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MHD FREE CONVECTION FLOW BETWEEN TWO PARALLEL POROUS WALLS WITH VARYING TEMPERATURE

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ABSTRACT: *The present paper analyzes the influence of the heat and mass transfer characteristics of a two dimensional steady laminar free convective flow of a viscous incompressible fluid between two parallel porous walls. Using the similarity variable, the partial differential equations were reduced to ordinary differential equations. The coupled ordinary differential equations were solved numerically using shooting method. The effect of various physical parameters, such as the Prandtl number, Grashof number, permeability parameter and ratio of the free stream velocity to parallel wall parameter on the boundary layer velocity and skin-friction coefficient are computed and discussed in detail.*
KEYWORDS: *Free convection, porous medium, similarity solution, MHD*

INTRODUCTION

The problem of fluid flow in an electromagnetic field has been studied for its importance in geophysics, metallurgy and aerodynamic extrusion of plastic sheets and other engineering processes such as in petroleum engineering, chemical engineering and heat exchangers. There are several situations where forced and natural convection occur with relatively comparable significance. This case is referred to as mixed convection heat transfer. Accurate knowledge of the overall convection heat transfer is important in many fields, including heat exchangers, hot water and stream pipes heaters, refrigerators and electrical conductors. Because of its industrial importance, this class of heat transfer has been the subject of many experimental and analytical studies (Bassam and Abu-Hijleh, (2002). Omowaye and Koriko (2010) discussed the Similarity solutions for free convection between two parallel porous walls at different temperatures. Seddeek and Salem (2005), investigated the heat and mass transfer distributions on stretching surface with variable viscosity and thermal diffusivity. Norfifah Backok and Anuar Ishak (2009) investigated the MHD Stagnation-point flow of a micro polar fluid with prescribed wall heat flux, Shankar et al (2010) discussed the radiation and mass transfer effects on unsteady MHD free convective fluid flow embedded in a porous medium with heat generation/absorption.

Present research field has attracted many researchers in resent years due to its astounding applications. Mahapatra and Gupta (2002) analyzed Stagnation-point flow towards a stretching surface. They reported in their research work that a boundary layer is formed when stretching velocity is less then the free stream velocity. The heat source/sink effects in thermal convection are significant where there may exist high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reaction. Tania et al (2010) has investigated the Effects of radiation, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. Sparrow and Cess (1961) provided one of the earliest studies using a similarity approach for stagnation point flow with heat source/sink which vary in time. Pop and Soundalgekar (1962) presented an unsteady free convection flow past an infinite plate with constant suction and heat source. Vajravelu and Hadjinicolaou (1993) analyzed the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Hossain et al. (2004) studied the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation/absorption. Alam et al. (2006) considered the problem of the free convection heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of magnetic field and heat generation. Chamkha (2004) investigated an unsteady convective heat and mass transfer past a semi-infinite porous moving plate with heat absorption. Hady et al. (2006) studied the problem of free convection flow convection flow along a vertical wary surface embedded in electrically conducting fluid saturated porous media in the presence of internal heat generation or absorption effect. Sharma and Singh (2010) have studied the effects of variable thermal conductivity and heat source/sink on MHD flow near a stagnation point on a linearly stretching sheet. Recently Makinde et al. (2011) studied Radiation effect on chemically reacting magneto hydrodynamics (MHD) boundary layer flow of heat and mass transfer through a porous vertical flat plate.

The objective of the present investigation is to study the influence of magnetic field on MHD free convective flow between parallel porous walls with variable temperature. In the problem formulation, the continuity, momentum, energy and concentration equations are reduced to some parameter problem by introducing suitable transformation variables. The equations that govern the flow are coupled and solve numerically using shooting techniques with the forth order Ranga-Kutta method. The effects of various governing parameters on the velocity, temperature, concentration are presented graphically and discussed quantitatively.

MATHEMATICAL ANALYSIS

Consider a two-dimensional steady laminar free convection flow of a viscous incompressible fluid between two parallel porous walls situated a distance L apart. The x-axis is taken along the wall and y – axis is transverse to the parallel wall. The fluid is injected in to the lower wall at y=0 and is sucked through the upper wall with uniform velocity, under the usual Boussineq's and boundary layer approximation, the governing equations for conservation of mass, momentum, energy and continuity respectively are as follows:

$$(\partial u/\partial x)+(\partial v/\partial y)=0 \quad (1)$$

$$u(\partial u/\partial x)+v(\partial v/\partial y)=(-1/\rho)(\partial p/\partial x)+v(\partial^2 u/\partial y^2)-(\sigma B_0^2/\rho)u+g\beta(T-T_\infty)-(v/K)u \quad (2)$$

$$u(\partial T/\partial x)+v(\partial T/\partial y)=(k/\rho c_p)(\partial^2 T/\partial y^2) \quad (3)$$

$$u(\partial C/\partial x)+v(\partial C/\partial y)=D(\partial^2 C/\partial y^2) \quad (4)$$

where: u and υ are the velocity components in the x and y directions respectively, T - the temperature of the fluid in the boundary layer, C - the species concentration in the boundary layer, g - the acceleration due to gravity, ν is the fluid kinematics viscosity, ρ - the density, σ - the electric conductivity of the fluid, β - coefficients of thermal expansion, k - the thermal conductivity, C_p - the specific heat at constant pressure, μ - the viscosity, B_o - the magnetic induction, D - the mass diffusivity.

The boundary conditions for the velocity, temperature and concentration fields are

$$u=0, v=0, \ T=T_\infty, \ C=C_\infty \quad \text{at } y=0$$

$$u\to u_\infty, \ T\to T_\infty, \ C\to C_\infty \quad \text{as } y\to\infty \quad (5)$$

where C >0 and u_∞ is the free stream velocity, T_∞ is free stream temperature, T_w is temperature of the horizontal wall.

The free-stream velocity $u_\infty=u(x)=bx$, where b is the stream velocity parameter. Equation (2)

$$(-1/\rho)(\partial p/\partial x)-(\nu/K)u_\infty = u_\infty(d u_\infty/dx) \quad (6)$$

Equating $\partial p/\partial x$ between Eq.(2) and (6), we get

$$u(\partial u/\partial x)+v(\partial v/\partial y)=u_\infty(\partial u_\infty/\partial x)+v(\partial^2 u/\partial y^2)+g\beta(T-T_\infty)+(v/K)(u_\infty-u)+(\sigma B_0^2/\rho)(u_\infty-u) \quad (7)$$

METHOD OF SOLUTION

Introducing the stream function $\psi(x,y)$ as defined by:

$$U=\partial\psi/\partial y, \ v=-\partial\psi/\partial x \quad (8)$$

The similarity variable

$$\eta=(c/v)^{1/2}y \text{ and } \psi(x,y)=(c/v)^{1/2}xf(\eta) \quad (9)$$

where: c = the parallel wall parameter, η = the similarity variable, υ - the kinematic viscosity, ψ = the stream function and the dimensionless temperature and concentration is given by

$$\theta=(T-T_\infty)/(T_w-T_\infty), \phi=(C-C_\infty)/(C_w-C_\infty) \quad (10)$$

Equations (8)-(10) in to Eq.(3)and (7), we get:

$$f'''+ff''-(f')^2-Gr\theta+(M+A)(\gamma-f')+\gamma^2=0 \quad (11)$$

$$\theta''+Pr f\theta'=0 \quad (12)$$

$$\phi''+Sc f\phi'=0 \quad (13)$$

where the Grashof numbe rGr=$(g\beta(T_w-T_\infty)/c^2x$, the permeability parameter A=1/Kc, the ratio of the free-stream velocity parameter to parallel wall parameter $\gamma=b/c$, the Prandtl number $Pr=\mu c_p/k$, Schmidt number $Sc=\nu/D$.

It is noted that Equation (1) is identically specified. The corresponding boundary conditions are reduced to:

$$f(0)=0, f'(0)=0, f'(\infty)=\gamma, \theta(0)=1, \theta(\infty)=0, \phi(0)=0, \phi(\infty)=1 \quad (14)$$

Skin-friction coefficient at the wall is given by

$$C_f=(\tau_w/\rho c_p(c\nu)^{1/2})=x f''(0)$$

where $\tau_w=\mu[(\partial u/\partial y)+(\partial v/\partial x)]_{y=0}$ is the shear stress at the wall.

The equations (11) - (13) are coupled and non-linear partial differential equations and hence analytical solution is not possible. Hence the dimensionless governing equations are solved numerically using the fourth-order Runge-Kutta method with shooting technique. The shooting method for linear equations is based on replacing the boundary value problem by two initial value problems, and solution of the boundary value problem is a linear combination between the solutions of the two initial value *problems. The shooting method for the nonlinear boundary value problem is similar to the linear case, except that the solution of the nonlinear problem can not be simply expressed as a linear combination between the solutions of the two initial value problems. The numerical computations have been done by the symbolic computation software MATHEMATICA. The fourth-order Runge-Kutta method is used to solve the initial value problems. The equation (13) being linear, solving it analytically, we directly get ϕ. The numerical approach is carried out in two stages. Solving the equation (12) by the nonlinear shooting method we obtain θ. Hence, equations (11) reduce to a system of linear equations with variable coefficients which could be solved by the linear shooting method to obtain f. The functions f, θ and ϕ are shown in figures.*

RESULTS AND DISCUSSION

Similarity solution for free convection flow between parallel porous walls at different temperatures has been considered. The parameter that govern the present flow situation are Grashof number (Gr), Prandlt number (Pr), the permeability parameter (A), the ratio of free stream velocity parameter to parallel wall parameter (γ) ,and Schmidt number (Sc). Also, comprehensive set of numerical results is displayed graphically in Fig.1-6 to illustrate the influence of the various physical parameters on the locally similarity solutions.

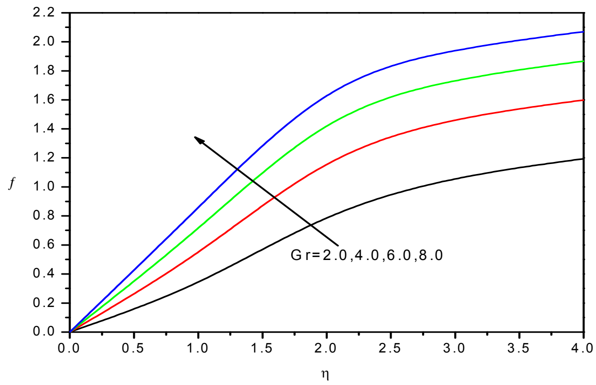

Fig .1(a) Variation of the velocity component f with Gr for M = 1.0, Pr = 0.71, γ = 0.1, A = 0.1, Sc = 0.6

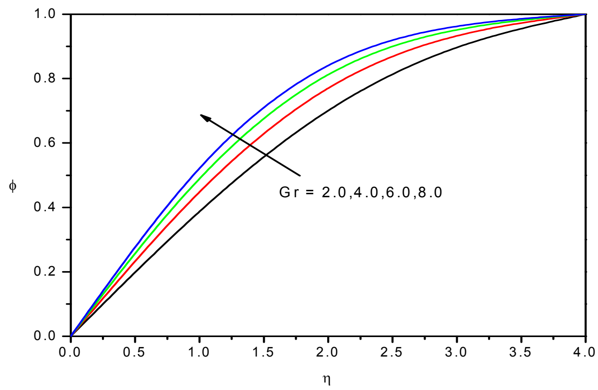

Fig.1 (b) Variation of the concentration ϕ with Gr for M=1.0, Pr = 0.71,γ = 0.1, A= 0.1, Sc = 0.62.

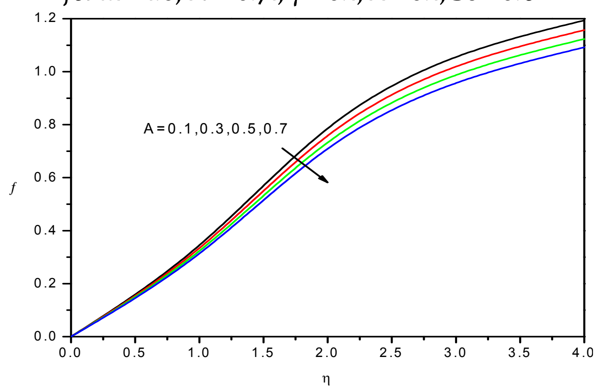

Fig .2(a) Variation of the velocity component f with A for Gr = 2.0, M=1.0, Pr = 0.71, γ = 0.1, Sc = 0.62.

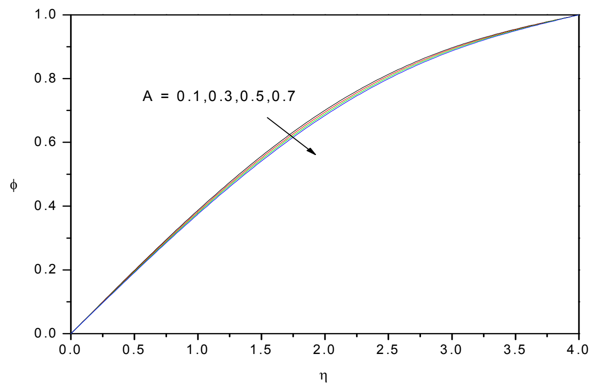

Fig.2 (b) Variation of the concentration ϕ with A for Gr = 2.0, M=1.0, Pr = 0.71, γ = 0.1, A= 0.1

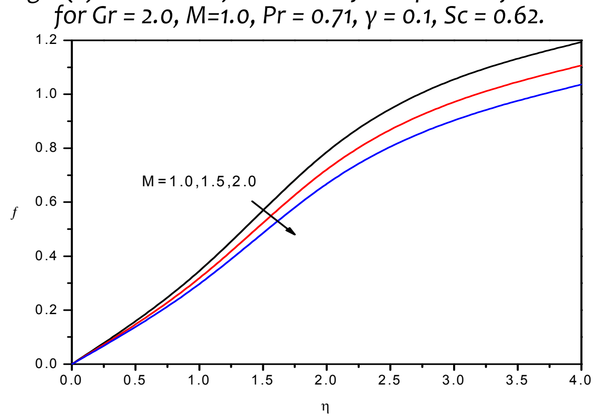

Fig .3(a) Variation of the velocity component f with M for Gr = 2.0, Pr = 0.71, γ = 0.1, A = 0.1, Sc = 0.62.

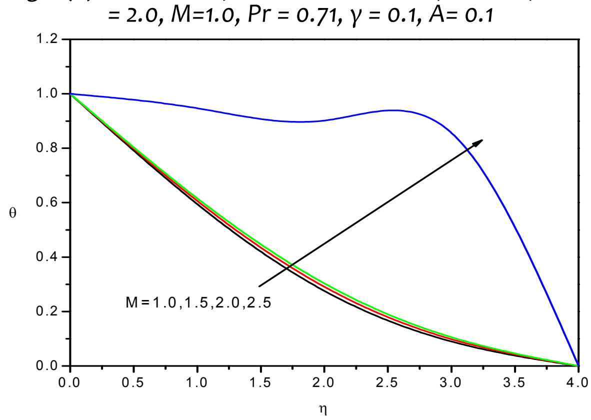

Fig.3 (b) Variation of the temperature ϑ with M for Gr = 2.0, Pr = 0.71, γ = 0.1, A= 0.1, Sc = 0.62.

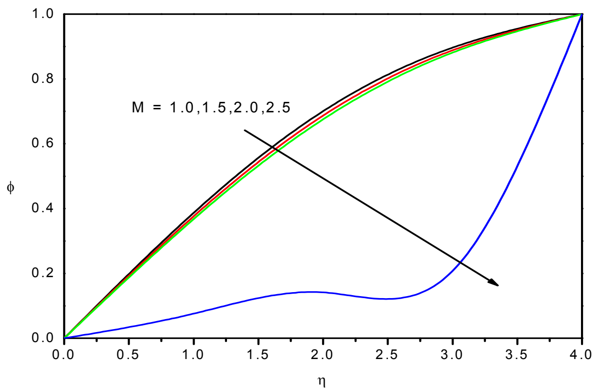

Fig.3(c) Variation of the concentration ϕ with M for Gr = 2.0, Pr = 0.71, γ = 0.1, A= 0.1, Sc = 0.62

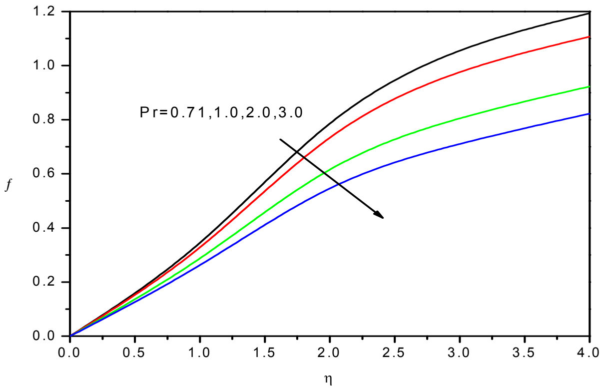

Fig .4(a) Variation of the velocity component f with Pr for Gr = 2.0, M=1.0, γ = 0.1, A = 0.1, Sc = 0.62

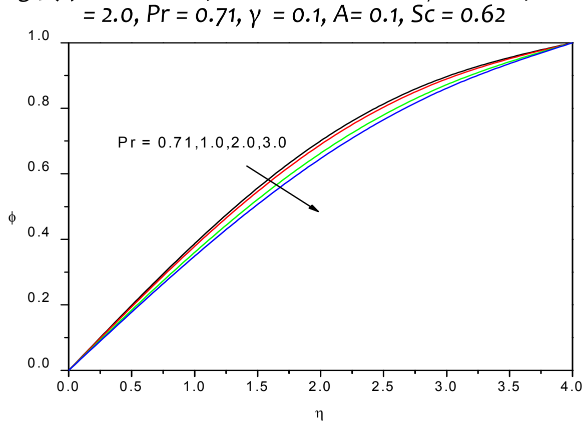

Fig.4(b) Variation of the concentration ϕ with Pr for Gr = 2.0, M=1.0, γ = 0.1, A = 0.1, Sc = 0.62.

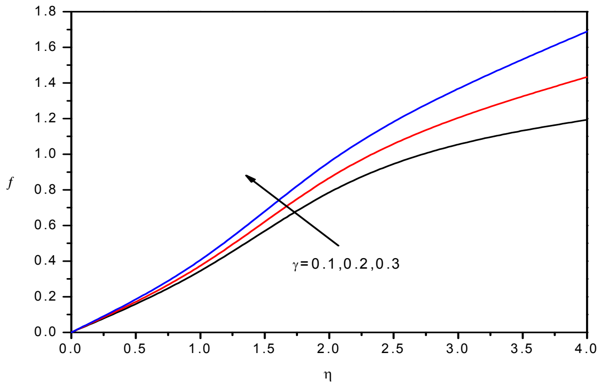

Fig .5(a) Variation of the velocity component f with γ for Gr=2.0,M=1.0, Pr = 0.71, A = 0.1, Sc = 0.62

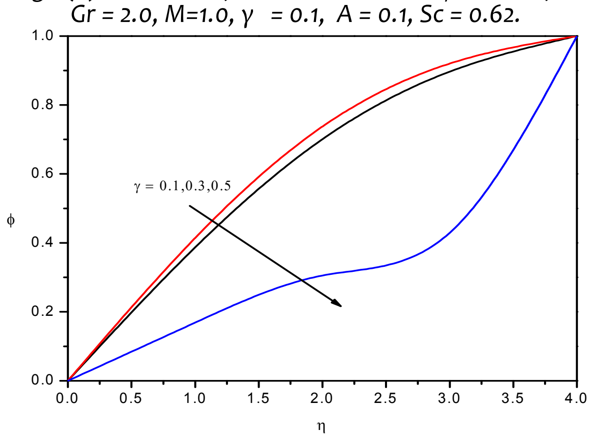

Fig .5(b) Variation of the concentration ϕ with γ for Gr=2.0,M=1.0, Pr = 0.71, A = 0.1, Sc = 0.62.

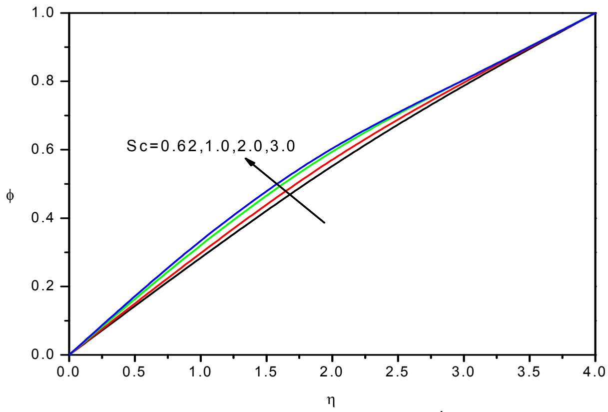

Fig.6 Variation of the concentration ϕ with Sc For Gr=2.0, M=1.0, Pr = 0.71,γ = 0.1, A= 0.1

Fig. 1(a) &1(b) illustrate the effect of varying the Grashof number on the velocity and concentration profiles. The increase in Grashof number (Gr) produces increase in the velocity and concentration of the fluid which agrees with natural phenomena because of the buoyancy force which assists the flow. This result is in agreement with that Omowaye and Koriko (2010), Sharma and singh (2009) obtained.

Fig.2 (a) & 2(b) revel the effect of permeability parameter (A) on the velocity, and Concentration profiles respectively. These Fig.2 (a) &2(b) confirm that the velocity and Concentration decreases with increase in permeability parameter. This is in excellent agreement with Omowaye and Koriko (2010), Pathak and Maheswari (2006), Kafoussians (1989) obtained.

For various values of the magnetic parameter M, the velocity component f and concentration are plotted in Fig.3 (a) &3(c). It can be seen that as M increases, the velocity and concentration decreases. As M increases, the Lorentz force, which opposes the flow, also increases and leads to enhanced deceleration of the flow. This result qualitatively agrees with the expectations, since the magnetic filed exerts a retarding force on the free convection flow. Fig.3 (b) shows that the temperature profiles for different values of magnetic parameter M. It is observed that the temperature increases with an increase in the magnetic parameter M.

Fig.4 (a) illustrates the velocity component f for different values of the Prandtl number Pr. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. From Fig.4 (b) it is observed that an increase in the Prandtl number results a decrease of the concentration with in the boundary layer.

The influence of the ratio of free stream velocity parameter to parallel wall parameter (γ) on the velocity f and concentration are plotted in Figs.5 (a) &5(b). As the ratio of free stream velocity parameter to parallel wall parameter (γ) increases the velocity increases and concentration decreases.

The influence of the Schmidt number Sc on the concentration profiles are plotted in Fig. 6. As the Schmidt number increases the concentration increases. This causes the concentration buoyancy effects to increases.

Table 1: Variation of f'', - θ', ϕ' at the wall with Gr, M for Pr = 0.71, b = 0.1, A = 0.1, Sc = 0.62

Gr	M	f''(O)	- θ'(0)	ϕ'(0)
1	1	0.717447	0.360711	0.347031
2	1	1.23316	0.415943	0.395955
3	1	1.70167	0.458034	0.433549
4	1	2.13914	0.492415	0.464453
5	1	2.55392	0.521675	0.490888
1	2	0.661677	0.344914	0.333069
2	2	1.12321	0.393232	0.375681
2	3	1.09994	0.0434236	0.653428

Table 2: Variation of f'', - θ', ϕ' at the wall with Pr, γ, A, Sc for Gr = 2.0, M = 1.0

Pr	γ	A	Sc	f''(O)	- θ'(0)	ϕ'(0)
0.71	0.1	0.1	0.62	1.23316	0.415943	0.395955
0.78	0.1	0.1	0.62	1.22336	0.428459	0.393697
1.0	0.1	0.1	0.62	1.19546	0.464667	0.387311
1.5	0.1	0.1	0.78	1.14441	0.5333877	0.375874
0.71	0.2	0.1	1.0	1.32573	0.431592	0.410139
0.71	0.3	0.1	0.62	1.43065	0.448311	0.425382
0.71	0.1	0.2	0.62	1.21981	0.413257	0.39355
0.71	0.1	0.3	0.62	1.20707	0.410675	0.391241
0.71	0.1	0.1	0.78	1.23316	0.415943	0.431141
0.71	0.1	0.1	1.0	1.23316	0.415943	0.476777
0.71	0.1	0.1	1.5	1.23316	0.415943	0.568391

The effect of various parameters on the functions f'', θ' and ϕ' at the plate surface is tabulated in Tables 1 and 2. It is observed that the magnitude of the wall temperature gradient increases as Prandtl number Pr or the ratio of free stream velocity parameter to parallel wall parameter (γ) increases, while it decreases as the magnetic parameter M or the permeability parameter A increases. The magnitude of the wall concentration gradient decreases as the magnetic field parameter M increases, while it increases with an increase in the Schmidt number Sc. Further more, the negative values of the wall temperature and concentration gradients, for all values of the dimensionless parameters, are indicative of the physical fact that the heat flows from the wall surface to the ambient fluid.

CONCLUSIONS

In this study, similarity solutions for free convective flow between two parallel porous walls with variable temperature are studied theoretically. A set of similarity equations governing the fluid velocity, temperature and concentration was obtained by using an appropriate similarity transformation. The dimensionless locally similar and non-linear ordinary differential equations are solved numerically by using shooting method. From the present numerical investigation we may conclude that:

The velocity component f decreases with an increase of Magnetic field parameter M, the permeability parameter A, Prandtl number Pr.

The velocity component f increases with an increase of Granshof number Gr and the ratio of free stream velocity parameter to parallel wall parameter γ.

The temperature component θ increases with an increase of Magnetic field parameter M.

The concentration component ϕ increases with an increase Schmidt number Sc, Grashof number Gr.

The concentration component ϕ decreases with an increase the permeability parameter A, the Prandtl number Pr.

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