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VIBRATION OF VISCO-ELASTIC ORTHOTROPIC PARALLELOGRAM PLATE WITH PARABOLICALLY THICKNESS VARIATION

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ABSTRACT: *A simple model presented here is to study the effect of parabolically thickness variation on vibration of visco-elastic orthotropic parallelogram plate having clamped boundary condition on all the four edges. Using the separation of variables method, the governing differential equation has been solved for vibration of visco- elastic orthotropic parallelogram plate. An approximate but quite convenient frequency equation is derived by using Rayleigh-Ritz technique with a two-term deflection function. Time period and deflection function at different point for the first two modes of vibration are calculated for various values of taper constants, aspect ratio and skew angle.*
KEYWORDS: Vibration, visco-elastic, orthotropic, parallelogram plate, parabolically thickness

INTRODUCTION

Visco-elastic behaviour may be defined as material response that exhibits characteristic of both a viscous fluid and an elastic solid. A visco-elastic material combines these two properties it returns to its original shape after being stressed, but does it slowly enough to oppose the next cycle of vibration. The degree to which a material behaves either viscously or elastically depends mainly on temperature and rate of loading (frequency). Many polymeric materials (plastic, rubbers, acrylics, silicones, vinyls, adhesive, etc.) having long chain molecules exhibit visco-elastic behaviour. The dynamic properties (shear modulus, extensional modulus, etc.) of linear visco-elastic materials can be represented by the complex modulus approach.

With the increasing interest in the aerospace industry for accurate design, analysis methods are becoming more critical in using advance composites for primary load bearing structures. Most designs and analysis techniques in the cases where the load is carried by the fibre, the assumption is satisfactory. However, transverse properties to the fibre and shear properties are matrix controlled and exhibit strong visco-elastic behaviour. In many exceptional cases where the time dependent behaviour of the transverse and shear stiffness may exhibit significantly affect on the material response must be examined. An extensive literature on the subject of vibration of plate dealing with various geometries and boundary conditions are available.

In the course of time, engineers have become increasingly conscious about the importance of the an-elastic behaviour of many materials and mathematical formulations have been attempted and applied for practical problems. Outstanding among them are the theories of ideally plastic and of visco-elastic materials. With the increasing use of plastics and new materials in the construction of equipment and structures, the development of the application of visco-elasticity is needed to permit rational design.

Oscillatory behavior of bodies have increasing importance in the field of engineering because most of the machines and engineering structures experience vibration and their design generally requires consideration for their dynamic behavior. The use of constructions of variable thickness serves two-fold requirements of safety and economy. Plates of variable thickness are often encountered in several engineering application and their use in telephone industry, nuclear reactor technology, naval structures, earth-quake resistant structures, aeronautical field and machine designs are very common.

Bambill, Rossit, Laura and Rossi [1] have analyzed transverse vibration of an orthotropic rectangular plate of linearly varying thickness and with a free edge. Dhotarad and Ganesan [2] have considered vibration analysis of a rectangular plate subjected to a thermal gradient. Amabili and Garziera [3] have considered transverse vibrations of circular, annular plates with several combinations of boundary conditions. Transverse vibration of skew plates with variable thickness has been discussed by

Singh and Saxena [4]. Ceribasi and Altay [5] introduced the free vibration analysis of super elliptical plates with constant and variable thickness by

Ritz method. Gupta, Ansari, and Sharma [6] have analyzed vibration analysis of non-homogenous circular plate of non linear thickness variation by differential quadrature method. Jain and Soni [7] discussed the free vibrations of rectangular plates of parabolically varying thickness. Singh and Saxena [8] discussed the transverse vibration of rectangular plate with bi-directional thickness. Free vibrations of non-homogeneous circular plate of variable thickness resting on elastic foundation are discussed by Tomar, Gupta and Kumar [9]. Yang [10] has considered the vibration of a circular plate with varying thickness. Gupta, Ansari and Sharma [11] discussed the vibration of non-homogeneous circular mindlin plates with variable thickness.

A survey of literature on vibration problems of skew plates shows that the vibration of skew plates has received rather less attention than that given to the other type i.e. rectangular, circular and elliptic plates. Recently, Gupta, Khanna and Gupta [12] study the vibration of clamped visco-elastic rectangular plate having bi-direction exponentially thickness variations'

Sufficient work [13,14] is available on the vibration of a rectangular plate of thickness variable in one direction, but none of them done on parallelogram plate. A simple model presented here is to study the effect of parabolic thickness variation on vibration of visco-elastic orthotropic parallelogram plate having clamped boundary conditions on all the four edges . The hypothesis of small deflection and linear, orthotropic visco-elastic properties are made. Using the separation of variables method, the governing differential equation has been solved for vibration of visco - elastic orthotropic parallelogram plate. An approximate but quite convenient frequency equation is derived by using Rayleigh-Ritz technique with a two-term deflection function. It is assumed that the visco-elastic of the plate is of the "Kelvin Type". Time period and deflection function at different point for the first two modes of vibration are calculated for various values of taper constant, aspect ratio, and skew angle.

PARALLELOGRAM PLATE AND ANALYSIS

The parallelogram plate R be defined by the three number a, b and ϑ as shown figure 1. The special case of rectangular plate follows by putting $\vartheta=0^o$, here

$$\xi=x-y\tan\vartheta, \quad \eta=y\sec\vartheta \qquad (1)$$

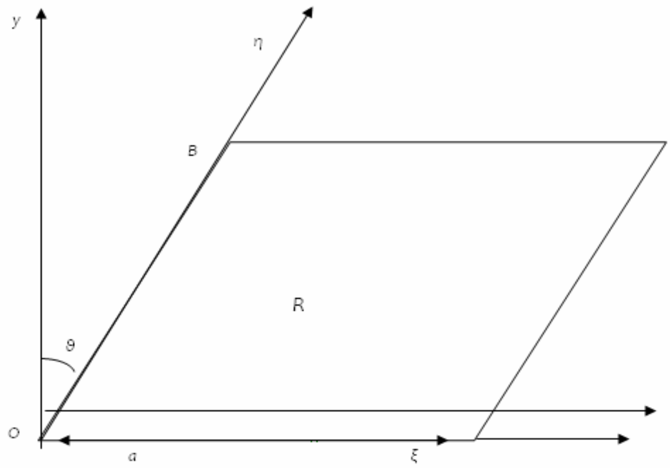

Figure 1. The parallelogram plate R

Bending and twisting moments of visco-elastic orthotropic parallelogram plate of variable thickness are related to displacement by

$$M_\xi= - \check{D} [D_\xi w,_{\xi\xi} + D_1\sec^2\vartheta\{ \sin^2\vartheta\, w,_{\xi\xi}-2\sin\vartheta\, w,_{\xi\eta}+ w,_{\eta\eta}\}],$$

$$M_\eta= - \check{D}[D_1 w,_{\xi\xi} + D_\eta\sec^2\vartheta\{ \sin^2\vartheta\, w,_{\xi\xi}-2\sin\vartheta\, w,_{\xi\eta}+ w,_{\eta\eta}\}],$$

and
$$M_{\xi\eta}=-2\check{D}D_{\xi\eta}\sec\vartheta[w,_{\xi\eta} -\sin\vartheta\, w,_{\xi\xi}]. \qquad (2)$$

where, D_ξ and D_η are flexural rigidities of plate, $D_{\xi\eta}$ is torsional rigidity of plate, $\check{D}$ is unique rhelogical operator.

The governing differential equation of transverse motion of visco-elastic orthotropic parallelogram plate of variable thickness, ξ- and η- co-ordinates, is [15]

$$\check{D}[(D_\xi + D_\eta \tan^4\vartheta + 2H \tan^2\vartheta)w,_{\xi\xi\xi\xi} - 4(\sec\vartheta) (D_\eta \tan^3\vartheta + H \tan\vartheta) w,_{\xi\xi\xi\eta} + (\sec\vartheta)(6D_\eta \tan^2\vartheta + 2H)w,_{\xi\xi\eta\eta} - 4 (D_\eta \tan\vartheta. \mathrm{Sec}^3\vartheta)w,_{\xi\eta\eta\eta} + (D_\eta \sec^4\vartheta)w,_{\eta\eta\eta\eta} + 2 (H,_\xi \tan^2\vartheta - H,_\eta \tan\vartheta + D_{\xi,\xi}-D_{\eta,\eta} \tan^3\vartheta)w,_{\xi\xi\xi} - 2(\sec\vartheta)(2H,_\xi \tan\vartheta - H,_\eta \tan\vartheta - 3D_{\eta,\eta} \tan^2\vartheta)w,_{\xi\xi\eta} + 2\sec^2\vartheta(H,_\xi - 3D_{\eta,\eta} \tan\vartheta)w,_{\xi\eta\eta} + 2(D_{\eta,\eta} \sec^3\vartheta) w,_{\eta\eta\eta} + (D_{\xi,\xi\xi} + D_{\eta,\xi\xi} \tan^4\vartheta - 2D_{\eta,\xi\eta} \sec\vartheta.\tan^3\vartheta + D_{\eta,\eta\eta} \sec^2\vartheta \tan^2\vartheta + 3D_{1,\xi\xi} \tan^2\vartheta - 2D_{1,\xi\eta} \sec\vartheta \tan\vartheta + D_{1,\eta\eta} \sec\vartheta + 4D_{\xi\eta,\xi\xi} \tan^2\vartheta - 4D_{\xi\eta,\xi\eta} \sec\vartheta \tan\vartheta)w,_{\xi\xi} - 2D_{\eta,\xi\xi} \sec\vartheta \tan^3\vartheta + 4D_{\eta,\xi\eta} \sec^2\vartheta \tan^2\vartheta - 2D_{\eta,\eta\eta} \sec^3\vartheta \tan\vartheta - 2D_{1,\xi\xi}. \mathrm{Sec}\vartheta \tan\vartheta - 4D_{\xi\eta,\xi\xi} \sec\vartheta\tan^2\vartheta + 4D_{\xi\eta\xi\eta}\sec^2\vartheta)w,_{\xi\eta} + (D_{\eta,\xi\xi}\sec^2\vartheta\tan^2\vartheta - 2D_{\eta,\xi\eta}\sec^3\vartheta\tan\vartheta + D_{\eta,\eta\eta}\sec^4\vartheta + D_{1,\xi\xi} \sec^2\vartheta)w,_{\eta\eta}] + \rho h w,_{tt}=0. \qquad (3)$$

A comma followed by a suffix denotes partial differential with respect to that variable. The solution of eq.(3) can be taken in the form of products of two functions as for free transverse vibration of the visco-elastic orthotropic parallelogram plate, w(ξ, η, t) can be expressed as

$$w(\xi,\eta,t) = W(\xi,\eta)T(t) \qquad (4)$$

where T (t) is the time function and W is the maximum displacement with respect to time t.

Substituting eq.(4) into eq.(3),one obtains

$$\check{D}[(D_\xi + D_\eta \tan^4\vartheta + 2H \tan^2\vartheta)W_{,\xi\xi\xi\xi} - 4(\sec\vartheta)(D_\eta \tan^3\vartheta + H\tan\vartheta)W_{,\xi\xi\xi\eta} + (\sec\vartheta)(6D_\eta \tan^2\vartheta + 2H)W_{,\xi\xi\eta\eta} - 4(D_\eta \tan\vartheta. \sec^3\vartheta)W_{,\xi\eta\eta\eta} + (D_\eta \sec^4\vartheta)W_{,\eta\eta\eta\eta} + 2(H_{,\xi}\tan^2\vartheta - H_{,\eta}\tan\vartheta + D_{\xi,\xi}\text{-}D_{\eta,\eta}\tan^3\vartheta)W_{,\xi\xi\xi} - 2(\sec\vartheta)(2H_{,\xi}\tan\vartheta - H_{,\eta}\tan\vartheta - 3D_{\eta,\eta}\tan^2\vartheta)W_{,\xi\xi\eta} + 2\sec^2\vartheta(H_{,\xi} - 3D_{\eta,\eta}\tan\vartheta)W_{,\xi\eta\eta} + 2(D_{\eta,\eta}\sec^3\vartheta)W_{,\eta\eta\eta} + (D_{\xi,\xi\xi} + D_{\eta,\xi\xi}\tan^4\vartheta - 2D_{\eta,\xi\eta}\sec\vartheta.\tan^3\vartheta + D_{\eta,\eta\eta}\sec^2\vartheta\tan^2\vartheta + 3D_{1,\xi\xi}\tan^2\vartheta - 2D_{1,\xi\eta}\sec\vartheta\tan\vartheta + D_{1,\eta\eta}\sec\vartheta + 4D_{\xi\eta,\xi\xi}\tan^2\vartheta - 4D_{\xi\eta,\xi\eta}\sec\vartheta\tan\vartheta)W_{,\xi\xi} - 2D_{\eta,\xi\xi}\sec\vartheta\tan^3\vartheta + 4D_{\eta,\xi\eta}\sec^2\vartheta\tan^2\vartheta - 2D_{\eta,\eta\eta}\sec^3\vartheta\tan\vartheta - 2D_{1,\xi\xi}.\sec\vartheta\tan\vartheta - 4D_{\xi\eta,\xi\xi}\sec\vartheta\tan^2\vartheta + 4D_{\xi\eta\xi\eta}\sec^2\vartheta)W_{,\xi\eta} + (D_{\eta,\xi\xi}\sec^2\vartheta\tan^2\vartheta\ {}_{-\eta,\xi\eta}\sec^3\vartheta\tan\vartheta + D_{\eta,\eta\eta}\sec^4\vartheta + D_{1,\xi\xi}\sec^2\vartheta)W_{,\eta\eta}]/\rho hW = -T_{,tt}/\check{D}T \quad (5)$$

The preceding equation is satisfied if both of its sides are equal to a constant. Denoting this constant by p^2, we get two equations;

$$\check{D}[(D_\xi + D_\eta \tan^4\vartheta + 2H \tan^2\vartheta)W_{,\xi\xi\xi\xi} - 4(\sec\vartheta)(D_\eta \tan^3\vartheta + H\tan\vartheta)W_{,\xi\xi\xi\eta} + (\sec\vartheta)(6D_\eta \tan^2\vartheta + 2H)W_{,\xi\xi\eta\eta} - 4(D_\eta \tan\vartheta. \sec^3\vartheta)W_{,\xi\eta\eta\eta} + (D_\eta \sec^4\vartheta)W_{,\eta\eta\eta\eta} + 2(H_{,\xi}\tan^2\vartheta - H_{,\eta}\tan\vartheta + D_{\xi,\xi}\text{-}D_{\eta,\eta}\tan^3\vartheta)W_{,\xi\xi\xi} - 2(\sec\vartheta)(2H_{,\xi}\tan\vartheta - H_{,\eta}\tan\vartheta - 3D_{\eta,\eta}\tan^2\vartheta)W_{,\xi\xi\eta} + 2\sec^2\vartheta(H_{,\xi} - 3D_{\eta,\eta}\tan\vartheta)W_{,\xi\eta\eta} + 2(D_{\eta,\eta}\sec^3\vartheta)W_{,\eta\eta\eta} + (D_{\xi,\xi\xi} + D_{\eta,\xi\xi}\tan^4\vartheta - 2D_{\eta,\xi\eta}\sec\vartheta.\tan^3\vartheta + D_{\eta,\eta\eta}\sec^2\vartheta\tan^2\vartheta + 3D_{1,\xi\xi}\tan^2\vartheta - 2D_{1,\xi\eta}\sec\vartheta\tan\vartheta + D_{1,\eta\eta}\sec\vartheta + 4D_{\xi\eta,\xi\xi}\tan^2\vartheta - 4D_{\xi\eta,\xi\eta}\sec\vartheta\tan\vartheta)W_{,\xi\xi} - 2D_{\eta,\xi\xi}\sec\vartheta\tan^3\vartheta + 4D_{\eta,\xi\eta}\sec^2\vartheta\tan^2\vartheta - 2D_{\eta,\eta\eta}\sec^3\vartheta\tan\vartheta - 2D_{1,\xi\xi}.\sec\vartheta\tan\vartheta - 4D_{\xi\eta,\xi\xi}\sec\vartheta\tan^2\vartheta + 4D_{\xi\eta\xi\eta}\sec^2\vartheta)W_{,\xi\eta} + (D_{\eta,\xi\xi}\sec^2\vartheta\tan^2\vartheta - 2D_{\eta,\xi\eta}\sec^3\vartheta\tan\vartheta + D_{\eta,\eta\eta}\sec^4\vartheta + D_{1,\xi\xi}\sec^2\vartheta)W_{,\eta\eta}] - \rho p^2 hW = 0 \quad (6)$$

and

$$T_{,tt} + p^2\check{D}T = 0 \quad (7)$$

Eqs.(6) and (7) are the differential equation of motion for visco-elastic orthotropic parallelogram plate of variable thickness and time function for visco-elastic orthotropic parallelogram plate of free vibration, respectively .

EQUATION OF MOTION

The expressions for the strain energy, V_{max}, and kinetic energy, T_{max}, in the visco-elastic orthotropic parallelogram plate when executing transverse vibration of mode shape W (ξ, η) are [16]

$$V_{max} = (1/2)\int_0^a\int_0^b [D_\xi(W_{,\xi\xi})^2 + D_\eta(W_{,\xi\xi}\tan^2\vartheta - 2W_{,\xi\eta}\tan\vartheta\sec\vartheta + W_{,\eta\eta}\sec^2\vartheta)^2 + 2D_1W_{,\xi\xi}(W_{,\xi\xi}\tan^2\vartheta + 2W_{,\xi\eta}\tan\vartheta\sec\vartheta + W_{,\eta\eta}\sec^2\vartheta) + 4D_{\xi\eta}(-W_{,\xi\xi}\tan\vartheta + W_{,\xi\eta}\sec\vartheta)^2]\cos\vartheta\, d\eta\, d\xi \quad (8)$$

and

$$T_{max} = (1/2)\rho p^2 \int_0^a\int_0^b (hW^2\cos\vartheta)\, d\eta\, d\xi \quad (9)$$

Assuming thickness variation of visco-elastic orthotropic parallelogram plate parabolically in ξ-direction only, as

$$h = h_o\{1 + \beta(\xi/a)^2\} \quad (10)$$

where β is the taper constant in ξ-direction and $h_o = h|_{\xi=0}$.

The flexural rigidities (D_ξ and D_η) and torsional rigidity ($D_{\xi\eta}$) of the plate can now be written as

$$D_\xi = E_1h_o^3(1+\beta(\xi/a)^2)^3/12(1-\nu_\xi\nu_\eta),$$
$$D_\eta = E_2h_o^3(1+\beta(\xi/a)^2)^3/12(1-\nu_\xi\nu_\eta),$$
$$D_{\xi\eta} = G h_o^3(1+\beta(\xi/a)^2)^3/12$$
$$\text{and } D_1 = \nu_\xi D_\eta = \nu_\eta D_\xi \quad (11)$$

SOLUTION AND FREQUENCY EQUATION

Rayleigh-Ritz technique requires that maximum strain energy be equal to the maximum kinetic energy. So it is necessary for the problem consideration that

$$\delta(V_{max} - T_{max}) = 0 \quad (12)$$

for arbitrary variations of W are satisfying relevant geometrical boundary conditions.

For a visco-elastic orthotropic parallelogram plate clamped (c) along all the four edges, the boundary conditions are

$$W = W_{,\xi} = 0 \text{ at } \xi = 0, a \text{ and } W = W_{,\eta} = 0 \text{ at } \eta = 0, b \quad (13)$$

and the corresponding two-term deflection function is taken as [241]

$$W = [(\xi/a)(\eta/b)(1-\xi/a)(1-\eta/b)]^2[A_1 + A_2(\xi/a)(\eta/b)(1-\xi/a)(1-\eta/b)] \quad (14)$$

which is satisfied Eq.(12),

Now assuming the non-dimensional variable as

$$X = \xi/a,\ Y = \eta/a,\quad \hbar = h/a,\quad \hat{W} = W/a \quad (15)$$

$$E^*_1 = E_1/(1-\nu_\xi\nu_\eta),\ E^*_2 = E_2/(1-\nu_\xi\nu_\eta),\ E^* = \nu_\xi E^*_2 = \nu_\eta E^*_1 \quad (16)$$

and component of E^*_1, E^*_2, E^* and G are E^*_1, $E^*_2 sec\vartheta$, $E^* sec\vartheta$ and $G\, sec\vartheta$ respectively ξ- and η- direction. Using eqs.(10), (11), (15) and (16) in eqs.(8) and (9), then substituting the values of T_{max} & V_{max} from eqs.(8) and (9) in eq(12), one obtains

$$(V_1-\lambda^2 T_1)=0 \qquad (17)$$

where,

$$V_1=\int_0^1\int_0^{b/a} (1+\beta X^2)^3[\{\cos^4\vartheta+(E^*_2/E^*_1)\sin^4\vartheta+2(E^*/E^*_1)\sin^2\vartheta\cos^2\vartheta+4(G_0/E^*_1)\sin^2\vartheta\cos^2\vartheta\}\hat{W}^2_{,XX}+(E^*_2/E^*_1)\hat{W}^2_{,YY}$$
$$+4\{(E^*_2/E^*_1)\sin^2\vartheta+(G_0/E^*_1)\cos^2\vartheta\}\hat{W}^2_{,XY}+2\{(E^*_2/E^*_1)\sin^2\vartheta+(E^*/E^*_1)\cos^2\vartheta\}\times\hat{W}_{,XX}\hat{W}_{,YY}-$$
$$\{(E^*_2/E^*_1)\sin^3\vartheta+2(E^*/E^*_1)\sin\vartheta\cos^2\vartheta+2(G_0/E^*_1)\sin\vartheta\cos^2\vartheta\}\}\hat{W}_{,XX}\hat{W}_{,XY}-4\{(E^*_2/E^*_1)\sin\vartheta\hat{W}_{,YY}\hat{W}_{,XY}]dYdX \qquad (18)$$

and

$$T_1=\int_0^1\int_0^{b/a} [(1+\beta X^2)\,\hat{W}2]\,dYdX \qquad (19)$$

$$\text{where, } p^2=(E^*_1 h_0^2/12a^2\rho\cos^5\vartheta)\,\lambda^2 \qquad (20)$$

Here limit of X and Y is 0 to 1 and 0 to b/a respectively.

But equation (17) involves the unknown A_1 and A_2 arising due to the substitution of W (ξ, η) from eq (14). These two constants are to be determined from eq (17), as follows:

$$\partial\,(V_1-\lambda^2T_1)/\partial A_n=0, \quad n=1, 2 \qquad (21)$$

Equation (21) simplifies to the form

$$b_{n1}A_1+b_{n2}A_2=0,\ n=1, 2 \qquad (22)$$

where b_{n1}, b_{n2} (n = 1, 2) involve parametric constants and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (22) must be zero. So gets the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}=0 \qquad (23)$$

Here, $b_{11}=2(F_1-\lambda^2B_1)$, $b_{12}=b_{21}=(F_2-\lambda^2B_2)$, and $b_{22}=2(F3-\lambda^2B_3)$

where

$$F_1=(a/b)^5\{\cos^4\vartheta+(E^*_2/E^*_1)\sin^4\vartheta+2(E^*/E^*_1)\sin^2\vartheta\cos^2\vartheta+4(G_0/E^*_1)\sin^2\vartheta\cos^2\vartheta\}(2/1575)+$$
$$(16/11025)\beta+(4/3675)\beta^2+(113/363825)\beta^3\}+(a/b)(E^*_2/E^*_1)\{(2/1925)\beta+(4/10725)\beta^2+(2/1575)+(4/75075)\beta^3\}+$$
$$4(a/b)^3\{(E^*_2/E^*_1)\sin^2\vartheta+(G_0/E^*_1)\cos^2\vartheta\}\{(4/11025)+(4/11025)\beta+(26/121275)\beta^2+(32/675675)\beta^3\}+$$
$$2(a/b)^3\{(E^*_2/E^*_1)\sin^2\vartheta+(E^*/E^*_1)\cos^2\vartheta\}\{(2/675675)\beta^3+(8/121275)\beta^2+(1/3675)\beta+(4/11025)\}],$$

$$F_2=[(a/b)^5\{\cos^4\vartheta+(E^*_2/E^*_1)\sin^4\vartheta+2(E^*/E^*_1)\sin^2\vartheta\cos^2\vartheta+4(G_0/E^*_1)\sin^2\vartheta\cos^2\vartheta\}\{(1/8085)+(1/8085)\beta+$$
$$(37/1981980)\beta^3+(4/53361)\beta^2\}+(a/b)(E^*_2/E^*_1)\{(1/216580)\beta^3+(1/10010)\beta+(6/175175)\beta^2+(1/8085)\}+$$
$$4(a/b)^3\{(E^*_2/E^*_1)\sin^2\vartheta+(G_0/E^*_1)\cos^2\vartheta\}\{(1/23100)\beta+(1/42900)\beta^2+(1/22050)+(1/210210)\beta^3\}+2(a/b)^3\{(E^*_2/E^*_1)\sin^2\vartheta$$
$$+(E^*/E^*_1)\cos^2\vartheta\}\{(/150150)\beta^2+(4/121275)\beta+(1/22050)\}],$$

$$F_3=[(a/b)^5\{\cos^4\vartheta+(E^*_2/E^*_1)\sin^4\vartheta+2(E^*/E^*_1)\sin^2\vartheta\cos^2\vartheta+4(G_0/E^*_1)\sin^2\vartheta\cos^2\vartheta\}\{(1/220220)\beta+5/2004002)\beta^2+$$
$$(17/30060030)\beta^3+(1/210210)\}+(a/b)(E^*_2/E^*_1)\{(3/2382380)\beta^2+(2/525525)\beta+(1/210210)+(1/6172530)\beta^3\}+$$
$$4(a/b)^3\{(E^*_2/E^*_1)\sin^2\vartheta+(G_0/E^*_1)\cos^2\vartheta\}\{(1/592900)+(3/1926925)\beta+(3/3853850)\beta^2+(3/20158600)\beta^3\}+$$
$$2(a/b)^3\{(E^*_2/E^*_1)\sin^2\vartheta+(E^*/E^*_1)\cos^2\vartheta\}\{(1/3853850)\beta^2+(3./524123600)\beta^3+(1/592900)+(19/15415400)\beta\}],$$

$$B_1=(a/b)^5\{(1/396900)+(1/1455300)\beta\},B_2=(a/b)^5\{(1/3841992)+(1/14270256)b\},$$
$$B_3=(a/b)^5((1/541080540)b+(1/144288144)\},$$

Form eq. (23), one can obtains a quadratic equation in p^2 from which the two values of p^2 can found. After determining A_1 & A_2 from eq. (22), one can obtain deflection function W. Choosing A_1=1, one obtains A_2= $(-b_{11}/b_{12})$ and then W comes out as

$$W=[XY(a/b)(1-X)(1-Ya/b)]^2[1+(-b_{11}/b_{12})XY(a/b)(1-X)(1-Ya/b)]. \qquad (24)$$

Differential equation of time function and it is solution. Time function of free vibration of visco-elastic orthotropic parallelogram plate is defined by the general ordinary differential equation (7). Their form depends on the visco-elastic operator Ď. For Kelvin, s model, one has

$$\check{D}=\{1+(\check{n}/G)\,(d/dt)\} \qquad (25)$$

where, ň is visco-elastic constant and G is shear modulus.

The governing differential equation of time function of visco-elastic orthotropic parallelogram plate, by using eq. (25) in eq. (7), is given as

$$T_{,tt} + p^2(ň/G)T_{,t} + p^2T = 0 \quad (26)$$

Solution of eq.(26) comes out as

$$T(t) = e^{kt}(C_1 Cos k_1 t + C_2 Sin k_1 t) \quad (27)$$

where,

$$k = -p^2\, ň/2G \quad (28)$$

and $$k_1 = p\{1 - (pň/2G)^2\}^{1/2} \quad (29)$$

Let us take initial conditions as

$$T=1 \text{ and } dT/dt=0 \text{ at } t=0 \quad (30)$$

Using initial conditions from eq. (30) in solution of diff.eq. (27), one obtains

$$T(t) = e^{rt}[\cos(st) + (-r/s)\sin(st)] \quad (31)$$

where $r = -(p^2 ň / 2G)$ and $s = p(G^2 - p^2 ň^2)/2G$.

Thus, deflection w may be expressed, by using Eq.(24) and (31) in eq.(4), as

$$w = [XY(a/b)(1-X)(1-Ya/b)]^2[1 - b_{11}/b_{12}) XY(a/b)(1-X)(1-Ya/b)][e^{rt}\{\cos(st) - r/s\sin(st)] \quad (32)$$

Time period of vibration of the plate is given by

$$K = 2\pi / p, \quad (33)$$

where p is frequency given by eq.(23).

RESULTS AND DISCUSSION

Time period and deflection are computed for viscoelastic orthotropic parallelogram plate whose thickness varies parabolically for different value of skew angle(ϑ), taper constant(β), and aspect ratio(a/b) at different points for first two mode of vibration. The orthotropic material parameters have been taken as [16] $E^*_2/E^*_1=0.01$, $E^*/E^*_1=0.3$, $G/E^*_1=0.0333$, $ň/G=0.000069$, $E^*_1/\rho=3.0\times10^5$ and $h_0=0.01$ meter.

All the results are presented in the tables (1-19).

The value of time period (K) for β=0.6, ϑ=45° have been found to decrease 35.88787359% for first mode and 34.73847213% for second mode in comparison to rectangular plate at fixed aspect ratio (a/b=1.5).

The value of time period (K) for β=0.6, ϑ=45° have been found to decrease 20.54410444% for first mode and 21.2286094% for second mode in comparison to parallelogram plate of uniform thickness at fixed aspect ratio (a/b=1.5).

TABLE 1. Time Period K (in Seconds) for different taper constant (β) and a constant aspect ratio (a/b = 1.5) for all X, Y.

β	ϑ=0		ϑ=45		ϑ=75	
	1st mode	2nd mode	1st mode	2nd mode	1st mode	2nd mode
0.0	0.143648	0.037357	0.091306	0.024076	0.014511	0.003831
0.2	0.132780	0.034464	0.084639	0.022299	0.013535	0.003581
0.4	0.122520	0.031664	0.078329	0.020576	0.012605	0.003339
0.6	0.113158	0.029060	0.072548	0.018965	0.011741	0.003111
0.8	0.104763	0.026695	0.067340	0.017493	0.010952	0.002899

TABLE 2. Time Period K (in Seconds) for different angle (ϑ) and a constant aspect ratio (a/b=1.5)

ϑ	β=0.0		β=0.2		β=0.6	
	1st mode	2nd mode	1st mode	2nd mode	1st mode	2nd mode
0	0.143648	0.037357	0.132780	0.034464	0.113158	0.029060
15	0.138373	0.036062	0.127948	0.033285	0.109118	0.028094
30	0.121441	0.031827	0.112404	0.029416	0.096057	0.024902
45	0.091306	0.024076	0.084639	0.022299	0.072548	0.018965
60	0.051489	0.013611	0.047825	0.012643	0.041150	0.010824
75	0.014511	0.003831	0.013535	0.003581	0.011741	0.003111

TABLE 3. Time Period K (in seconds) for different aspect ratio (a/b) for all X,Y

a/b	β=0.0, ϑ=0		β=0.0, ϑ=45		β=0.6, ϑ=0		β=0.6, ϑ=45	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.5	0.172915	0.042600	0.119167	0.029503	0.133324	0.032325	0.092018	0.022424
1.0	0.160377	0.040555	0.106813	0.027355	0.124812	0.031079	0.083521	0.021097
1.5	0.143648	0.037357	0.091306	0.024076	0.113158	0.029060	0.072548	0.018965
2.0	0.125747	0.033392	0.075799	0.020293	0.100377	0.026435	0.061261	0.016345
2.5	0.108496	0.029151	0.062002	0.016658	0.087768	0.02348	0.050893	0.013674

Table(1) show the results of time period(K) for different values of taper constant(β) and fixed aspect ratio (a/b=1.5) for three values of skew angle (θ) i.e. θ=0°, θ=45° and θ=75° for first two mode of vibration. It can be seen that the time period (K) decrease when taper constant (β) increase for two mode of vibration at θ=0°, θ=45° and θ=75°.

Table(2) show the results of time period(K) for different values of skew angle(θ) and fixed aspect ratio (a/b=1.5) for three values of taper constant (β) i.e. β=0.0 , β=0.2 and β=0.6 for first two mode of vibration. It can be seen that the time period (K) decrease when skew angle (θ) increase for two mode of vibration at β=0.0, β=0.2 and β=0.6.

Table (3) shows the results of time period (K) for different values of aspect ratio (a/b) and fixed taper constant (β=0.0 and β=0.6) for two values of skew angle (θ) i.e. θ=0° and θ=45° for first two mode of vibration. It can be seen that the time period (K) decrease when aspect ratio (a/b) increase for two mode of vibration.

The value of deflection (w) for β=0.6 and θ=45° have been found to increase 14.19166057% for first mode and 1.06666667% for second mode in comparison to parallelogram plate of uniform thickness for initial time 0.K at X=0.2, Y=0.4 and a/b=1.5.

The value of deflection (w) for β=0.6 and θ=45° have been found to decrease 4.75899939% for first mode and 0.524934383% for second mode in comparison to rectangular plate for initial time 0.K at X=0.2, Y=0.4 and a/b=1.5.

The value of deflection (w) for β=0.6 and θ=45° have been found to increase 11.91193538% for first mode and decrease 6.028368794% for second mode in comparison to parallelogram plate of uniform thickness for time 5.K at X=0.2, Y=0.4 and a/b=1.5.

The value of deflection (w) for β=0.6 and θ=45° have been found to decrease 7.906675308% for first mode and 11.96013289% for second mode in comparison to rectangular plate for time 5.K at X=0.2, Y=0.4 and a/b=1.5.

Tables (4-11) show the results of deflection (w) for different values of X , Y and fixed taper constant (β=0.0 and β=0.6) , and aspect ratio (a/b=1.5) for two values of skew angle (θ) i.e. θ=0° and θ=45° for first two mode of vibration with time 0.K and 5.K. It can be seen that deflection (w) start from zero to increase then decrease to zero for first two mode of vibration (except second mode at Y=0.2, Y=0.4) and second mode of vibration deflection (w) at (Y=0.2, Y=0.4) start zero to increase then decrease then increase then decrease and finally become to zero for different value of X.

TABLE 4. Deflection w for different X, Y and β=0.0,θ=0 at Initial time 0.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.2	0.001087	0.000393	0.001413	0.000376	0.000204	0.000149	0.001537	0.002573
0.4	0.0024	0.000057	0.003108	-0.000389	0.000456	0.000271	0.003527	0.007024
0.6	0.0024	0.000057	0.003108	-0.000389	0.000456	0.000271	0.003527	0.007024
0.8	0.001087	0.000393	0.001413	0.000376	0.000204	0.000149	0.001537	0.002573
1.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TABLE 5. Deflection w for different X, Y and β=0.6,θ=0 and a/b=1.5 at Initial time 0.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.2	0.001239	0.000396	0.001639	0.000381	0.000216	0.000150	0.001310	0.002568
0.4	0.002913	0.000067	0.003874	-0.000374	0.000496	0.000272	0.002761	0.007009
0.6	0.002913	0.000067	0.003874	-0.000374	0.000496	0.000272	0.002761	0.007009
0.8	0.001239	0.000396	0.001639	0.000381	0.000216	0.000150	0.001310	0.002568
1.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TABLE 6. Deflection w for different X, Y and β=0.0,θ=45 and a/b=1.5 at Initial time 0.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.001057	0.000392	0.001367	0.000375	0.000202	0.000149	0.001582	0.002574
0.4	0.002298	0.000053	0.002956	-0.000394	0.000447	0.000271	0.003680	0.007030
0.6	0.002298	0.000053	0.002956	-0.000394	0.000447	0.000271	0.003680	0.007030
0.8	0.001057	0.000392	0.001367	0.000375	0.000202	0.000149	0.001582	0.002574
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 7. Deflection w for different X, Y and β=0.6,ϑ=45 and a/b=1.5 at Initial time 0.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.001187	0.000395	0.001561	0.000379	0.000212	0.000150	0.001388	0.002570
0.4	0.002736	0.000063	0.003610	-0.00038	0.000482	0.000272	0.003025	0.007016
0.6	0.002736	0.000063	0.003610	-0.00038	0.000482	0.000272	0.003025	0.007016
0.8	0.001187	0.000395	0.001561	0.000379	0.000212	0.000150	0.001388	0.002570
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 8. Deflection w for different X, Y and β=0.0,ϑ=0 and a/b=1.5 at time 5.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.001037	0.000328	0.001347	0.000314	0.000195	0.000125	0.001465	0.002144
0.4	0.002289	0.000047	0.002964	-0.000324	0.000434	0.000226	0.003363	0.005853
0.6	0.002289	0.000047	0.002964	-0.000324	0.000434	0.000226	0.003363	0.005853
0.8	0.001037	0.000328	0.001347	0.000314	0.000195	0.000125	0.001465	0.002144
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 9. Deflection w for different X, Y and β=0.6,ϑ=0 and a/b=1.5 at l time 5.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.001167	0.000313	0.001543	0.000301	0.000203	0.000118	0.001233	0.002031
0.4	0.002742	0.000053	0.003647	-0.000295	0.000467	0.000215	0.002600	0.005544
0.6	0.002742	0.000053	0.003647	-0.000295	0.000467	0.000215	0.002600	0.005544
0.8	0.001167	0.000313	0.001543	0.000301	0.000203	0.000118	0.001233	0.002031
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 10. Deflection w for different X, Y and β=0.0,ϑ=45 and a/b=1.5 at time 5.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.000981	0.000295	0.001269	0.000282	0.000187	0.000113	0.001468	0.00194
0.4	0.002132	0.00004	0.002743	-0.000297	0.000415	0.000204	0.003415	0.005297
0.6	0.002132	0.00004	0.002743	-0.000297	0.000415	0.000204	0.003415	0.005297
0.8	0.000981	0.000295	0.001269	0.000282	0.000187	0.000113	0.001468	0.00194
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 11. Deflection w for different X, Y and β=0.6,ϑ=45 and a/b=1.5 at time 5.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.001081	0.000276	0.001421	0.000265	0.000193	0.000104	0.001263	0.001794
0.4	0.002491	0.000044	0.003286	-0.000265	0.000439	0.00019	0.002754	0.004898
0.6	0.002491	0.000044	0.003286	-0.000265	0.000439	0.00019	0.002754	0.004898
0.8	0.001081	0.000276	0.001421	0.000265	0.000193	0.000104	0.001263	0.001794
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

The value of deflection (w) for β=0.6 and $\vartheta=45^{o}$ have been found to increase 3.216374269% for first mode and 0.302114804% for second mode in comparison to parallelogram plate of uniform thickness for initial time 0.K at X=0.2, Y=0.4 and aspect ratio (a/b=0.5).

The value of deflection (w) for β=0.6 and $\vartheta=45^{o}$ have been found to decrease 0.703234881% for first mode and no effect for second mode in comparison to rectangular plate for initial time 0.K at X=0.2, Y=0.4 and aspect ratio (a/b=0.5).

The value of deflection (w) for β=0.6 and $\vartheta=45^{o}$ have been found to increase 1.238390093% for first mode and decrease 7.604562738% for second mode in comparison to parallelogram plate of uniform thickness for time 5.K at X=0.2, Y=0.4 and aspect ratio (a/b=0.5).

The value of deflection (w) for β=0.6 and $\vartheta=45^{o}$ have been found to decrease 2.967359050% for first mode and 8.988764045% for second mode in comparison to rectangular plate for time 5.K at X=0.2, Y=0.4 and aspect ratio (a/b=0.5).

Tables (12-19) show the results of deflection (w) for different values of X , Y and fixed taper constants (β=0.0 and β=0.6) , and aspect ratio (a/b=0.5) for two values of skew angle (θ) i.e. θ=0° and θ=45° for first two mode of vibration with time 0.K and 5.K. It can be seen that deflection (w) start from zero to increase then decrease to zero for first two mode of vibration (except second mode at Y=0.6 and 0.8) and second mode of vibration deflection (w) at (Y=0.6 and Y=0.8) start zero to increase then decrease then increase then decrease and finally become to zero for different value of X

TABLE 12. Deflection w for different X, Y and β=0.0,θ=0 and a/b=0.5 at Initial time 0.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.000213	0.00015	0.000688	0.000331	0.001203	0.000397	0.001585	0.000381
0.4	0.000486	0.000272	0.001585	0.000381	0.002791	0.000069	0.003692	-0.000371
0.6	0.000486	0.000272	0.001585	0.000381	0.002791	0.000069	0.003692	-0.000371
0.8	0.000213	0.00015	0.000688	0.000331	0.001203	0.000397	0.001585	0.000381
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 13. Deflection w for different X, Y and β=0.6,θ=0 and a/b=0.5 at Initial time 0.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.000217	0.000150	0.000711	0.000332	0.001255	0.000398	0.001663	0.000383
0.4	0.000500	0.000272	0.001663	0.000383	0.002966	0.000073	0.003953	-0.000365
0.6	0.000500	0.000272	0.001663	0.000383	0.002966	0.000073	0.003953	-0.000365
0.8	0.000217	0.000150	0.000711	0.000332	0.001255	0.000398	0.001663	0.000383
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 14. Deflection w for different X, Y and β=0.0,θ=45 and a/b=0.5 at Initial time 0.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.000212	0.000150	0.000684	0.000331	0.001193	0.000396	0.001570	0.000381
0.4	0.000484	0.000272	0.001570	0.000381	0.002755	0.000068	0.003639	-0.000373
0.6	0.000484	0.000272	0.001570	0.000381	0.002755	0.000068	0.003639	-0.000373
0.8	0.000212	0.000150	0.000684	0.000331	0.001193	0.000396	0.001570	0.000381
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 15. Deflection w for different X, Y and β=0.6,θ=45 and a/b=0.5 at Initial time 0.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.000216	0.00015	0.000706	0.000332	0.001243	0.000398	0.001645	0.000383
0.4	0.000497	0.000272	0.001645	0.000383	0.002927	0.000072	0.003895	-0.000367
0.6	0.000497	0.000272	0.001645	0.000383	0.002927	0.000072	0.003895	-0.000367
0.8	0.000216	0.00015	0.000706	0.000332	0.001243	0.000398	0.001645	0.000383
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 16. Deflection w for different X, Y and β=0.0,θ=0 and a/b=0.5 at time 5.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.000205	0.000128	0.000662	0.000282	0.001157	0.000338	0.001524	0.000325
0.4	0.000467	0.000232	0.001524	0.000325	0.002683	0.000058	0.003549	-0.000317
0.6	0.000467	0.000232	0.001524	0.000325	0.002683	0.000058	0.003549	-0.000317
0.8	0.000205	0.000128	0.000662	0.000282	0.001157	0.000338	0.001524	0.000325
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 17. Deflection w for different X, Y and β=0.6,θ=0 and a/b=0.5 at l time 5.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.000206	0.000121	0.000674	0.000267	0.00119	0.00032	0.001577	0.000309
0.4	0.000474	0.000219	0.001577	0.000309	0.002812	0.000059	0.003748	-0.000294
0.6	0.000474	0.000219	0.001577	0.000309	0.002812	0.000059	0.003748	-0.000294
0.8	0.000206	0.000121	0.000674	0.000267	0.00119	0.00032	0.001577	0.000309
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 18. Deflection w for different X, Y and β=0.0,θ=45 and a/b=0.5 at time 5.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.000201	0.000119	0.000646	0.000263	0.001126	0.000315	0.001482	0.000302
0.4	0.000457	0.000216	0.001482	0.000302	0.002602	0.000054	0.003437	-0.000296
0.6	0.000457	0.000216	0.001482	0.000302	0.002602	0.000054	0.003437	-0.000296
0.8	0.000201	0.000119	0.000646	0.000263	0.001126	0.000315	0.001482	0.000302
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 19. Deflection w for different X, Y and β=0.6,θ=45 and a/b=0.5 at time 5.k

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.000200	0.000110	0.000654	0.000243	0.001151	0.000291	0.001523	0.000280
0.4	0.000460	0.000199	0.001523	0.000280	0.002710	0.000053	0.003606	-0.000268
0.6	0.000460	0.000199	0.001523	0.000280	0.002710	0.000053	0.003606	-0.000268
0.8	0.000200	0.000110	0.000654	0.000243	0.001151	0.000291	0.001523	0.000280
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

CONCLUSIONS

The Rayleigh-Ritz technique has been applied to study the effect of the taper constants on the vibration of clamped visco-elastic isotropic parallelogram plate with parabolically varying thickness on the basis of classical plate theory.

On comparison with [17], it is concluded that:

Time period K is more for non-uniform thickness in case of parabolic variation as comparison to linear variation. Deflection w is less for non-uniform thickness in case of parabolic variation as comparison to linear variation. In this way, authors concluded that parabolic variation is more useful than linear variation.

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