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INFLUENCE OF THE SUSPENSION ELASTIC CHARACTERISTICS UPON THE RAILWAY VEHICLES MOTION IN A CURVE

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ABSTRACT: *Influence of the elastic characteristics of the wheelsets steered system upon the curving of the bogie is herein investigated. To solve the equilibrium equations of the quasi-static movement of a bogie in a circular curve, a new approach is suggested, based on defining the secant and chord limit positions. This is how the issue of non-determining the bogie position is solved, due to the clearence in the track. The curving position of the bogie is established by relying on the unbalanced centrifugal forces corresponding to such limit positions. By implementing the method of solving the equilibrium equations, the case of a bogie with elastic steered wheelsets is being analysed, via reporting the results to the ones for a bogie with fixed wheelsets. It has been shown that the performance of the bogie with elastic steered wheelsets while moving in a curve is conditioned by a set of parameters, thus pointing at the influence of the curve radius, velocity and the bogie axle base. This will lead to situations when the performance of a bogie with fixed wheelsets is superior to the bogie with elastic steered wheelsets.*
KEYWORDS: *railway vehicle, bogie, suspension, curve, limit positions*

INTRODUCTION

Curves are critical areas in the railway track, raising issues regarding circulation safety, running behaviour [1, 2], comfort of passengers, noise [3], wear of the rolling surfaces and of the wheel flanges, as well as the rails, track skidding or the wear of its components [4, 5, 6].

When bogies are designed, the objective is to provide hunting stability in tangent track, without any harm done to the vehicle dynamic behavior during curve circulation. In order to improve the running behavior, solutions have been constantly searched for so that the vehicles comply with these requirements, but the ‚conflict' between moving in a curve and stability in the alignment is still an ongoing topic for the research in this field.

The bogies with elastic steered of the wheelsets have been thought of to solve the above-mentioned problem. While the wheelsets have the ability to radially orient themselves in a curve and transversally move against the track axle, the conditions of having a skid-free rolling ('pure rolling') are present. As a result, the conditions related to the wear decrease in the wheel's and rail's rolling profile in a curve are rendered.

An important role in guaranteeing the performance regarding the transversal stability and improved results in the motion in a curve is played by the primary suspension elasticity. A correct design of suspension in correlation with the wheels' rolling profile and the vehicle body may result into solving the conflict given by the guidance in a curve and stability in the alignment [7]. The relevant adoption of the rigidity of springs in the wheelsets steered system can lead to the desired performance during curve motion. It should be mentioned that, for a better curving, the required values of the suspension rigidities do not allow reaching a transversal stability to be sufficient to the circulation in alignment. To meet the requirements of a good curving at small radius and a high critical speed in the alignment is not an easy task with bogies of classical construction, but the need is for achieving a ‚conciliating bogie' [8, 9, 10].

The use of the latest technologies relying on bogies with a controlled creeping wheelset will help reach a good compromise between the stability in the alignment and the wheelsets curving, thus providing safety against derailment, decrease in rolling surfaces wear and of the wheels and rails guide flanges and increase of the interval between two successive wheels reprofilings [11]. While using this technological innovation, the same conclusion is drawn – that the magnitude of the critical speed in tangent track and the ability of radial orientation of the wheelsets in a curve depend a great deal on the elastic characteristics of the bogie primary suspension [12].

The paper examines the influence of the elastic characteristics of the wheelsets suspension upon curving of a bogie, by comparing the results to a bogie with fixed wheelsets.

The simulation of motion in a curve is difficult, mainly due to the nonlinearities of the wheel-rail contact. It is about the nonlinearity derived from the clearance of the wheelset in the track and the one given by the geometry between wheels and rails [1,13]. Likewise, the friction force has a nonlinear variation, depending on the creeping [14]. Under these circumstances, the issue of non-determination of *the bogie position in a stationary behavior emerges. To overcome such difficulty, a new approach is required for the equilibrium equations – to introduce the concepts of limit positions that are defined as the positions in which the rear wheelset of a vehicle is in a purely geometric contact with either the inner rail (the secant limit position) or the outer rail (the chord limit position). Starting with the unbalanced centrifugal forces that allow reaching the two limit positions, the position of the bogie in the track may be directly established afterwards, in dependence of the effective unbalanced centrifugal force. This method helps investigate the performance of motion in a curve for a elastic steered wheelset bogie versus a bogie with fixed wheelsets, the velocity behavior and the bogie axle base.*

THE MECHANICAL MODEL AND THE EQUILIBRIUM EQUATIONS

Figure 1 shows the mechanical model of a two-wheelset bogie in a stationary motion with a constant velocity V along a circular curve of radius R. The wheelsets are linked to a body frame of the bogie via linear springs of stiffness k_x, k_y in the longitudinal and lateral direction, respectively. The bogie base is 2a, the transversal base of the suspension 2b and the distance between the rolling circles of the wheels is 2e.

The position of each wheelset while moving along a curve is determined by the displacements y_1 and y_2 respectively of the wheelsets' centre in respect to its local reference moving frame and the attack angles α_1 and α_2 reported to the radial position (fig. 2). Similarly, the position of the body frame of the bogie, reduced to its transversal axle, is given by the lateral displacement y_b against the track axle and the rotation α_b, compared to the tangent position to the curve. In a conventional way, the positive sign of the angles $\alpha_{1,2}$ and α_b is considered to show the rotation of the wheelsets compared to the radial position, namely of the bogie frame against the tangent direction to the curve, towards the curve exterior (counterclockwise direction). Also, the lateral displacements $y_{1,2}$ and y_b towards the exterior of the curve will have a positive sign.

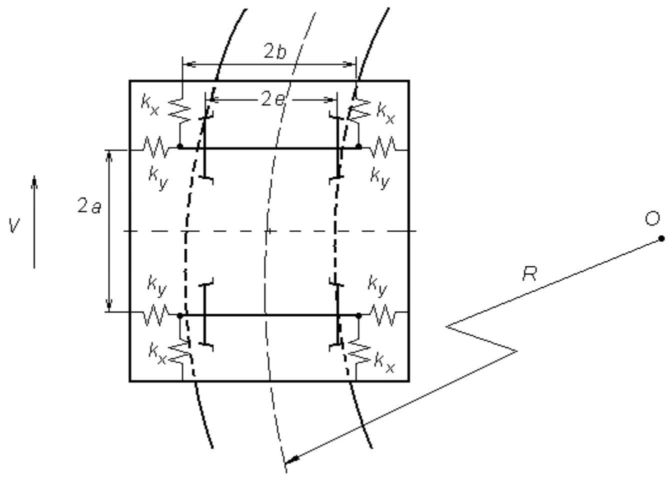

Figure 1. Bogie in a quasi-static behavior during motion in a circular curve

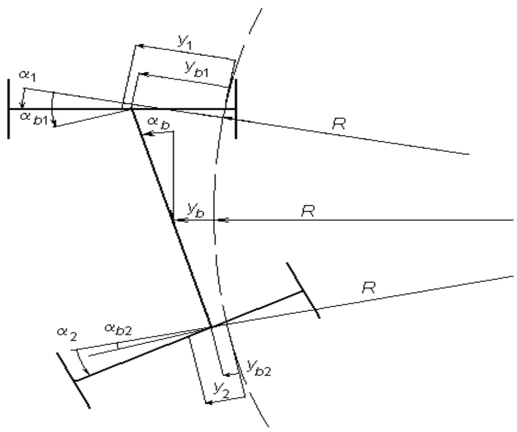

Figure 2. Coordinates of the bogies in the curve

It is supposed that the attacking wheel of the leading wheelset is in contact with the flange of the high rail. The position of the rear wheelset may be in contact with the high rail or the low rail or even between both rails, depending on the equilibrium position of the bogie. The three cases are known as the so-called the chord position, the secant position and the free position, respectively.

If the wheelset clearance in the track is σ, corresponding to the three positions in the curve, then the lateral displacements of the wheelsets centres will be as follows:

- *for the chord position $y_1 = \sigma/2$, $y_2 = \sigma/2$;*
- *for the secant position $y_1 = \sigma/2$, $y_2 = -\sigma/2$;*
- *for the free position $y_1 = \sigma/2$, $-\sigma/2 < y_2 < -\sigma/2$.*

It has been considered that the two wheelsets are free, as no traction or braking forces are being applied from the exterior. The possible load transfers are overlooked and the hypothesis that the bogie wheels are equally loaded with the static load Q_0 is being adopted. To make the equilibrium equations simpler, the action of the centring force is neglected.

Likewise, the assumption is that the entire load on wheel acts on the supporting point during the exhaustion of the clearence in the track. The effect of the wheel flange is replaced by a guidance roller that introduces the leading forces P_1 and P_2, corresponding to the two wheelsets. This hypothesis has been adopted by Heummann [15] and recommended by Sebeşan [2] and others.

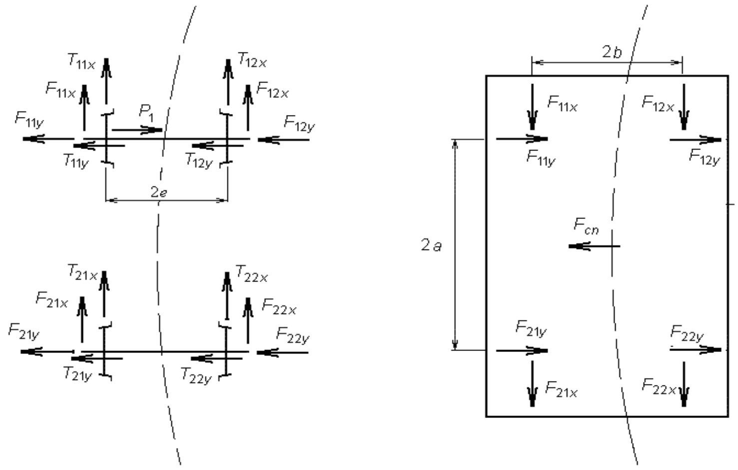

Figure 3. Forces acting on wheelsets and bogie frame in a curve, free position

The position of the bogie curving is given by the action of the friction forces, the elastic forces, of the unbalanced centrifugal force and by the leading forces when the wheelset clearence is completed in the track.

Figure 3 shows the forces acting on the wheelsets and on the bogie frame, for the free position. The elastic forces between the wheelsets and the bogie frame $F_{ijx,y}$ are visible, the friction forces on the longitudinal direction T_{ijx} and transversal T_{ijy}, the unbalanced centrifugal force F_{cn} and the leading force P_1.

It should be mentioned that it is only the leading force P_1 acting on the bogie when in a free position. For the extreme curving position, there will be the leading force P_2 on the second wheelset, as a reaction of the inner rail when the vehicle is in the secant position or as a reaction of the outer rail for the chord position (figure 4).

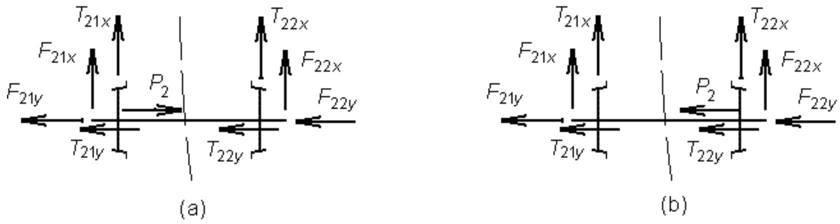

Figure 4. Forces acting on the bogie's second wheelset while curving in the chord position (a) and in the secant position (b).

The elastic forces depend on the rigidities k_x and k_y of the elastic elements in the wheelsets' steered system and on the relative position between the bogie frame and the wheelsets

$$F_{i1x} = -F_{i2x} = bk_x(\alpha_{bi} - \alpha_i)\text{, for } i = 1, 2; \quad (1)$$

$$F_{i1y} = F_{i2y} = k_y(y_{bi} - y_i)\text{, for } i = 1, 2, \quad (2)$$

where $$\alpha_{bi} = \alpha_b \pm \frac{a}{R},\ y_{bi} = y_b + \frac{a^2}{2R} \pm a\alpha_b \quad (3)$$

represent the rotation angles and the transversal displacements against the track wheelset, of the bogie frame, reduced to its longitudinal axle, against the two wheelsets.

The friction forces can be calculated by using the non-linear formula given by Chartet [16]

$$T_{ijx} = -\frac{\varkappa\nu_{ijx}Q_o}{\sqrt{1+(\varkappa\nu_{ij}/\mu)^2}},\ T_{ijy} = -\frac{\varkappa\nu_{ijy}Q_o}{\sqrt{1+(\varkappa\nu_{ij}/\mu)^2}}, \quad (4)$$

where κ stands for the creepage coefficient, μ is the adherence coefficient and ν_{ij} is the creepage of the wheel 'ij' with the components ν_{ijx} and ν_{ijy} on the longitudinal and transversal directions.

Should the non-linearity of the geometric contact is considered, namely the variations in the rolling effective radius and of the contact angles on both wheels, for the displacements y_1 and y_2 of the two wheelsets compared to the median position, the longitudinal creepage are written as such

$$\nu_{ijx} = \pm\left[\frac{e}{R} - \frac{\Delta r_{ijx}}{r}\right], \quad (5)$$

where $$\Delta r_{i1} = r_{i1} - r,\ \Delta r_{i2} = r - r_{i2}, \quad (6)$$

with r_{ij} – the radius of the rolling circles that depend on the wheelset displacement and r – the wheel radius when the wheelset takes a central position between the rails.To calculate the variations of the *radiuses in the effective rolling circles for the transversal displacement of the wheelset Δr_{ij}, the method of the contact curve can be applied [1].*

The creepage transversal components are

$$\nu_{yzij} = -\alpha_i \,. \tag{7}$$

The unbalanced centrifugal force derives from the distribution on the bogie of the unbalanced centrifugal force that acts upon the vehicle carbody and the unbalance centrifugal force acting upon the bogie. This force depends on the curve radius R, the superelevation of the track in the curve h and on the velocity V

$$F_{cn} = Mg\left(\frac{V^2}{gR} - \frac{h}{2e}\right), \tag{8}$$

where M stands for half of the vehicle mass, and g represents the acceleration of gravity.

Should the vehicle travels at the so-called equilibrium velocity in the below equation

$$V = V_o = \sqrt{\frac{ghR}{2e}}\,, \tag{9}$$

then the unbalanced centrifugal force is annuled.

When the velocity is smaller than the equilibrium velocity, the unbalanced centrifugal force acts towards the inner curve, thus deplacing the bogie into that direction. Should the velocity is higher than the equilibrium velocity, the unbalanced centrifugal force will point to the outer curve, and so will the bogie. The equilibrium equations for the bogie free position are as below

$$\sum_{j=1}^{2}(T_{1jy} + F_{1jy}) - P_1 = 0\,; \tag{10}$$

$$\sum_{j=1}^{2}(T_{2jy} + F_{2jy}) = 0\,; \tag{11}$$

$$\sum_{j=1}^{2}(-1)^{j+1}(eT_{1jx} - bF_{1jx}) = 0\,; \tag{11}$$

$$\sum_{j=1}^{2}(-1)^{j+1}(eT_{2jx} - bF_{2jx}) = 0\,; \tag{13}$$

$$F_{cn} - \sum_{i=1}^{2}\sum_{j=1}^{2}F_{ijy} = 0\,; \tag{14}$$

$$\sum_{i=1}^{2}\sum_{j=1}^{2}\left[(-1)^{j+1}bF_{ijx} + (-1)^{i+1}aF_{ijy}\right] = 0\,. \tag{15}$$

Likewise, the above equations represent the equilibrium requirement for the chord position, should the equation (11) rewrites as

$$\sum_{j=1}^{2}(T_{2jy} + F_{2jy}) - P_2 = 0 \tag{16}$$

or for the secant position, but the relation (11) reads as below

$$\sum_{j=1}^{2}(T_{2jy} + F_{2jy}) + P_2 = 0\,. \tag{17}$$

After replacing the friction forces and the elastic ones in the equilibrium equations, a system of non-linear equations is obtained

$$Aq = B\,, \tag{18}$$

*where **q** stands for the column vector of the bogie displacements and of the leading force(s), **A** is a matrix depending on the wheelsets' displacements and **B** is a column vector of the free terms, including the unbalanced centrifugal force.*

*Corresponding to the three situations of the bogie curving, the vector **q** will be:*

- the bogie is in the free position

$$q = \begin{bmatrix}\alpha_1 & \alpha_2 & y_2 & y_b & \alpha_b & P_1\end{bmatrix}^T; \tag{19}$$

- the bogie is in the chord or secant position

$$q = \begin{bmatrix}\alpha_1 & \alpha_2 & y_b & \alpha_b & P_1 & P_2\end{bmatrix}^T. \tag{20}$$

*Also, the structure of matrix **A** and of the column vector **B** will change, in dependence of those three cases.*

The nonlinear equation (18) can be solved with an iterative method.

DETERMINING THE CURVING POSITION

It should be noticed that the issue of the bogie curving is nonlinear, due to the wheelset clearance in the track. This is why the equilibrium equations need to be changed in dependence on the bogie position, namely secant, free or chord. Moreover, for every case, the equilibrium equations are nonlinear due to wheels/rails contact, including the contact geometry and the friction coefficient.

In other words, for a certain value of the unbalanced centrifugal force, the position of the bogie cannot be directly determined, due to the non-linearity of the wheelset clearance in the track. This issue can be taken care of by introducing the limit positions of the wheelset in the track, which is the secant and chord limit positions. This is how it can be calculated the limit centrifugal force at which the bogie shifts from the secant position to the free and to the free to the chord position. And the speed ranges, where the corresponding equilibrium equations are valid, can be delimitated.

To calculate the limit unbalanced centrifugal force F_{cns} that defines the bogie placement in the curve in a secant position, it is considered that the bogie is in a free position ($P_2 = 0$) but the second wheelset has exhausted the clearance in the track $y_2 = -\sigma/2$. If the bogie is in a free position in a curve ($P_2 = 0$), but the second wheelset is displaced compared to the track axle by $y_2 = \sigma/2$, then the bogie finds itself at the limit between the free and chord positions, while the limit unbalanced centrifugal force is now F_{cnc}.

*The limit values of the unbalanced centrifugal force are calculated starting from the equilibrium equations that correspond to the bogie's free position (10-15), rewritten in the matriceal form (18), where the column vector **q** is as follows*

- for the limit secant position

$$q = \begin{bmatrix} \alpha_1 & \alpha_2 & y_b & \alpha_b & P_1 & F_{cns} \end{bmatrix}^T ; \tag{21}$$

- for the limit chord position

$$q = \begin{bmatrix} \alpha_1 & \alpha_2 & y_b & \alpha_b & P_1 & F_{cnc} \end{bmatrix}^T . \tag{22}$$

Should the values of the limit centrifugal forces are known, the bogie position can be derived from the below criteria:

- *the bogie is in the secant position for $F_{cn} < F_{cns}$*
- *the bogie is in the free position for $F_{cns} < F_{cn} < F_{cnc}$*
- *the bogie is in the chord position for $F_{cnc} < F_{cn}$.*

For a certain unbalanced centrifugal force F_{cn}, the appropriate equilibrium equation can be selected, based on the correlations above. With an iterative solution for this equation, the parameters defining the positions of wheelsets and bogie frame are obtained, as well as the leading force(s).

THE NUMERICAL APPLICATION

In reliance on the equilibrium equations and the method of finding their solution, presented in the previous section, the influence of the suspension elastic characteristics will be calculated upon the curving position of a bogie, a position that is defined by the displacements and rotations of the wheelsets and of the bogie frame. The analysis is carried out in correlation with the curve radius, velocity and the bogie axle base.

The bogie parameters being considered are as follows: $M = 23500$ kg, $k_x = k_y = 70$ MN/m, $2a = 2.56$ m, $2b = 2$ m, $2e = 1.5$ m, $2r = 0.92$ m, $Q_o = 57{,}63$ kN, $\mu = 0.36$, $\kappa = 188$.

It is ascertained that the bogie wheels have a CFR S78 wear profile, and the track has UIC60 rails. The track super-elevation has the maximum admitted value $h = 150$ mm, no matter the curve radius. Similarly, a super-elevation $I_{adm} = 70$ mm is taken into account.

It is said that the bogie moves at the maximum velocity admitted by the track parameters

$$V = 0.291\sqrt{R(h+I)} , \tag{23}$$

where for $h = 150$ mm and $I = 70$ mm, it becomes $V = 4.32\sqrt{R}$.

To display the influence of the elastic characteristics of the wheelsets steering, the results of the numerical simulations will be compared to the ones from a bogie with the same parameters, but with fixed wheelsets against the frame.

The position in which the bogie places while in a curve with radiuses between 150 ... 3000 m can be determined from figure 5. It can be noticed that, for curve radiuses of up to 260 m, the bogie with elastic steered wheelsets places in the secant position ($F_{cn} < F_{cns}$), and for radiuses higher than 260 m, in the free position ($F_{cns} < F_{cn} < F_{cnc}$) – see figure 5, a. If the bogie wheelsets are fixed against the frame (fig. 5, b) the bogie has the secant position for curves with radiuses shorter than 385 m; for a R between 385 m and 1620 m, it will have a free position. Should the curve radius is longer than 1620 m, the bogie with fixed wheelsets will be in a chord position($F_{cn} < F_{cnc}$).

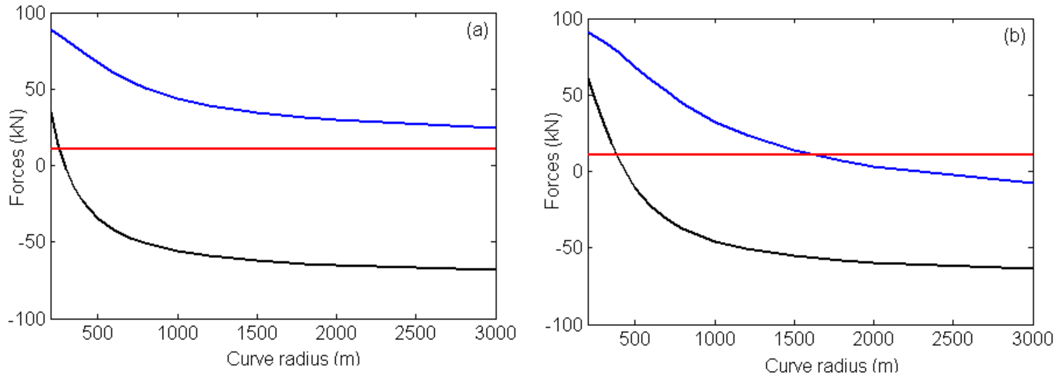

Figure 5. The unbalanced centrifugal force acting upon the bogie and the unbalanced centrifugal force for the secant and chord limit positions, (a) bogie with elastic steered wheelsets; (b) bogie with fixed wheelsets: F_{cnc}; F_{cns}; F_{cn}.

Moreover, the verification of the curving position for both bogies observes the chart in figure 6, where the leading forces P_1 and P_2 are presented while acting on the two wheelsets of the bogie. It is evident that the bogie with the elastic steered wheelsets is in the free position for radiuses between 260 m and 2250 m. For values of R higher than 2250 m, the attacking wheel of the first wheelset draws apart from the rail (P_1 < 0). The other previous observations rely on the values of the steeering forces. Thus, the secant position of the two bogies is characterized by the value of the negative values P_2 (for R between 150 and 260 m for the bogie with elastic steered wheelsets or for values of R between 150 ... 385 m – for the bogie with fixed wheelsets). The location of the bogies in a free position is made obvious by the zero values of the leading force P_2. When the bogie with fixed wheelsets reaches the chord position, P_2 has positive values for R between 1620 and 3000 m.

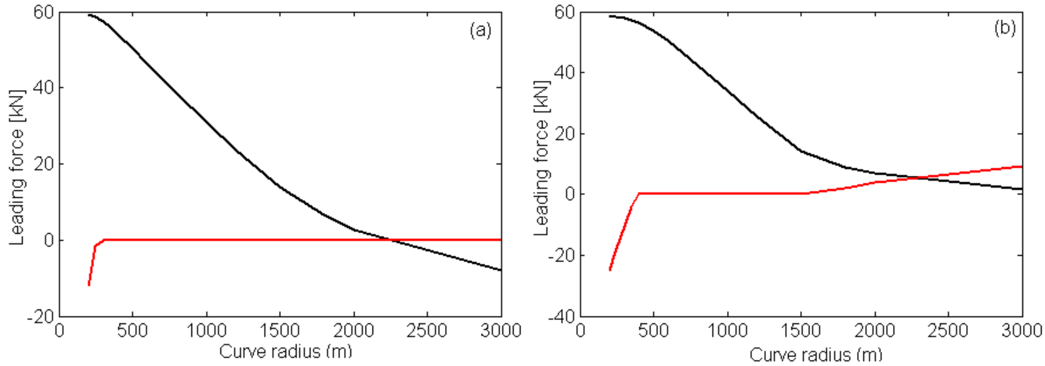

Figure 6. The leading forces, (a) bogie with steered wheelsets; (b) bogie with fixed wheelsets: P_1; P_2.

For a contrastive analysis between the two type of bogies, the numerical simulations will consider curve radiuses shorter than 2000 m, which fulfills the requirement that the attacking wheel of the leading wheelset be in contact with the flange of the outer rail, for both bogies.

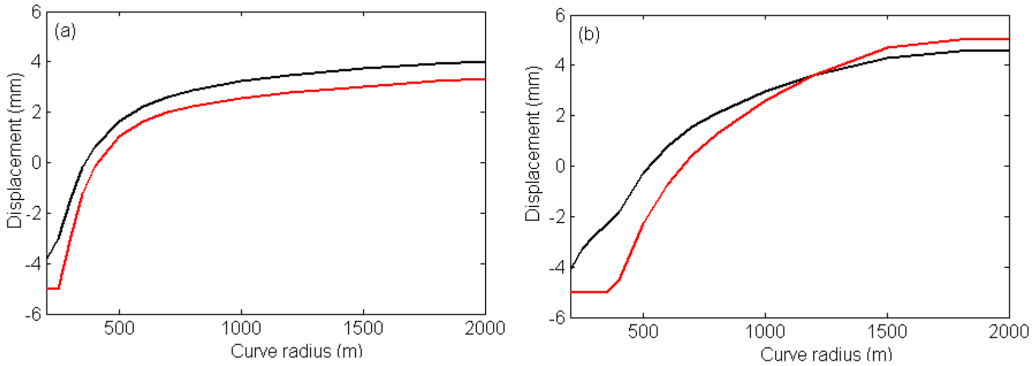

Figure 7. The influence of curve radius, (a) bogie with elastic steered wheelsets; (b) bogie with fixed wheelsets: y_b; y_2.

In figure 7, the transversal displacements of the bogie frame against its center and of the rear wheeelset center against the track axle are shown, in dependence on the curve radius. For curves with a radius shorter than 260 m, where both bogies are in the secant position, the lateral displacement to the curve interior of the wheelset center against the track axle will have the maximum value allowed by the play in the track y_2 = -5 mm. For R longer than 260 m, the bogie with elastic steered wheelsets are in a *free position during curving, and the wheelset displacement against the track axle starts lowering while the curve radius gets longer. For R = 410 m, the wheelset is not any longer displaced against the track axle (y_2 = 0), and for smaller values in the radius, the wheelset will laterally displace against the track axle towards the outer rail, with y_2 increasing along with R. The same observations are valid for the bogie with fixed wheelsets, except that it will be in the secant position for values of curve radius up to 385 m and that the displacement y_2 is zero for R = 664 m. When comparing the values derived for the two types of bogies, it can be noticed that, for an R between 150 ... 600 m, the wheelset displacement is larger for a bogie with fixed wheelsets. On the other hand, should the curve radius is within 600 - 1000 m, then the displacement of the bogie with fixed wheelsets is smaller.*

As for the displacement of the bogie frame against the track axle for the two types of bogies, the observations are similar with the displacement of the wheelsets. It should be mentioned that the center of the bogie, reduced to its longitudinal axle, is not displaced against the track axle unless the bogie with elastic steered wheelsets is curving in a curve of R = 360 m, and the one with fixed wheelsets in a curve with radius of 525 m. In terms of the magnitude of y_b at both bogies, while curving in curves with radiuses between 450 and 1200 m, the values registered for the bogie with fixed wheelsets are smaller than the one with elastic steered wheelsets.

The influence of the curve radius upon the magnitude of the rotation angles of the two wheelsets against the radial position and of the longitudinal axle center of bogie frame against the tangent position in a curve is presented in figure 8. The results derived for curves with shorter radiuses than 250 m, where both bogies are placed in a secant position, are herein detailed. It can be noticed that the rotation angles of both wheelsets decrease along with an increase in the curve radius. Similarly, for an R between 150...250 m, the angle α_1 is smaller for the bogie with fixed wheelsets and the rotation angle of the second wheelset is bigger.

In reference to the bogie with elastic steered wheelsets, an increase in the curve radius turns into a smaller rotation angle α_1. The second wheelset is in a radial position when curving in curves with radiuses of 220 or 550 m. Its rotation angle has different values, as a function of R interval. Thus, for R shorter than 220 m, α_2 lowers while the radius goes up and continues to go down after locating in a free position for R between 260 m and 550 m; for higher values of the curve radius, it will change its rotation sense and draw apart from the radial position. The rotation angle of the bogie frame lowers while the curve radius goes up, going close to zero for high values of the curve radius.

As far as the bogie with fixed wheelsets is concerned, and based on the derived results, similar conclusions with the ones for the bogie with elastic steered wheelsets can be drawn. The following mentions should be made: the first wheelset does not have a radial position, no matter the curve radius where the bogie is; the second wheelset can roll with no transversal skid in curves with R between 330 m and 675 m. For radiuses longer than 1620 m, where the bogie is in the chord position, the longitudinal axle of the bogie frame is tangent to the curve (α_b = 0) and the rotation angles of the wheelsets against the radial direction become equal and of contrary directions; in this case, the angle α_2 is smaller than the one in the second wheelset in the bogie with elastic steered wheelsets.

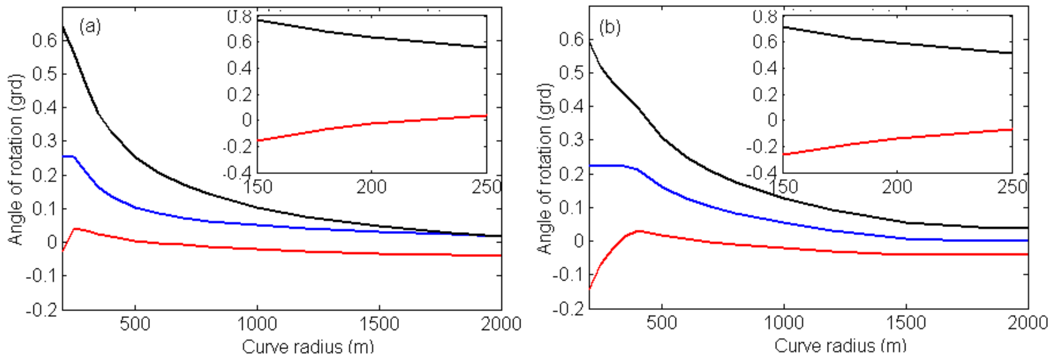

Figure 8. The influence of the curve radius, (a) bogie with elastic steered wheelsets; (b) bogie with fixed wheelsets: —— α_b; —— α_1; —— α_2.

Figure 9 shows the influence of velocity upon the quasi-static movement of the bogie while moving along a curve of a 1200 m radius; at the maximum velocity corresponding to the super-elevation insufficiency (V =150 km/h) (as seen in the figure 5 results), both the bogie with elastic steered wheelsets and the bogie with fixed wheelsets are in a free position. It is evident that the variation in velocity will not change this position. The displacement of the second wheelset and of the bogie frame against the track axle increases along with the velocity, due to the unbalanced centrifugal force that shares the same behavior.

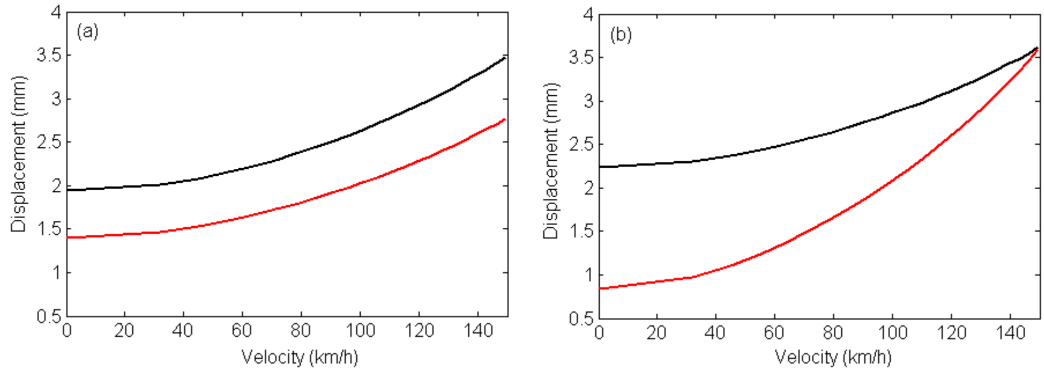

Figure 9. The influence of velocity, (a) bogie with elastic steered wheelsets; (b) bogie with fixed wheelsets: —— y_b; —— y_2 .

Upon comparing the results for the two types of bogies, it can be confirmed that the displacement of the second wheelset against the curve axle (for speeds up to circa 90 km/h), is smaller than for the bogie with fixed wheelsets. At a maximum velocity, though, y_2 is higher by 0.8 mm than in the bogie with elastic steered wheelsets. The displacement of the bogie frame is larger for the bogie with fixed wheelsets within the entire range of velocities.

As seen in figure 10, no matter what type of bogie, the first wheelset of the bogie rotates, independent on velocity, towards the outer curve, and the rotation angle against the radial position α_1 gets smaller while velocity goes up. At the latter wheelset, this rotation direction maintains itself until velocity reaches circa 108 km/h, when the wheelset rotates towards the inner curve and, at the same time, deviates from the radial position by increasing the angle α_2. It is worthwhile mentioning that, for both bogies, at V = 108 km/h, the second wheelset is able to roll with no transversal skid through its location into a radial position ($\alpha_2 = 0$). A further comparison of the two types of bogies shows that the rotation angle of the first wheelset is wider for the bogies with fixed wheelsets than in the one with elastic steered wheelsets – differences are visible at small velocities. For the rear wheelset, the differences in the two types of bogies are insignificant.

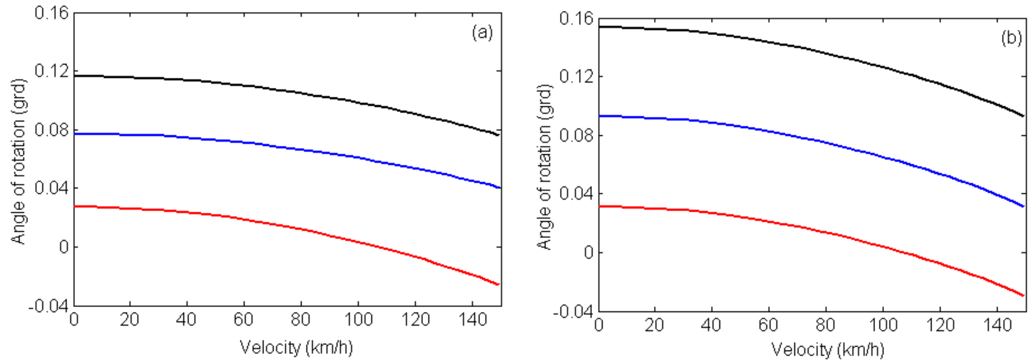

Figure 10. The influence of velocity, (a) bogie with elastic steered wheelsets; (b) bogie with fixed wheelsets: —— α_b; —— α_1; —— α_2

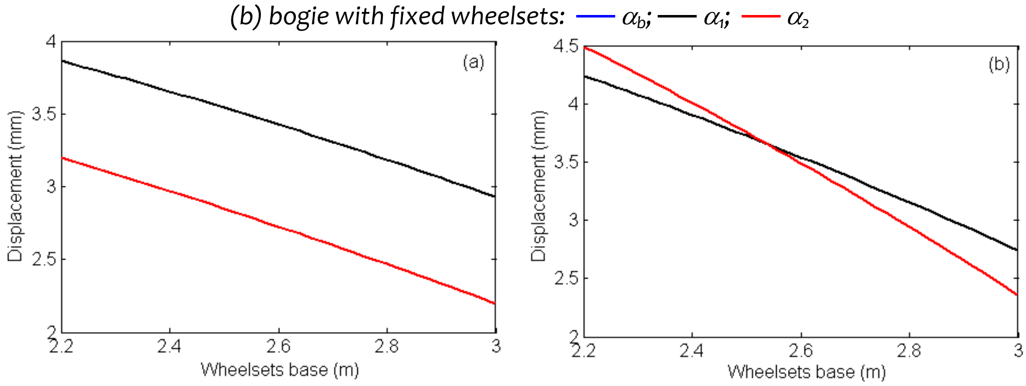

Figure 11. The influence of bogie axle base, (a) bogie with elastic steered wheelsets; (b) bogie with fixed wheelsets: —— y_b; —— y_2 .

The influence of the bogie axle base upon the lateral displacements of the bogie frame and the rear wheelset in a curve with R = 1200 m, while bogie is moving at the maximum velocity allowed by the track parameters (V = 150 km/h) is described in figure 11. As seen above (fig. 9), these movement contexts trigger higher lateral displacements y_b and y_2 for the bogie with fixed wheelsets against the frame. It can be noticed that an increase in the axle base results into smaller displacements for both *types of bogies. In comparison with the reference value of the axle base (2a = 2.56 m), if increased to 3 m, a lowering of y_2 by 0.58 mm at the bogie with steered wheelsets and by 1.23 mm for the one with fixed wheelsets. Similarly, the bogie frame will displace laterally (versus the reference context), with 0.54 mm less if the bogie has elastically steered wheelsets and 0.87 mm for the bogie with fixed wheelsets.*

In the curve with a 1200-meter radius, where both bogies are in a free position (fig. 12, a and b), the magnitude of the bogie axle base leads to an increase of the rotation angles of the two wheelsets against the radial position and of the rotation angle of the bogie frame against the tangent position to the curve.

While comparing the two types of bogies, for the bogie with fixed wheelsets it can be noticed that the rotation angles of the wheelsets against the radial position, as well as the rotation angle of the bogie frame against the tangent position to the curve are higher, no matter the magnitude of the axle base, versus the corresponding ones for the bogie with steered wheelsets. But these differences are not significant.

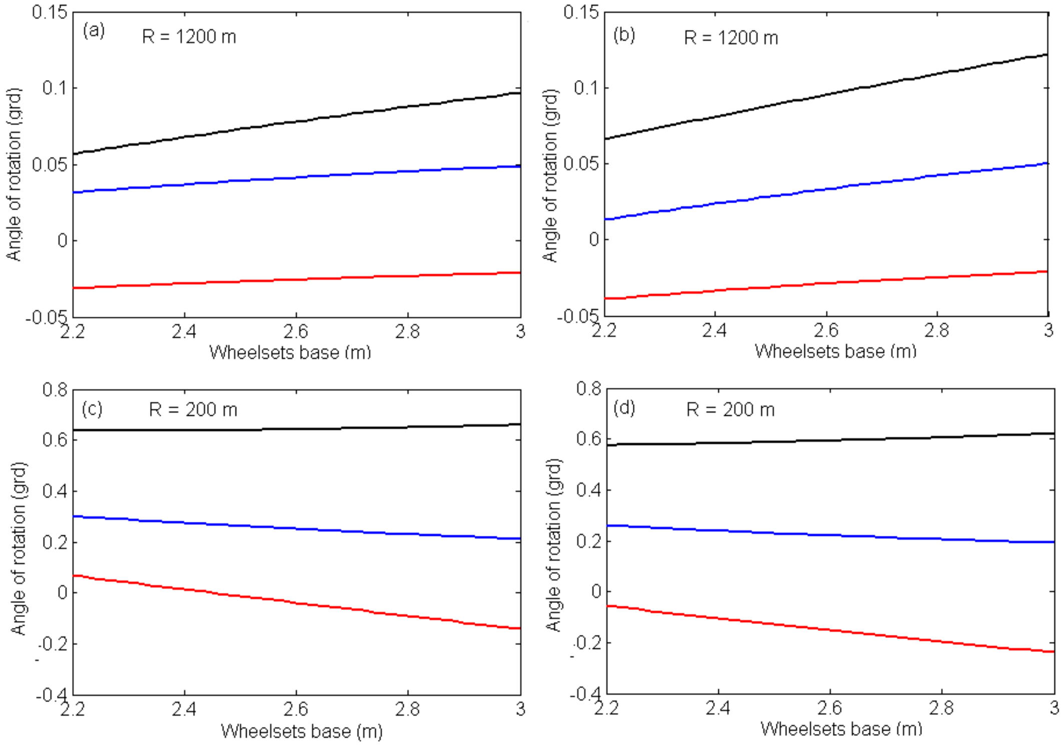

Figure 12. The influence of bogie axle base, (a), (c) bogie with elasstic steered wheelsets; (b), (d) bogie with fixed wheelsets: —— α_b; —— α_1; —— α_2.

Should the bogie is curving in a curve with a 200-meter radius, moving at the allowed maximum velocity of 61 km/h, as previously shown, this will set in a secant position (fig. 12, c and d). By increasing the axle base, for both types of bogies, the rotation angle of the first wheelset goes up with no effect. At the second wheelset, for the bogie with steered wheelsets, an increase in the axle base over the value of 2.45 mwill change its rotation direction, namely towards the inner curve.

For smaller value of 2a than the one mentioned, the rotation angle α_2 will increase and the wheelset will draw apart from the position that would provide rolling without any transversal skid. This is valid only for the 2.45-meter bogie axle base, for which α_2 becomes zero. In a further comparison of the results derived for the two types of bogies, it can be noticed that while curving in small-radius curves, irrespective of the axle base magnitude, the rotation angle of the first wheelset against the radial position and the rotation angle of the bogie frame against the tangent position in the curve are smaller for the bogie with fixed wheelsets, even though these differences are very small.

CONCLUSIONS

The study of moving in a curve in quasi-static behavior of a bogie exemplifies an important theoretical issue with application in safety against derailment, wear of the rolling surface, stability of the rolling track, etc.

This paper intoduces three non-linear aspects to be considered for the study of a two-axlebogie moving along a curve. Firstly, it is the non-linearity given by the wheelset clearence in the track, raising questions in terms of establishing the curving position of the bogie for a certain velocity.

In order to overcome this difficulty, a new approach is suggested, based on the concept of limit position. The method of limit positions, which is a solution to determine the curving position of the *bogie, starts from the idea of calculating the value of the unbalanced centrifugal force that provides the reaching of the two bogie limit positions, namely the secant and the chord positions.*

The other two non-linear aspects of the issue of moving along a curve come from the contact geometry and from the characteristic of the friction coefficient in dependence of creepage. This is the reason why the equilibrium equations for any of the bogie curving positions are non-linear.

The non-linear model has been applied for the study of moving in a curve of a bogie with elastic steered wheelsets and one with fixed wheelsets.

The numerical simulations have allowed some conclusions regarding the influence of the suspension elastic characteristics upon the curving position and of the accompanying geometrical coordinates. The analyses were made in correlation with the curve radius, velocity and the bogie axle base.

It has been shown that the performance of the bogie with elastic steered wheelsets while moving in a curve is not always superior to the one with fixed wheelsets. This thing depends on the curve radius and velocity. Thus, for the small-radius curves, the fixed wheelsets will be rolling with smaller transversal skid than the elastic steered ones. This is a result of the rotation angles compared to the radial position, smaller for the bogies with fixed wheelsets than the ones with elastically steered wheelsets.

Similarly, for certain velocity intervals, the results derived for the two types of bogies, showed insignificant differences.

By assuming the elastic characteristics of the elements of linking the wheelsets in correlation to the moving behavior, the vehicle characteristics and the track parameters can turn into improving the performance of the moving along a curve of the bogies with elasic steered wheelsets.

REFERENCES

[1.] Sebeşan, I., Mazilu, T, Vibrațiile vehiculelor feroviare (Vibrations of the railway vehicles), Matrixrom, Bucureşti, 2010.
[2.] Sebeşan, I., Dinamica vehiculelor de cale ferată (Dynamic of the railway vehicles), Ed. MatrixRom, Bucureşti, 2011.
[3.] Mazilu, T., Confortul la materialul rulant (Comfort at the rolling stock), Ed. MatrixRom, București, 2003.
[4.] Sun, Y.Q., Simson, S., Nonlinear three-dimensional wagon-track model for the investigation of rail corrugation initiation on curved track, Vehicle System Dynamics, vol. 45, 113-132, 2007.
[5.] Long, X.Z., Wei, Q.C., Zheng, F.Z., Dynamic analysis of railway transitions curves, Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, vol. 224, 1-14, 2010.
[6.] Gonzalez, F., Perez, J., Vinolas, J., Alonso, A., Use of active leading in railway bogies to reduce rail corrugation on curves, Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, vol. 221, 509-519, 2007.
[7.] Wickens, A.H., Fundamentals of Rail Vehicle Dynamics – Guidance and Stability, Swets&Zertlinger, Netherlands, 2005.
[8.] Joly, R., Étude de la stabilité transversale a d'un véhicule ferroviare circulant à grande vitesse, Rail International, 2, 1971.
[9.] Joly, R., Circulation d'un véhicule ferroviare en courbe de faible rayon – Bogie de conception classique, Bogie à essieux auto-orientée, Rail International, 4, 1988.
[10.] Joly, R., Pyrgidis, C., Circulation d'un véhicule ferroviare en courbe –Efforts de guidaje, Rail International, 1990.
[11.] Messouci, M., Steady state motion of rail vehicle with controlled creep forces on curved track, ARPN Journal of Engineering and Applied Sciences, vol.4, no.2, 2009.
[12.] Cataldi-Spinolaa, E., Glockerb, Ch., Stefanellic, R., Götschd, M., Influence of the Wheel Diameter on the Curve Squealing of Railway Vehicles, Eoronoise, Naples, 2003.
[13.] Arrus, P., de Pater, A.D., The stationary motion of a one-axle vehicle along a circular curves with real rail and wheel profiles, Vehicle System Dynamics, vol. 37, 29-58. 2002.
[14.] Kalker, J.J., On the rolling contact of two elastic bodies in the presence of dry friction, PhD Thesis, Delft Univ. of Technology, 1967.
[15.] Heumann, H., Grundzüge der Führung der Schienenfahrzeuge, Sonderdruck aus Elektriche Bahnen, Olandenburg, München, 1954.
[16.] Chartet, M., La théorie statique du déraillement d'un essieu, Revue Générale des Chemins de Fer, s. 365, 1950.