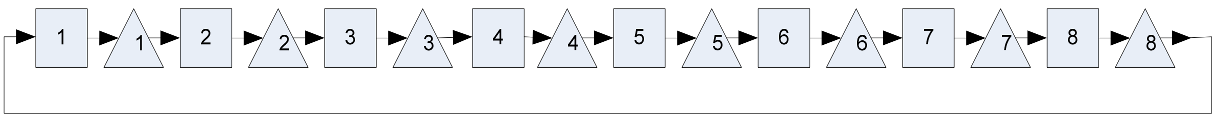

Figure 2- Schematic description of a closed line

PERFORMANCE ANALYSIS

Our computation for the closed lines is based on the method proposed in [3]. Gershwin and Werner suggest decomposing the series of stages into modules of two successive stages (namely, the upstream and downstream) separated by the buffer. For instance the line in Figure 2 is analyzed in modules formed by stage pairs (1,2), (2,3),..(7,8) and (8,1). The performance analysis of every pair is based upon an earlier work [4]. The method in [3] provides the performance parameters to be combined for closed lines in the way proposed earlier for the open production lines in [5].

The discrete time stochastic model of a given module is in the form of a Markov chain. Its states are defined by the triple (n, α_1, α_2) evolving in transitions at equally spaced intervals. Here n is the number of parts in the buffer, α_i is the state of the stages (i=1 for the upstream, i=2 for the downstream). α_i shows whether a stage is operational or in one of the failure modes. Random failures in different modes with given probabilities are assumed to occur independently. Basic assumptions and solution approach for the steady state probabilities can be found in [4]. The critical part in the assumptions is that the upstream stage stops operating if the intermediate buffer is full, no matter whether the downstream stage is operational or not. Thus stages are not allowed to serve as temporary buffers.

A module is busy producing parts in all states where n is positive and α_2 indicates that the downstream stage is not down. Let p(n, α_1, α_2) denote the steady state probability of being in any state (α_1, α_2). Hence efficiency, E, of the module is given by

$$E = \sum_{n>0,\alpha_2=\{operational\}} p(n, \alpha_1, \alpha_2) \quad (2)$$

Starvation occurs when the downstream stage is operational but the intermediate buffer is empty. Hence its percentage is given by

$$S = \sum_{n=0,\alpha_2=\{operational\}} p(n, \alpha_1, \alpha_2) \quad (3)$$

Similarly blocking percentage is given by (N is the capacity of the buffer)

$$B = \sum_{n=N,\alpha_1=\{operational\}} p(n, \alpha_1, \alpha_2) \quad (4)$$

Successive modules (like. modules (1,2), (2,3) and (3,4) in the above example) affect one another in the direction of flow through their starvation. For instance module (1,2) starving for long enough causes modules (2,3) and (3,4) to suffer from lack of part flow with some likelihood for each. These effects are reflected as additional failure modes to every stage in addition to the stage's own failure mode. Blocking in the modules lying downstream has an effect in the reverse direction of part flow. For instance module (4,5) getting blocked for long enough causes modules (1,2) and (2,3) to suffer from this lockup. Blocking effects from downstream stages are similarly reflected as additional failure modes to every stage in addition to the stage's own failure mode [3,5].

The starvation effect reflected as an additional failure mode to a downstream stage has inspired our proposed segment aggregation approach.

APPROXIMATION WITH SEGMENT AGGREGATION

We take a closed line and form a number of linear segments from several consecutive stages. Each of these linear segments forms an open line starting and ending with a stage from the original line. Buffer for the last stage is omitted due to its triviality. This is illustrated in Figure 3 below. Two segments are formed on an 8-stage closed line. Segment 1 is an open line with 3 stages and segment 2 is an open line with 4 stages.

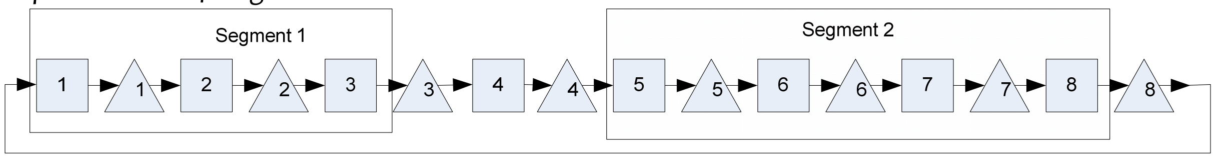

Figure 3 – Formation of line segments on a closed line

Consider the last stage of a given segment. Starvation caused by the upstream stages can be handled as an additional failure mode. We aggregate effects of all preceding stages on the last stage's starvation as an additional random cause for its failure. Let us denote this new failure mode to occur with probability p_f. We also aggregate the probability of resuming operation after this failure mode and denote that by $Wrepair_f$.

$$Wrepair_f = \sum_{i \in \{all\ preceding\ stages\}} r_i \frac{S_i}{S} \quad (5)$$

In the above identity r_i is the probability of repair for the failure in stage i and S_i is the starvation caused specifically by stage i on the last stage. Hence $Wrepair_f$ is the average of repair probabilities weighted by their individual effects on the starvation in the last stage. Now the equality,

$$p_f = \frac{S\, Wrepair_f}{E} \quad (6)$$

is used by the preservation of flow before and after a breakdown as it is suggested in the related works [3,5]. Thus the last stage will get into the failure state given it was operational as often as it resumes operation given that it has failed due to a starvation.

In equations (5) and (6) is r_i's are given system parameters and S_i's, S and E are computed for the segment taking it as an open line (using the approach in [5]).

REVISED NUMBER OF CARRIAGES

A closed line is characterized by limited the number of circulating carriages. Although empirical work in [3] emphasizes the insensitivity of the line efficiency within a wide range of this parameter, it is still needed in performance analysis. Aggregation of linear segments as their last stage causes a number of these carriages to be absorbed (hence missing) for later analysis. Our approximation requires assessing the proper number (of carriages) resulting from this removal.

We find the sum of the average buffer sizes in the analytical solutions for linear segments. Buffer capacities in our analytical solutions for the open lines are taken one larger than they are in the true closed line to allow the stages serve as temporary buffers (ref. Section 2.1). Hence the sum of average buffer sizes is an indicator for the number of carriages kept busy, on the average, by all the stages and buffers except the last stage on the respective line segments. On the other hand, open lines face no blocking at their upstream hence a raw part is assumed always available to the first stage. However, in the closed line case this ample supply of raw parts is not true. So, number of carriages kept busy has to be reduced for the limited supply of raw parts due to the cycling in closed lines.

Efficiency, E, of the closed line is bounded from above by the minimal isolated stage efficiency, E_s, defined for the individual stages as

$$E_s = \frac{1}{1 + \frac{f}{r}} \tag{7}$$

where f is the failure and r is the repair probability, respectively, at a generic stage s. Thus, E ≤ Minimum $_{s \, \varepsilon \, \{all \, stages\}}$ $\{E_s\}$. Hence the sum of the average buffer sizes in the analytical solutions is multiplied by Minimum $_{s \, \varepsilon \, \{all \, stages\}}$ $\{E_s\}$ to infer an approximate number of carriages removed from the true total present in the closed line.

RESULTS

Our proposed approach is tested on three different closed line configurations. Data used in these configurations is shown in Table 1. We made data resemble a recently studied industrial case of an automated automotive body assembly line.

Isolated efficiencies in the case we refer to were in the range [0.887,0.982].

Table 1 - Closed Line Configurations tested

Configuration	No. of Stages	Failure and repair (f , r) probabilities	Capacities of succeeding buffer stocks
A	9	{(0.01,0.3),(0.02,0.3),(0.01,0.3),(0.03,0.4),(0.02,0.45),(0.02,0.3),(0.042,0.33),(0.02,0.45),(0.098,0.36)}	{3,3,6,3,3,6,5,4,6}
B	9	{(0.03,0.4),(0.02,0.45),(0.02,0.37),(0.042,0.33),(0.02,0.45),(0.098,0.36), (0.013,0.45),(0.02,0.45),(0.04,0.54)}	{3,3,6,5,4,6,4,4,6}
C	13	{(0.013,0.45),(0.02,0.45),(0.04,0.54),(0.02,0.36),(0.0098,0.36),(0.008,0.45),(0.01,0.45),(0.042,0.33),(0.02,0.36),(0.0098,0.36),(0.0042,0.33),(0.008,0.45),(0.01,0.45)}	{4,4,6,3,3,3,5,6,3,5,3,3,6}

We first defined the segments and solved the open line configurations. Segmentation in configurations A and B were of the form {3, 3, 3} corresponding to 3 successive stages in each segment with their respective succeeding buffer stock capacities taken from the last column of Table 1. Segemnt formation is arbitrary. The reason for the choice of few stages (3 in the experiments) in defining the segments is to have an accurate aggregation. Fewer stages in aggregation yields more accurate aggregate results, but increases the burden on closed line calculations to be performed later.

The succeeding buffer following the last stage in every segment (like the third entry, 6, for configuration A) was irrelevant as it had no impact on the segment performance. Similarly, segmentation in configuration C was in the form {3, 5, 5}. This way, 5 distinct open line segments were identified

Analytical solutions and the simulated results are compared for the open line approximations and these are shown in Table 2. Simulation model is run on the Educational Version Release 2.00b of WITNESS. Buffer capacities are taken one unit larger in the analytical solutions due to the conventions in WITNESS allowing stages to store parts besides the buffers. Simulation results are from one replication in each case with a run length of 1 million cycles after the 100,000 cycles for the warm-up period.

Table 2 - Analytical solutions compared with simulation results for open line segments

		Approximate Analytical Solution			Simulation Result	
Open line segment	Intermediate Buffer Capacities	Failure Probability (starvation)	Repair Probability (resuming)	Sum of Average Buffer Sizes	Failure Probability (starvation)	Repair Probability (resuming)
{(0.01,0.3),(0.02,0.3),(0.01,0.3)}	{3,3}	0.022	0.3	3	0.017	0.262
{(0.03,0.4),(0.02,0.45), (0.02,0.37)}	{3,3}	0.028	0.423	2.8	0.024	0.36
{(0.042,0.33),(0.02,0.45), (0.098,0.36)}	{5,4}	0.034	0.336	3.2	0.032	0.317
{(0.013,0.45),(0.02,0.45), (0.04,0.54)}	{4,4}	0.009	0.45	6.3	0.007	0.349
{(0.02,0.36),(0.0098,0.36), (0.008,0.45),(0.01,0.45), (0.042,0.33)}	{3,3,3,5}	0.006	0.411	8	0.005	0.345
{(0.02,0.36),(0.0098,0.36), (0.0042,0.33),(0.008,0.45), (0.01,0.45)}	{3,5,3,3}	0.046	0.347	8.8	0.041	0.304

It can be seen form Table 2 that order of magnitudes is identical for the failure probabilities to mimic starvation. Repair probabilities, although close, are not exactly identical. This is believed to be the result of aggregating all stages by weighting as in (5).

We took the true number of carriages in the closed line to be half of the total buffer capacities in the line. It is mentioned in [3] that numerical tests indicate the superior performance of such a setting although performance is pretty flat around it ([6] is another study on buffer sizing). It was 19 in configurations A and B, and 28 in configuration C. Thus in the approximate closed line solution we took 9 carriages in all configurations.

We simulated the three closed line configurations in 6 replications each 200 000 cycles long following a warm-up period of 100 000 cycles in the beginning of the first replication. Line status were maintained however all statistics were reset between replications.

Table 3 – Performance Measurement Experiments

Configuration	Average Efficiency in Simulation Replications						Mean Simulated Efficiency	Approximate Efficiency Computed
A	0.811	0.813	0.813	0.81	0.812	0.809	0.811	0.808
B	0.818	0.821	0.816	0.818	0.817	0.816	0.818	0.82
C	0.819	0.814	0.814	0.813	0.812	0.811	0.814	0.809

Table 3 shows the close matches between the approximate efficiencies computed by the proposed approach and the means of the respective replication averages.

CONCLUSIONS

Analytical solutions for closed lines with stages failing at random pose computational difficulties especially for real sized cases. An approximation to aggregate consecutive stages as open linear segments and to represent their impact as a single failure mode is proposed. The discrete time stochastic model is in the form of a Markov chain and our idea is based on the relations borrowed from a well known decomposition scheme.

A series of numerical tests with data akin to a real automotive manufacturer's body line is reported. Extensive simulation results and our approximation demonstrate the acceptable accuracy achievable from the proposed aggregation.

There are areas that can be improved further. Namely the approach in aggregating different repair rates of the collapsed stages and the method of revising the total number of carriages in the closed line to account for the aggregation are candidates for further study. We think aggregation may also be a viable approach in analyzing closed lines with merging branches.

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