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DETERMINATION OF FRICTION COEFFICIENT IN TRANSITION FLOW REGION FOR WATERWORKS AND PIPELINES CALCULATION

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ABSTRACT: *Analysis was done on recent results in obtaining efficient formula for the friction coefficient, particularly in the transition flow region. Accuracy and complexity of 15 explicit approximations of the Colebrook-White equation for determining the friction coefficient has been studied. Maximum relative error was determined, for each approximation, and given in the table together with their complexity and complexity index. It was demonstrated that these approximations obtained by fitting the Moody diagram obtained using the C-W formula that yielded from Nikuradse's measurements are unsuccessful in transition flow region and cover only tthe turbulent flow above Re = 4000. Investigations are described, that have succeeded in eliminating these drawbacks and determine a formula for the friction coefficient for all Re numbers ($0 \le Re \le 10^8$), and all values of the relative roughness that covers all six curves of Nikuradse's measurements and that is more precise than the C-W formula and all up to date published equations. This formula doesn't require constraint for its use and is reccomended for efficient calculation of hydraulic losses in waterworks and other closed pipelines.*
KEYWORDS: *hydraulic losses, friction coefficient, relative roughness, flow in pipelines*

INTRODUCTION

Hydraulic calculations of waterworks irrespective on the degree of science development and proficiency of computer methods, still have certain indetermination. This is true, particularly in water networks where, due to technologic constraints, velocity of the flow is rather small. The focus is on unsolved problems, arising in determining the most appropriate formula for hydraulic losses, due to friction in the fluid flow. This whole topic, can be divided into three sections, two of which are completely determined in the analytical sense, while the third one is full with problems, that recently provoque strong discussions. As it is known, there are no problem regarding laminar flow and developed turbulence. The (third) unresolved section is the transition region between these two, laminar and developed turbulent, flow regimes.

Before 1939 when Colebrook-White [1] eq'n was published, for turbulent regime in smooth pipes, Prandtl equation was widely used implicit in friction factor. Prandtl derived a formula from the logarithmic velocity profile and available experimental data on smooth pipes:

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{2.51}{Re\sqrt{\lambda}}\right) = 2\log\left(Re\sqrt{\lambda}\right) - 0.8 \qquad (1)$$

The development of approximate equations, for calculations of friction factor in rough pipes, began with Nikuradse's turbulent pipe flow investigations, in 1932. and 1933. The results of his experiments are shown in figure 1. The tests were conducted in the region $500 < Re < 10^6$ for 6 different relative roughnesses ε/D defined by average roughness and pipe diameter. For turbulent regime in rough pipes von Karman's relation was widely used:

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{\varepsilon}{3.71D}\right) = 1.14 - 2\log\left(\frac{\varepsilon}{D}\right) \qquad (2)$$

Greek ε is the equivalent Nikuradse's sand-grain roughness value for the inner surface of pipe (or the so called uniform roughness). Prandtl's and von Karman's relations are known as NPK (Nikuradse–Prandtl–Karman) equations [2]. Colebrook later performed the experiments upon sixteen spun concrete-lined pipes and six spun bitumastic-lined pipes ranging in the diameter from 101.6 to 1524 mm with average surface roughness values between .043 and .254 mm.

In an attempt to classify the data available at the time and those from experiment conducted by himself and his colleague White developed a curve fit to describe transitional roughness:

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{\varepsilon}{3.71D} + \frac{2.51}{Re\sqrt{\lambda}}\right) \qquad (3)$$

Colebrook-White equation describes a monotonic change in the friction factor from smooth to fully rough as it is clearly observed in figure 2. It is valid especially for commercial steel pipes. Colebrook-White equation is also basis for widely used Moody's [14] chart. Many do seem to believe that the

Moody's diagram has surprisingly good properties. In fact, all it is a plot of solutions of nonlinear transcendental Colebrook-White (C-W) equation. In principle, Moody diagram is used for solution of three types of problems, i.e. a problem in which head loss is unknown, in which volume flow rate is unknown and in which diameter is unknown. Solving for unknown head loss, with Moody's diagram, is relatively straightforward but the use of implicit Colebrook-White formula complicates solving all three types of problems.

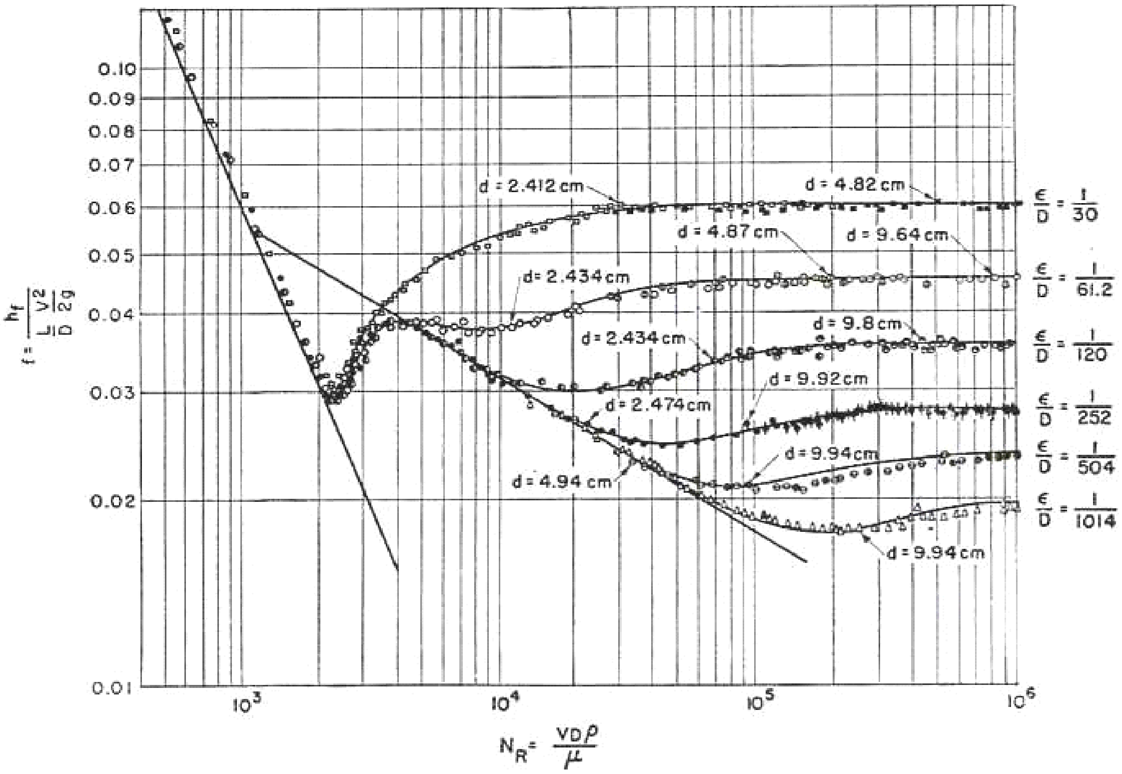

Figure 1. Experimental results of Nikuradse [14]

ACCURACY AND COMPLEXITY OF EXPLICIT APPROXIMATIONS TO COLEBROOK-WHITE EQUATION

There were some early expressions of Colebrook-White equation in explicit form which were not particularly accurate, but in the years 1973–1984 there was a flurry of activity obtaining more accurate approximations, that appeared mainly in the chemical engineering literature.

The equations, describing friction in the transition regime, were analysed by many authors (Barr, Swamee & Jain, ... etc). Barr, e.g. replaced the C-W equation with his own formula, with ± 1 % error. The equation holds in the same interval as the C-W equation, i.e. only for $4x10^3 < Re < 10^8$ [3]. Swamee & Jain have proposed their own equation, which is confined to an even narrower region i.e. $5x10^3 < Re < 10^8$ and to relative roughness $10^{-6} < \varepsilon/D < 10^{-2}$ [4]. Maximal percentage (relative) errors of all the available approximations over the entire range of applicability of Colebrook-White equation is present in the first column of table 1.

For many applications, simpler but less accurate explicit equation will be sufficed. Sometime, simplicity is sacrificed for - excessive accuracy. To find balance between these two extremes Zigrang and Sylvester [8] introduced "complexity" as the number of (all) algebraic notations calculator key strokes required to solve some equation. Complexity index was defined as the quotient of key strokes required for an approximation and the least complex one. Complexity and complexity index of all the available explicit approximations are given in table 1, too.

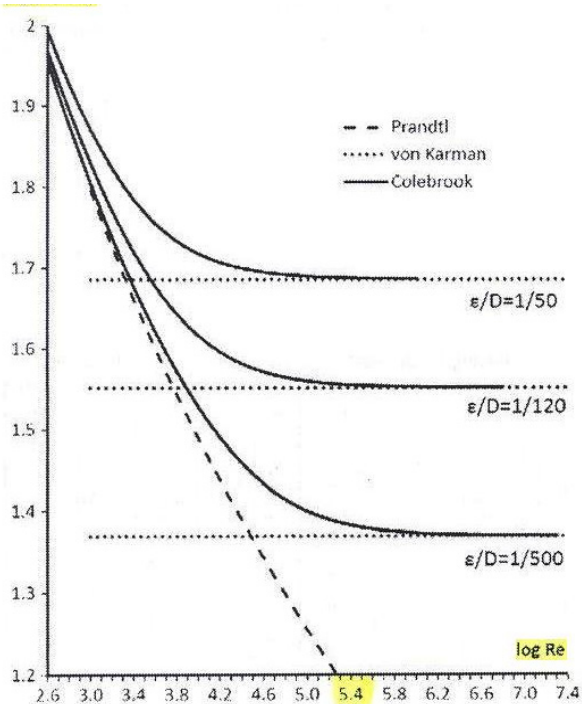

Figure 2. Colebrook-White equation and its components [2]

Table 1. Maximum relative error and complexity index of approximations to C-W equation

	δ_{max} (%)	complexity (–)	complexity index (–)
Romeo et al. [11]	0.134	125	4.05
Buzelli [15]	0.138	104	3.38
Vatankhah et al. [16]	0.147	77	2.51
Barr [3]	0.277	80	2.59
Serghides [17]	0.354	107	3.47
Chen [7]	0.356	91	2.97
Sonnad & Goudar [12]	0.803	67	2.18
Papaevangelou et al. [18]	0.825	67	2.18
Zigrang & Sylvester [8]	1.007	47	1.53
Haaland [9]	1.408	35	1.13
Jain [6]	2.044	35	1.13
Swamee & Jain [4]	2.044	36	1.17
Manadilli [10]	2.065	44	1.42
Churchill [5]	2.172	31	1.00
Avci & Karagoz [19]	4.186	47	1.53

Although C-W equation and approximations to this equation have extremely wide application, one should not disregard the constraints, in the interval of Re numbers - $4000 \le Re \le 10^8$. The upper limit of the interval is not a problem but the lower limit (Re = 4000) is. Obviously from the plot in figure 3, showing comparison of C-W equation and Nikuradse's data, value at the lower limit is too high which confines the equation to be applicable at very high Re numbers.

If we compare the values of the friction coefficients, obtained by the C-W formula and those of the other authors, with Nikuradse's measurements, we discover that the obtained values in the range $1000 \le Re \le 10000$ are not realistic. Lower or higher values of friction coefficients ceertainly lead to large errors in hydraulic calculations and obtaining unrealistic flowrates.

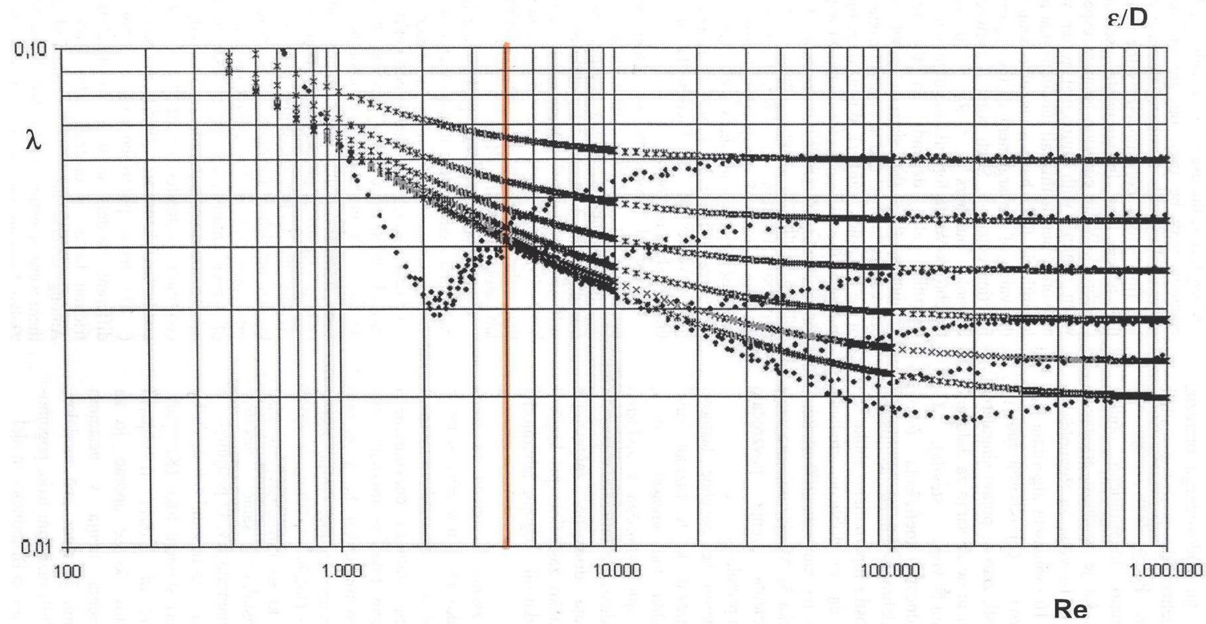

Figure 3. Colebrook-White equation and Nikuradse's measurements [2]

All cited authors, were trying to approximate the C-W equation by fitting the points that are given in Moody's diagram. None of them recognized the fact, that Moody's diagram was in fact formed from data obtained by using the C-W formula which is in fact an approximation of the Nikuradse's harp and not a too sucessful one (which is widely recognised) due to high degree of inconsistence with the original in the region of transition to turbulence. Also all the cited approximations cover only turbulent flow region, and not right after critical Re number but far distantly above 4000. By the authors of articles [2,13], these shortcomings have been eliminated, and the friction coefficient has been determined for all Re numbers.

DETERMINATION OF FRICTION COEFFICIENT USING SWITCHING FUNCTIONS

In the experimentally obtained diagram s.c. Nikuradse's harp, are clearly distinguished four regions, showing the friction coefficient (function) performance, strongly different from the other regions. The authors of [13] searched appropriate functions for each of these regions.

To eliminate the influence of particular functions, outside their region of validity, switching functions have been introduced, that switch the particular formula on and off, when needed. Smoothness and continuity is assured only by multiply differentable functions. The possible solutions have been chosen in the form of exponential function e^x and all its derivatives.

In figure 1 is obvious that the friction coefficient has four functional regions, clearly defined three regions (I, III i IV), and the unclear II critical region. The first, laminar region, is defined by a linear function in log-log diagram, representing the simple correlation:

$$\lambda = \frac{64}{Re} \tag{4}$$

The fourth is the fully developed turbulence region, where the friction coefficient depends only on relative roughness. Prandtl and Karman [14] represented this dependence as:

$$\lambda = \frac{0.25}{\left(\log\left(\frac{\varepsilon}{3.71D} \right) \right)^2} \tag{5}$$

The third is the transitional region and turbulence is still developing so the friction coefficient depends on both Re and ε/D. On Moody's diagram, i.e. on Nikuradse's harp it is obvious that if we travel leftwards, i.e. from high towards low Re numbers, λ rises slowly. Envelope of the rising is the line describing the performance of the friction coefficient in the hydraulic smooth flow regime and that has been expressed in the analytical form by Blasius [14] as:

$$\lambda = \frac{0.3164}{\sqrt[4]{Re}} \tag{6}$$

The second i.e. critical region is in the 2100 ≤ Re ≤ 5000 interval and can be seen as well on the Nikuradse's diagram (figure 1) as the line connecting the laminar (I) and the transitional (III) flow. In this region the friction coefficient depends on both Re and relative roughness.

In general, to describe four regions one needs – four characteristic functions. However, it was supposed that the II i.e. critical region (between laminar and turbulent regions) does not have a characteristic describing function, but that is merely a result of overlapping of laminar and turbulent regions. Special equations were formed solely for the three flow regimes (regions I, III & IV) and were connected by using three switching functions. Only two functions are not sufficient because in the developed turbulence in zone IV Blasius term for zone III should be switched off, and it is necessary to be switched off in the zone II i.e. critical region. However, for the end regions only one switching function is obviously sufficient.

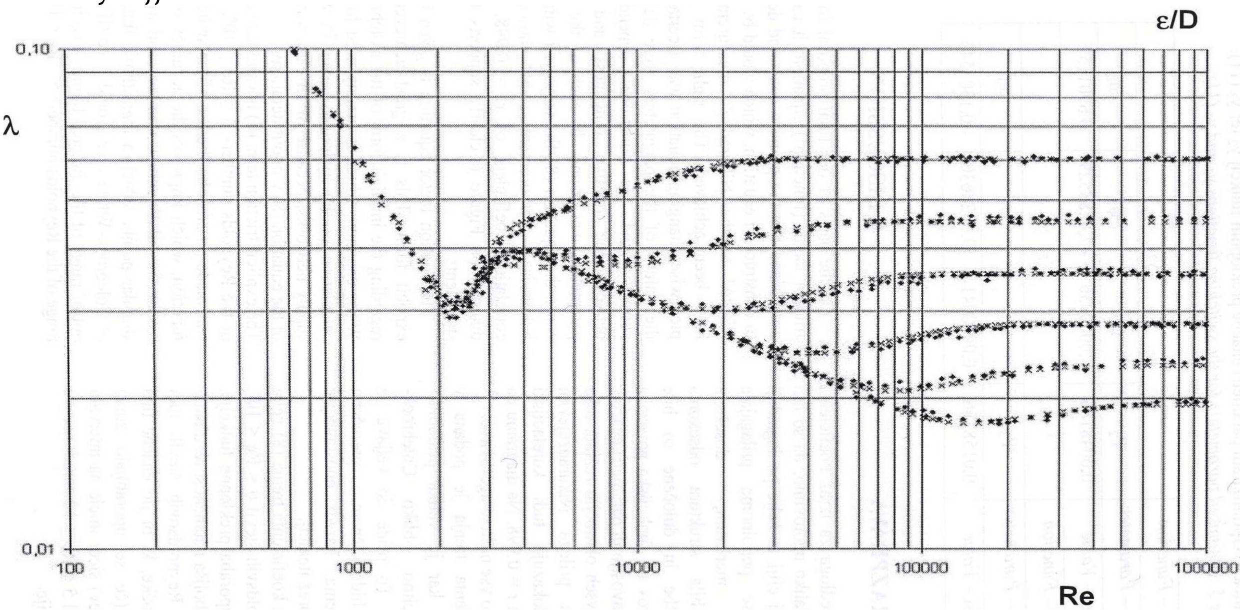

Figure 4. New (generalized) equation for λ and Nikuradse's results [13]

Following this logic the friction coefficient equation consists of three terms. The first term of the equation covers the region of laminar flow, the second term is the equation for transition region, and the third term is the equation for the hydraulically rough region. As expected, the simple sum of these three equation terms does not bring fruitful results. Therefore introduced were the switching functions which enabled the optimal approximation of Nikuradse's results. The analytical expression for the friction coefficient therefore takes the form:

$$\lambda = \frac{a}{\text{Re}} \cdot (1 - y_1) + \frac{b}{\text{Re}^{\beta}} \cdot (y_1 - y_3) + \frac{c}{\log^2\left(\frac{\varepsilon}{k_\varepsilon \cdot D}\right)} \cdot y_2 \quad (7)$$

The switching function y1 after the performed analysis is represented by the equation:

$$y_1 = e^{-e^{-(\gamma \text{Re} + \delta)}} \quad (8)$$

The switching functions y2 and y3, dependent on relative roughness and Re number, are:

$$y_2 = e^{-e^{-\left(\left(\Psi_2 \frac{\varepsilon}{D} + \Omega_2\right)\text{Re} + \left(\psi_2 \frac{\varepsilon}{D} + \omega_2\right)\right)}} \quad (9)$$

$$y_3 = e^{-e^{-\left(\left(\Psi_3 \frac{\varepsilon}{D} + \Omega_3\right)\text{Re} + \left(\psi_3 \frac{\varepsilon}{D} + \omega_3\right)\right)}} \quad (10)$$

Diagram of the newly proposed (generalized) friction equation (7) is shown in figure 4.

Discussion of the results

One can conclude that the authors of [13] have fulfilled the firstly posed goal on constructing these equations that would follow the Nikuradse's data, completely. The authors have formed the structure of an equation, different by each sector and determined the values of parameters and the functional relationship, between the relative roughness and the equation's parameters. This equation (7) in all flow regions follows the Nikuradse's results perfectly which is proved by the correlation coefficient $r = 0.998$. In figure 4 all these characteristics can be observed.

The friction coefficient equation is outlined in its explicit form which is definitely a large step ahead, in comparison with the implicit Colebrook-White equation. It is important, especially when performing hydraulic calculation of waterworks systems, due to less complex solution of the novel generalized equation (7) in explicit form and especially without iterations.

The explicit friction coefficient equation (7) is valid: for Re numbers $0 \le \text{Re} \le 10^8$. This was enabled by using the switching functions which increase the accuracy of results for all values of Re numbers, and eliminate all the singular points that the Colebrook-White type equations would have outside their own prescribed limits in the interval (range) $5 \le \text{Re} \le 10$.

Conclusions

Accuracy and complexity of 15 explicit approximations of the Colebrook-White equation for determining the friction coefficient has been studied. Maximum relative error was determined for each approximation and outlined in the table, together with its complexity and complexity index. It has been concluded that all authors tried to approximate the C-W equation by fitting points that are given in Moody's diagram, that was obtained by using the C-W formula itself, which is in fact an approximation of Nikuradse's measurements. These attempts haven't been too successful, due to strong inconsistence with the original in transition to turbulence region. Also, all these approximations cover only the turbulent flow region, but only above Re=4000.

The shortcomings have been eliminated by the authors of article [13] by obtaining the friction coefficient function valid for all Re numbers. In the experimentally obtained Nikuradse's plot one clearly observes four regions, with different shapes of the friction coefficient function in each of them. Many different performances means that a simple function, i.e. dependence of friction coefficient λ on Re number and on relative roughness cannot be found. Inappropriate for the transition region construction in the form of C-W equation additionally confirms that fact. Therefore, it was reasonable to search the functions for each of these regions separately.

Based on the digitalized Nikuradse's measurements the analytical expression for the friction coefficient function has been obtained in explicit form, defined for Re numbers $0 \le \text{Re} \le 10^8$ and all values of relative roughnesst. New equation follows all six curves of the Nikuradse's harp and is more accurate than C-W equation and all up to date known published equations.

In this manner, a very high value of the correlation coefficient, $r = 0.998$, was attained. The new equation has no constraint on applicability since it is valid at all Reynolds numbers and all values of relative roughness. Therefore, it has been concluded, that the new equation can be fully recommended for hydraulic calculation of waterworks and other closed pipelines.

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