ANNALS OF FACULTY ENGINEERING HUNEDOARA
– INTERNATIONAL JOURNAL OF ENGINEERING
Tome X (Year 2012) – FASCICULE 3 (ISSN 1584 – 2673)

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VERTICAL SUSPENSION DAMPING OPTIMIZATION ON THE RUNNING BEHAVIOUR CRITERION OF THE RAILWAY VEHICLE

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ABSTRACT: *The paper presents a method to optimise the vertical suspension damping of a railway vehicle by applying the criterion of the minimisation of both bogie and car body acceleration, considering the random track irregularity. This issue is interesting from the view point of improving the vehicle design so as to increase safety and comfort of the railway transportation. To this end, the vehicle model takes into account only the bounce and pitch of the car body and bogies for the sake of simplicity. The frequency-domain response of the vehicle is calculated considering the influence of the wheelbase and bogie spacing filter effects. The RMS value of the car body and bogie acceleration is determined and, starting from this, the optimal damping of the suspension is calculated. Finally, the vertical vibration behaviour of the railway vehicle is studied.*
KEYWORDS: *railway vehicle, vibration, optimal damping, random irregularity, acceleration*

INTRODUCTION

The running behaviour of a railway vehicle reflects the vibration level to which the passengers or the goods are exposed during travelling and, for that reason, it is described by the car body acceleration. Smooth running behaviour is a basic requirement to homologate any railway vehicle.

The vibration of the railway vehicles is mainly caused by the track irregularity in both vertical and horizontal plans. However, the two types of vibration are decoupled because the railway vehicle is a symmetric mechanical structure (inertial, elastic and geometrical) [1]. Narrowing it down to the vertical vibration, the bounce and pitch vibrations have to be mentioned, critical for the running behaviour, safety and quality of the track [2-7].

The vehicle suspension has the key role to obtain a smooth running behaviour. In fact, the suspension stiffness is chosen in such a way that the vehicle natural frequencies be within particular frequency ranges. For instance, for a passenger car, the bounce low natural frequency is situated about 1 Hz. In these circumstances, the suspension damping becomes critical for the running behaviour improvement.

In this paper, the method of the vertical two-level suspension damping optimization is presented, following the criterion of the minimisation of both bogie and car body acceleration, due to the random track irregularity. To this end, the vehicle model takes into account only the bounce and pitch of the car body and bogies, for the sake of simplicity. However, the damping of the primary suspension is optimised by minimising the pitch acceleration of the bogie, due to its influence on the car body structural vibration (overlooked in here). Also, the influence of the wheelbase and bogie spacing filter effect is pointed out.

MECHANICAL MODEL

Figure 1 shows the mechanical model for studying the vertical vibrations of a two-level suspension railway vehicle. The model consists in 3 rigid bodies, representing the car body and the two suspended masses of the bogies. The suspension levels, two on each bogie, are modelled by Kelvin-Voigt systems. Solely the movements in a vertical plan will be taken into account, the bounce and the pitch, respectively. It is considered that the bogies pitch movement is decoupled from the one of the car body. And there is the assumption that the track irregularities are identical on both rails, with no rolling movement of the vehicle.

Similarly, as the vehicle natural frequencies are much smaller than the axles' on the track, the hypothesis of a perfectly rigid track will be adopted, where this track will impose vertical displacements to the vehicle, by axles, due to the longitudinal level.

The movement equations are as below:

- the movement equation of the car body bounce and pitch

$$m_c\ddot{z}_c + 2c_2(2\dot{z}_c - \dot{z}_{b1} - \dot{z}_{b2}) + 2k_2(2z_c - z_{b1} - z_{b2}) = 0; \quad (1)$$

$$m_c\ddot{\theta}_c + 2a_2c_2(2a_2\dot{\theta}_c - \dot{z}_{b1} + \dot{z}_{b2}) + 2a_2k_2(2a_2\theta_c - z_{b1} + z_{b2}) = 0; \quad (2)$$

- the movement equations of the bogies bounce and pitch

$$\begin{aligned} & m_b\ddot{z}_{b1,2} + 2c_2(\dot{z}_{b1,2} - \dot{z}_c \mp a_2\dot{\theta}_c) + 2k_2(z_{b1,2} - z_c \mp a_2\theta_c) + \\ & + 2c_1(2\dot{z}_{b1,2} - \dot{\eta}_{1,3} - \dot{\eta}_{2,4}) + 2k_1(2z_{b1,2} - \eta_{1,3} - \eta_{2,4}) = 0 \end{aligned} ; \quad (3)$$

$$J_b\ddot{\theta}_{b1,2} + 2a_1c_1(2a_1\dot{\theta}_{b1,2} - \dot{\eta}_{1,3} + \dot{\eta}_{2,4}) + 2a_1k_1(2a_1\theta_{b1,2} - \eta_{1,3} + \eta_{2,4}) = 0; \quad (4)$$

where z_c and θ_c are the car body bounce and pitch, $z_{b1,2}$ and $\theta_{b1,2}$ are the bogies bounce and pitch, m_c and $J_c = m_c i_c^2$ are the car body mass and the mass inertia moment, m_b and J_b the mass and the mass inertia moment of the bogies, $2k_1$ and $2c_1$ are the stiffness and damping of the primary suspension for an axle, $2k_2$ and $2c_2$ are the stiffness and damping of the secondary suspension for a bogie. Also, there is the notation η_i with i = 1 to 4 for the irregularity of the track opposite those four axles.

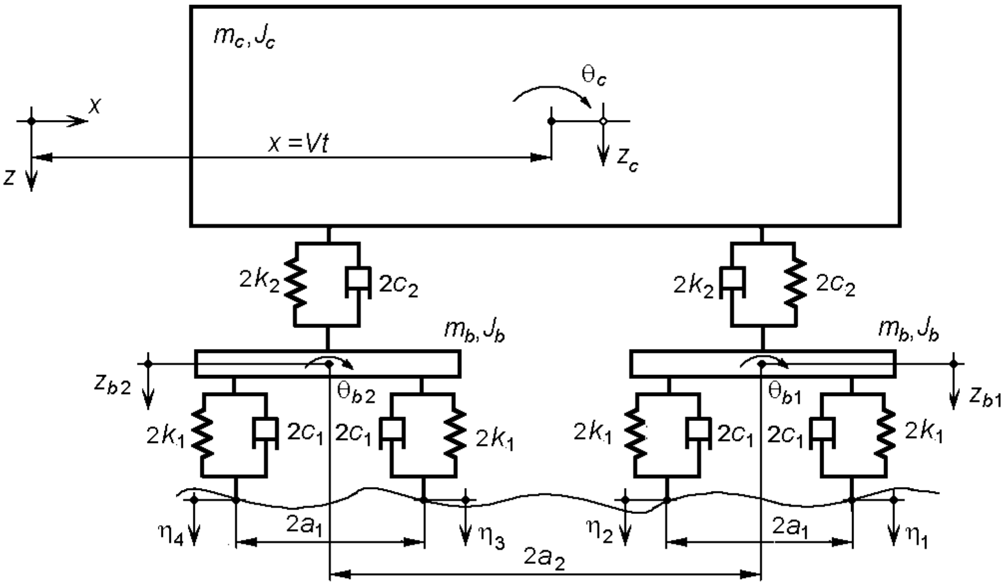

Fig. 1. The vehicle's mechanical model.

It has been noticed that the car body pitch and bounce are coupled via the bogies bouncing movement, i.e. the first four equations are coupled. Despite of this, introducing the following variable

$$z_c^* = a_2\theta_c, \; 2z_b^+ = z_{b1} + z_{b2}, \; 2z_b^- = z_{b1} - z_{b2}, \; z_{b1,2}^* = a_1 z_{b1,2}. \quad (5)$$

two independent systems will be obtained.

To find the frequency-domain response of the vehicle, the irregularities of the track are considered to have a sinusoidal shape, with the wavelength L and the amplitude η_0. At the velocity V over the sinusoidal irregularity on the track, this will induce an imposed movement to the vehicle, which becomes a time function of a angular frequency $\omega = 2\pi V/L$

$$\eta_{1,2} = \eta_0 \cos\frac{\omega}{V}(Vt \pm a_1 + a_2), \; \eta_{3,4} = \eta_0 \cos\frac{\omega}{V}(Vt \pm a_1 - a_2). \quad (6)$$

During the steady-state harmonic behaviour, all the variables of the system are harmonic, with the angular frequency being induced by the irregularity of the track. Upon introducing the complex variables associated to the real ones,

$$\overline{z}_c = \overline{Z}_c e^{i\omega t}, \; \overline{z}_c^* = \overline{Z}_c^* e^{i\omega t}, \; \overline{z}_b^\pm = \overline{Z}_b^\pm e^{i\omega t}, \; \overline{z}_{b1,2}^* = \overline{Z}_{b1,2}^* e^{i\omega t},$$

$$\overline{\eta}_{1,2} = \eta_0 e^{i\omega t} e^{i\frac{\omega}{V}(a_2 \pm a_1)}, \; \overline{\eta}_{3,4} = \eta_0 e^{i\omega t} e^{i\frac{\omega}{V}(-a_2 \pm a_1)}, \quad (7)$$

the frequency-response factors may be calculated:

- for the bounce movement in the car body centre

$$\overline{H}_c = \frac{\overline{Z}_c}{\eta_0} = \frac{(\omega_c^2 + \alpha_c\omega i)(\omega_b^2 + \alpha_b\omega i)}{(\omega_c^2 - \omega^2 + \alpha_c\omega i)\left[\mu\omega_c^2 + \omega_b^2 - \omega^2 + (\mu\alpha_c + \alpha_b)\omega i\right] - \mu(\omega_c^2 + \alpha_c\omega i)^2}\overline{H}_v^+; \quad (8)$$

- for the effect of car body pitch opposite the bogie

$$\overline{H}_c^- = \frac{\overline{Z}_c^-}{\eta_0} = \frac{\varepsilon_c(\omega_c^2 + \alpha_c\omega i)(\omega_b^2 + \alpha_b\omega i)}{(\varepsilon_c\omega_c^2 - \omega^2 + \varepsilon_c\alpha_c\omega i)\left[\mu\omega_c^2 + \omega_b^2 - \omega^2 + (\mu\alpha_c + \alpha_b)\omega i\right] - \mu\varepsilon_c(\omega_c^2 + \alpha_c\omega i)^2}\overline{H}_v^-; \quad (9)$$

- for the symmetrical bounce movement of the bogies

$$\overline{H}_b^+ = \frac{\overline{Z}_b^+}{\eta_0} = \frac{(\omega_c^2 - \omega^2 + \alpha_c\omega i)(\omega_b^2 + \alpha_b\omega i)}{(\omega_c^2 - \omega^2 + \alpha_c\omega i)\left[\mu\omega_c^2 + \omega_b^2 - \omega^2 + (\mu\alpha_c + \alpha_b)\omega i\right] - \mu(\omega_c^2 + \alpha_c\omega i)^2}\overline{H}_v^+; \quad (10)$$

- for the anti-symmetrical bounce movement of the bogies

$$\overline{H}_b^- = \frac{\overline{Z}_b^-}{\eta_o} = \frac{(\varepsilon_c\omega_c^2 - \omega^2 + \varepsilon_c\alpha_c\omega i)(\omega_b^2 + \alpha_b\omega i)}{(\varepsilon_c\omega_c^2 - \omega^2 + \varepsilon_c\alpha_c\omega i)\left[\mu\omega_c^2 + \omega_b^2 - \omega^2 + (\mu\alpha_c + \alpha_b)\omega i\right] - \mu\varepsilon_c(\omega_c^2 + \alpha_c\omega i)^2}\overline{H}_v^-; \quad (11)$$

- for the pitch movement of the bogies above the axle

$$\overline{H}_{b1,2}^* = \frac{\overline{Z}_{b1,2}^*}{\eta_o} = \frac{-\omega^2 + \varepsilon_b\alpha_b i\omega}{\varepsilon_b\omega_b^2 - \omega^2 + \varepsilon_b\alpha_b i\omega}\overline{H}_{pb1,2}, \quad (12)$$

where we have filtering factors due to the wheelbase and bogie spacing effect

$$\overline{H}_v^+ = \cos\frac{\omega}{V}a_1\cos\frac{\omega}{V}a_2, \ \overline{H}_v^- = i\cos\frac{\omega}{V}a_1\sin\frac{\omega}{V}a_2, \ \overline{H}_{pb1,2} = e^{\pm i\frac{\omega}{V}a_2} i\sin\frac{\omega}{V}a_1, \quad (13)$$

with $$\omega_c^2 = \frac{4k_2}{m_c}, \ \alpha_c = \frac{4c_2}{m_c}, \ \omega_b^2 = \frac{4k_1}{m_b}, \ \alpha_b = \frac{4c_1}{m_b}, \ \mu = \frac{m_c}{2m_b}, \ \varepsilon_c = \frac{a_2^2}{i_c^2}, \ \varepsilon_b = \frac{a_1^2}{i_b^2} \quad (14)$$

and $\alpha_c = 2\zeta_c\omega_c$, $\alpha_b = 2\zeta_b\omega_b$,

where $$\zeta_c = \frac{4c_2}{2\sqrt{4k_2m_c}}, \ \zeta_b = \frac{4c_1}{2\sqrt{4k_1m_b}} \quad (15)$$

are the damping ratios of the two-levels of suspension.

Next, it is considered that the defects of the track have a random behaviour and they may be shaped as the vertical track irregularity PSD which is described in spatial domain as

$$S(\Omega) = \frac{A}{(B+\Omega)^3}, \quad (16)$$

where: A and B are constant that depend on the track quality, and Ω is the wave number.

Since the track defects become an excitation factor for a vehicle travelling along the track with the velocity V, the irregularity PSD must be expressed in dependence with the angular frequency $\omega = V\Omega$. Thus, from equation (16), the track irregularity PSD is derived in terms of angular frequency and vehicle velocity

$$G(\omega) = \frac{AV^2}{(BV+\omega)^3}. \quad (17)$$

However, the RMS value of the defects does not depend on the velocity, according to the next equation

$$\sigma = \sqrt{\frac{1}{\pi}\int_0^\infty G(\omega)d\omega} = \sqrt{\frac{1}{\pi}\int_0^\infty \frac{AV^2}{(BV+\omega)^3}d\omega} = \frac{1}{\sqrt{2\pi}}\frac{\sqrt{A}}{B}. \quad (18)$$

Starting from the frequency-response factors of the vehicle and the PSD of the track defects, it may be calculated the acceleration PSD for the car body (at middle and above bogie) and the bogie pitch (above axle)

$$G_{cm}(\omega) = \omega^4 G(\omega)\left|\overline{H}_c(\omega)\right|^2, \ G_{cb}(\omega) = \omega^4 G(\omega)\left|\overline{H}_c(\omega) + \overline{H}_c^*(\omega)\right|^2, \ G_b^*(\omega) = \omega^4 G(\omega)\left|\overline{H}_b^*(\omega)\right|^2. \quad (19)$$

Finally, the RMS value for the car body and bogie acceleration is obtained

$$\sigma_{cm,cb} = \sqrt{\frac{1}{\pi}\int_0^\infty G_{cm,cb}(\omega)d\omega}, \ \sigma_b^* = \sqrt{\frac{1}{\pi}\int_0^\infty G_b^*(\omega)d\omega}. \quad (20)$$

NUMERICAL APPLICATION

This chapter showcases the results of a numerical application done for a passenger car that travels at different velocities on a track with random irregularity. The track irregularity PSD from equation (17) is taken into account with the constants A= $2.004\cdot10^{-6}$ and B = 0.36. The RMS value of the defects is 1.57 mm and this value corresponds to a track with the quality level between QN1 and QN2 [8]. The parameters of the vehicle model are as such: m_c= 30000 kg, i_c = 7 m, m_c= 2600 kg, i_b = 0,9 m, k_1 = 1 MN/m, k_2 = 400 kN/m, $2a_1$ = 2.6 m and $2a_2$ = 19.2 m.

Figure 2 shows the track excitation input gain for bounce vibration. Two velocities are taken into account, 120 and 240 km/h, respectively. It has to be mentioned that filtering comes from both the 'wheelbase filter' effect and 'bogie spacing filter' effect. The covering with dotted line corresponds to the influence of the wheelbase filter effect that introduces minimum values into the response of the car body at a $V/2a_1$ interval. For instance, at a 120 km/h velocity, the interval between two consecutive minimum values is of 12.82 Hz. The former occurs at the frequency of $V/4a_1$, respectively at 6.41 Hz (at 120 *km/h). On the other hand, the bogie spacing filter effect introduces minimum values at frequency intervals of $V/2a_2$, where the first minimum happens at $V/4a_2$. Since the bogie spacing is much bigger than the wheelbase, the intervals will be smaller. At the same velocity, it results that the first minimum due to the bogie spacing filter effect is at 0.87 Hz, then they happen at intervals of 1.74 Hz.*

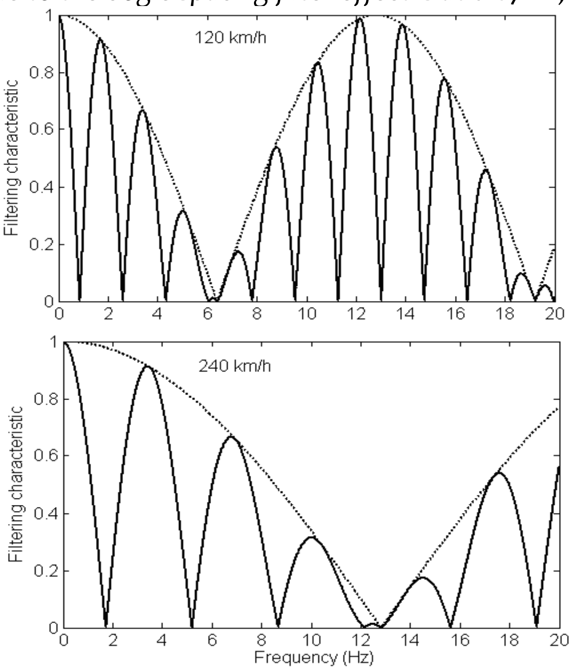

Figure 2. The filtering characteristic in the car body center (bounce).

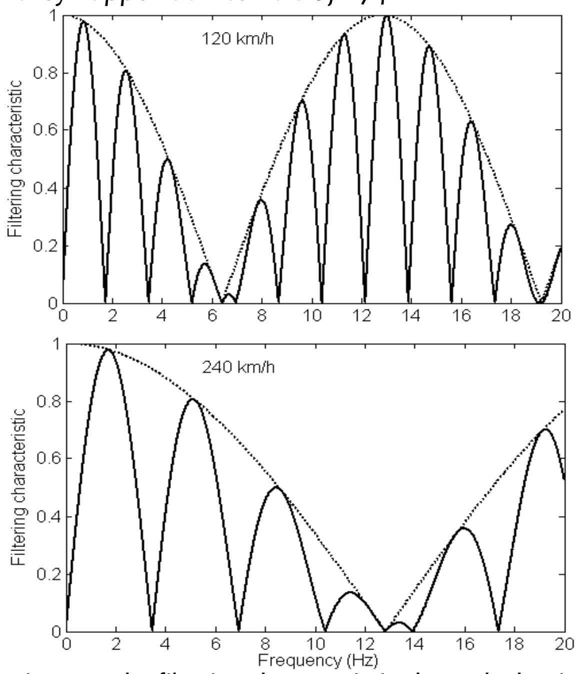

Figure 3. The filtering characteristic above the bogie (the car body pitch).

The track input to car body bounce is maximum when the wheelbase or the bogie spacing is an integer multiple of the track irregularity wavelength. In this situation, the two points defining the base will simultaneously reach maximum and minimum values of the track irregularities. While the velocity increases, the minimum and maximum values of the track input move towards higher frequencies. What is specific to the filtering effect is that the pass-by band of the first lobe goes up with the velocity.

Figure 3 presents the track excitation input gain for pitch vibration. Also, the covering agrees with the wheelbase filtering effect. The frequency intervals characteristics at which the minimum values occur are maintained. Similarly, it may be noticed the change of the characteristic shape while velocity increases. The essential difference is linked to the bogie spacing filter effect, and the minimum values here correspond to the maximum ones in the previous figure.

The filtering effect makes the system frequency-response be dependent on the velocity, a specific feature for vehicles.

In figure 4, the magnification factor of acceleration at the car body center and above the first bogie is displayed for 120, 180 and 240 km/h, taking into account the influence of the wheelbase and bogies spacing filtering effect. Also, the acceleration magnification factor calculated without the filtering effect is displayed. The damping constants are c_1 = 15.3 kNs/m, c_2 = 11 kNs/m. First, the car body response is lower due to the filtering effect. The local maximum is related to the natural frequencies. There is an influence of the low bounce resonance at circa 1 Hz, as well as of the low resonance of the car body pitch at 1.45 Hz, occurring above the bogie. The high bounce and pitch natural frequencies influence may be observed at about 6.9 Hz. The frequency intervals of the minimum response at the car body center correspond to the previous ones. The response is significantly modified, due to the velocity for the particular case of the resonance frequencies. The filtering effect is high at 180 km/h and low at 240 km/h. The car body response above the bogie has the minimum values only due to the wheelbase filtering effect, while the bogie spacing filter effect is practically levelled out. The response magnitude depends on the velocity. For instance, the maximum value at the low bounce natural frequency may be observed at 180 km/h and the minimum value at 120 km/h. As anticipated, the filtering effect is selective along the car body, and it depends on the vehicle velocity.

In the following, both primary and secondary damping is calculated, considering the acceleration minimization criterion. Figure 5 shows the acceleration bogie above axle due to the pitch vibration only versus the damping ration of the primary suspension for three velocities, 120, 180 and 240 km/h, respectively. The optimum values of the damping ratio are 0.365, 0.305 and 0.245 and they depend on velocity.

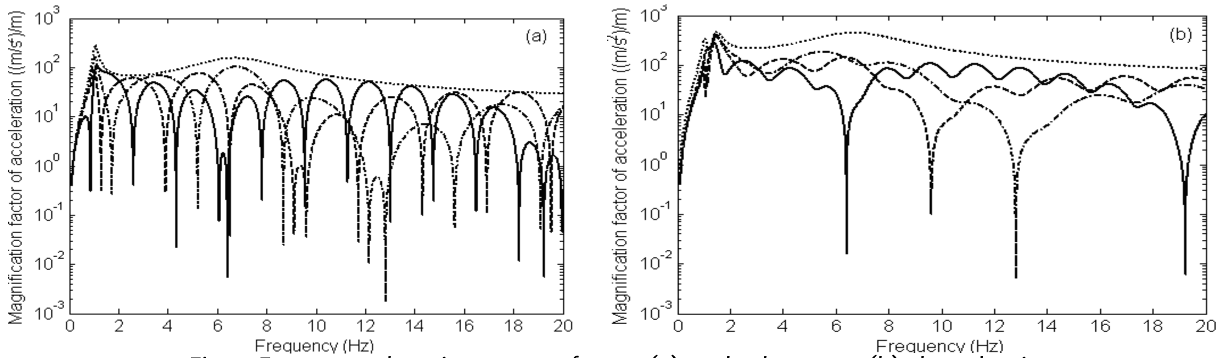

Fig. 4. Frequency-domain response factor: (a) car body center; (b) above bogie
—, 120 km/h, – – –, 180 km/h, – · – · –, 240 km/h, · · · ·, non-filtering effect.

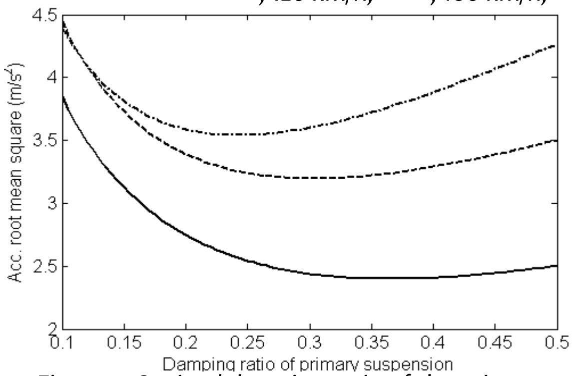

Figure 5. Optimal damping ratio of the primary suspension: —, 120 km/h, – – –, 180 km/h, – · – · –, 240 km/h.

Next, by taking these values for the primary damping ratio, the secondary damping ratio may be calculated, so that the car body acceleration is minimum.

The RMS value of the car body center acceleration for a ratio damping of the secondary suspension range of 0.01 – 0.50 is displayed in figure 6. The acceleration has been calculated for both non-filtering and filtering effect and for preceding velocities. The former aspect refers to the fact that the acceleration is overestimated if the filtering effect is neglected. The latter aspect, subsequent to the former, concerns the values of the optimal damping ratio that are different when the filtering effect is considered or not. For instance, the values of the optimal damping ratio are 0.165, 0.115 and 0.160 for the velocities of 120, 180 and 240 km/h, respectively, when the filtering effect is simulated and 0.195, 0.165 and 0.145 for case of the non-filtering.

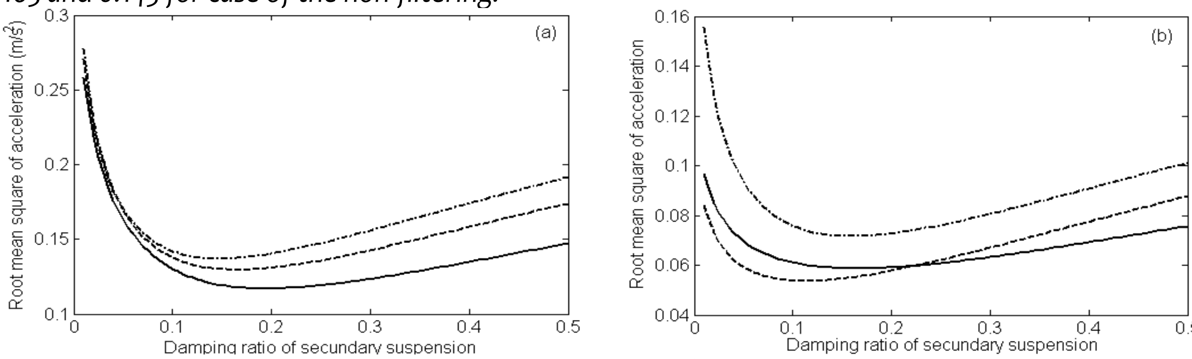

Figure 6. Optimal damping ratio of the secondary suspension (at middle): a) non-filtering effect; b) filtering. effect; —, 120 km/h, – – –, 180 km/h, – · – · –, 240 km/h.

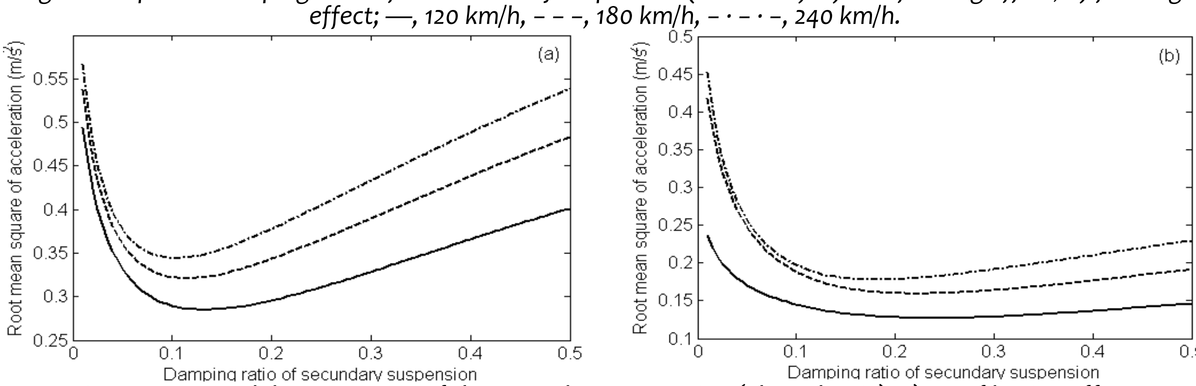

Figure 7. Optimal damping ratio of the secondary suspension (above bogie): a) non-filtering effect; b) filtering effect; —, 120 km/h, – – –, 180 km/h, – · – · –, 240 km/h.

Figure 7 presents the RMS value of the car body acceleration above the first bogie for the same secondary suspension damping ratio range and velocities. Also, both filtering and non-filtering cases have been considered. It is obvious this time that the acceleration is significantly higher than the preceding simulation. When comparing the results in figure 7 a and b, it may be observed again that the filtering effect leads to low acceleration values. There are big differences between the values of the *optimal damping ratio. When the filtering effect is taken into account, the damping ratio has to be 0.240, 0.220 and 0.180 for the three velocities considered. By overlooking the filtering effect, much smaller values are obtained, 0.135, 0.115 and 0.105, respectively.*

Finally, the vertical vibration behaviour of the vehicle may be estimated by considering the optimal damping values of the suspension previously calculated. For instance, the RMS value of the car body acceleration is presented in figure 8 for velocities between 40 and 240 km/h. The RMS value of the acceleration has been calculated along the car body for the optimal damping corresponding to 240 km/h, i.e. the primary suspension damping ratio of 0.245 and damping ratio of 0.180 for the secondary suspension. Generally speaking, the acceleration increases along to the velocity, from the car body centre to the car body ends. However, there is a particular velocity range around 90 km/h when the acceleration is higher because the filtering effect, derived from the wheelbase and bogie spacing, is less effective.

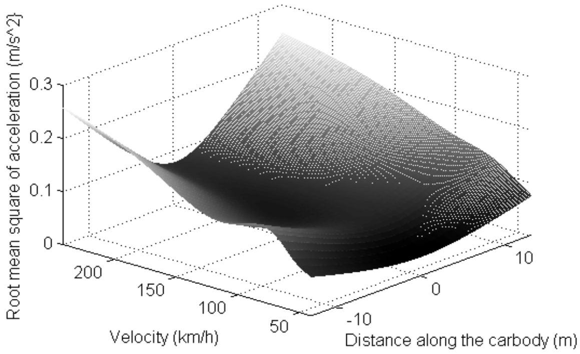

Fig. 8. The RMS value of the car body acceleration (ζ_b = 0.245, ζ_c = 0.180).

CONCLUSIONS

The vertical vibrations of the railway vehicles mainly derive from the irregularities of the track. These vibrations may inflict damage upon the running behaviour, safety, comfort and track quality. Due to that, the improvement of the vehicle design and its suspension especially is the only way to guarantee the smooth running behaviour. From this perspective, the choice of the suspension damping is a critical question. To solve this issue, this paper proposes a method to optimise the suspension damping based on the acceleration minimisation criterion.

To illustrate this method, the case of a vehicle on two-level suspension bogies running on a track with random irregularity is considered. The vehicle model consists of three rigid bodies (car body and the two bogie suspended masses) with six degrees of freedom, and four Kelvin-Voigt systems for the suspension.

The frequency-domain response of the vehicle depends on the velocity due to the wheelbase and bogie spacing filtering effect. The response magnitude is lower when the filtering effect is taken into account.

The optimal damping ratio of the primary suspension has been calculated via the minimisation of the bogie pitch acceleration in order to limit the influence on the car body's structural vibration. The damping ratio of the secondary suspension has to be obtained by applying the minimisation of the car body acceleration criterion. To this end, it is better to consider the car body acceleration above bogie where the vibration level is the highest. Thus, the vehicle running behaviour will be as smooth as possible. The optimal values of the damping ratio depend on the velocity and they decrease when the vehicle velocity increases.

The numerical simulation of the running behaviour of a railway vehicle with optimal damping has shown that the acceleration increases from the car body centre to its ends. Similarly, the filtering effect is selective and effective within a particular velocity range and, due to that, the car body acceleration exhibits a local maximum value at moderate velocities.

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