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MATHEMATICAL MODEL FOR CYLINDER PRESSURE IN A SPARK IGNITION ENGINE

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ABSTRACT: *The work is done drawing the diagram shown in coordinates "p-θ" for a spark ignition engines, Dacia, based on a mathematical model. Analytic model describing the work processes of the engine during its operation and it consist of a set of tuning parameters that all have a physical meaning. This parameters are closely connected to the ideal Otto cycle. Cylinder pressure traces contain information about the work and emission producing process which is valuable for the engine management system. For some diagnosis and control problems it would be beneficial to have information about the cylinder pressure available.*
KEYWORDS: *cylinder, model, pressure, engine, analytical, cycle, interpolation*

INTRODUCTION

In the case of the engines diagram shown in the following situation, [10]:

- *the design stage of the internal combustion engine for determining gas force with which bodies are sized motor mechanism;*
- *for existing engines in use to determine performance at regular intervals and to establish interventions need and maintenance.*

The diagram can be traces usually indicate coordinate „p–v" or „p-θ" and less common coordinate „T-θ". Parameters involved in traced this diagram characterize the work processes of the engine during its operation. The determinations of these parameters can be done through the thermal cycle of the internal combustion engine and the calculations can be done by two methods, [10]:

- *method of cvasideal cycle;*
- *method of the real cycle;*

The first method involves the calculation of the cvasi-ideal thermal cycle based on the theoretical cycle corrected calculation. It is based on simplifying assumptions. This gives some expeditives and results obtained is approximate [10].

The second method involves plotting the variation of pressure on energy balance relations, burning is describe with Vibe s law. This method is being closer to reality, [9], [10],[11] .

The work is done drawing the diagram shown in coordinates "p-θ" for a spark ignition engines, Dacia, based on a mathematical model which allows the pressure in the cylinders during its operation. Drawing the diagram based on the model will be achieved after an extensive analysis of the gas exchange process taking place in the cylinder ignition engine. This is drawn for a cylinder engine derived under discussion. In the case of the spark ignition engine combustion cycle of air and fuel mixture is achieved in a very short time and with a small change in a volume.

Gas exchange process taking place in the cylinders of internal combustion engines represented all phenomena accompanying exhaust and intake processes. They must be made so that in the cylinder is introduce as much fresh gas on amounts available and had to lose a small amount as the fresh gas cylinder exhaust gas cleaning, [9].

Cylinder pressure traces contain information about the work and emission producing process which is valuable for the engine management system. For some diagnosis and control problems it would be beneficial to have information about the cylinder pressure available, [4], [5].

Work in this direction has already been made for diesel engines in [1], where the differential equation for the cylinder temperature is reduced to a Riccati differential equation that is solved analytically, [4], [5].

MATHEMATICAL MODELING OF PROCESSES IN SPARK IGNITION ENGINES

Modeling a process requires conception, development and use of optimal combinations of assumptions and equations whose complexity depends on the nature of the phenomena studied. Based on the model obtained, the processes studied can be analyzed in detailed, [9]. Mathematical modeling process makes important contributions motor development at different stages, depending on the complexity of the mathematical model by, [9]:

- *better understanding the processes analyzed on model accuracy;*

- *predict engine behavior for a wide range of variation of structural and functional parameters of its; no need to create working models; identify trends and constraints involved in the conception and constructive energy solutions;*
- *valorification of mathematical models in design special programs to simulate complex processes in internal combustion engines;*

In this work is presented an analytical model for pressures in spark ignited engines, which is developed and validated in [4], [5] and this model was drawn shown for a spark ignition engine, Dacia.

Given that the most important functional processes in internal combustion engines are: gas exchange process, compression process, the combustion process of detente, the analytical model of this work considered these processes.

The idea behind this model is based on the observation that the ideal Otto cycle provides valuable information about the compression and expansion processes, [4], [5]. The real cycle is similar to the ideal Otto cycle and the similarities are largest early in the compression and late in the expansion. It is characterized by the compression and expansion phases which are well defined by the states of the fluid. Another idea which is find in this model is that the heat release analysis procedure based on pressure ratio management, developed by Matekunas [4],[6],[7],[8].

The model consists of a set of tuning parameters that all have a physical meaning. These parameters are closely connected to the ideal Otto cycle.

This model describes the cylinder pressure during the high pressure and combustion phase. The burn rate of the combustion has a big influence on the pressure and it is considered to be known. In the literature there are several approaches presented for estimating the burn rate which could be used directly in this model, [1], [2], [8].

Model description

The modeled pressure $p(\theta)$ is built up by two asymptotic traces and a interpolation between these. The cylinder pressure model is divided into four parts, [4], [5]:

- *the compression process is described by a politropic process. The politropic process encapsulates the heat transfer, so that there is no need to explicit include the heat transfer in the model;*
- *the expansion asymptote is also described by a politropic process and the reference point for expansion temperature and pressure is calculated using a constant volume combustion process;*
- *to describe the pressure ration is used the Vibe function;*
- *the gas exchange phase during ISA-IA the pressure is approximated by the intake manifold pressure and during the period SA-DSE the pressure is approximated by the exhaust manifold pressure. Between the phases the pressure can be determine through an interpolation using a cosine function.*

Model evaluation

The model consist of a set of parts of which the following, [4],[5]:

- *selection of compression pressure;*
- *selection of compression and expansion politropic coefficient;*
- *selection of expansion pressure;*
- *interpolation between firing and expansion.*

The analytic model of cylinder pressure is constituted of four parts as follows:

1. ISA intake valve closing time until the start of combustion IA is described by the equation:

$$p_c(\theta) = p_{ISA}\left(\frac{V_{ISA}}{V(\theta)}\right)^{k_c} \tag{1}$$

In relation (1), $V(\theta)$ reprezented the law of change in volume and is given by equation (2):

$$V(\theta)' = V_c + \frac{V_s}{2}\left(1 - \cos\theta + \frac{\lambda_b}{2}\sin^2\theta\right) \tag{2}$$

where: V(ϑ) – cylinder volume in the moment "ϑ"; V_c – volume of displacement; V_s – minimum volume of firing chambre; $\lambda_b = \frac{R_{manivelei}}{L_{bielei}}$;

2. The start of combustion until IA to SA corresponding to the end of combustion is described by ecuation (3):

$$p(\theta) = vibe(\theta)p_c(\theta) + (1 - vibe(\theta))p_D(\theta) \tag{3}$$

3. Time corresponding to the end of combustion SA until DSE which represented the moment when the exhaust is opening valve, described by ecuation (4):

$$p_D(\theta) = p_3\left(\frac{V}{V(\theta)}\right)^{k_D} \tag{4}$$

4. Interpolation until DSE and the moment when intake valve closing ISA + 720°, after which the process begins again.

The compression process can be modeled with good approximation through a process polytropic. This process is characterized by compression polytropic exponent k_c. In the modelling that takes place between the time when intake valve closing ISA and the start of combustion IA.

The equations describing the pressure and temperature in the compression process are:

$$p_c(\theta) = p_{ISA}\left(\frac{V_{ISA}}{V(\theta)}\right)^{k_c} \tag{5}$$

$$T_c(\theta) = T_{ISA}\left(\frac{V_{ISA}}{V(\theta)}\right)^{k_c - 1} \tag{6}$$

Equation (5) and (6) describe the pressure and the temperature to the point of ignition.

Combustion process in internal combustion engines, both spark ignition and compression ignition quantified by combustion law. In the case of spark ignition engine, combustion modeling process can be evaluated by a Vibe function, grade I.

The is the modeling processes using Vibe function was done in several papers in the field, [9], [11].

$$vibe(\theta) = 1 - \exp\left(-a\left(\frac{\theta - \theta_{IA}}{\theta_{SA} - \theta_{IA}}\right)^{m+1}\right) \tag{7}$$

Form factors "a" and "m" are determined according to the angles of combustion equation (8) and (9):

$$m = \frac{\ln\left(\dfrac{\ln(1-0{,}1)}{\ln(1-0{,}85)}\right)}{\ln(\Delta\theta_d) - \ln(\Delta\theta_d + \Delta\theta_b)} - 1 \tag{8}$$

$$a = -\ln(1-0{,}1)\left(\frac{\Delta\theta}{\Delta\theta_d}\right)^{m+1} \tag{9}$$

Burn duration can be calculated knowing the corresponding phase rapide angle firing noted with θ_d and the corresponding phase firing moderate angle with θ_b. This approach has been treated in works [4], [5], [6]. The relationship for determining the combustion is: $\Delta\theta \cong 2\theta_d + \theta_b$.

Relaxation process can be modeled with good approximation through a polytrope process. This process is characterized by a polytropic exponent of relaxation k_D. In the modelling this process takes place between intake valve closing time ISA and the start of combustion IA.

$$p_D(\theta) = p_3\left(\frac{V_3}{V(\theta)}\right)^{k_D} \tag{10}$$

$$T_D(\theta) = T_3\left(\frac{V_3}{V(\theta)}\right)^{k_D - 1} \tag{11}$$

The parameters p_3, V_3 and T_3 are related condition "3" for the Otto cycle. Pressure p_3 can be described by the relation:

$$p_3 = \frac{p(\theta_{IA})}{T(\theta_{IA})}\left(T(\theta_{IA} + \Delta T_{comb})\right) \tag{12}$$

where: $\Delta T_{comb} = \dfrac{(1-x_R)\cdot Q_i \cdot \eta_A(\lambda)}{\left(\lambda\left(\dfrac{m_a}{m_c}\right)_{st} + 1\right)\cdot c_v}$; $\quad c_v \cong \dfrac{0{,}287}{k_D - 1}$; $\quad \eta_A(\lambda) = 0{,}95 \min\ (1;\ 1{,}2\cdot\lambda - 0{,}2)$

where: Q_i- lower calorific value of fuel; λ- excess air coefficient; x_R-residual gas mass ratio; V(ϑ)-cylinder volume in the moment ϑ; ϑ - the crankshaft rotation angle measured from the lower dead point during gas exchange.

SOLVING THE MODEL EQUATION AND DRAWING DIAGRAMS

For trace the diagram, we considered the specific case of an ignition engine Dacia with four stroke, for which we used in calculations following dates: cylinder diameter, D = 73 mm; stroke cylinder: S = 77 mm; rod length: L = 128 mm; connecting rod length, 176 mm compression ratio: 8,5;

Admision valve: advance of opening the intake valve: 22^o;

delay intake valve closing : 62^o;

Evacuation valve: advance of opening the exhaust valve open 60^o;

delay closing the exhaust valve: 20^o.

Lower calorific value of fuel: Q_i = 43 500 kJ/kg; excess air coefficient: $\lambda = 0{,}96$ residual gas mass ratio: x_R = 0,063. Compression polytrope exponent k_c =1,34. Polytrope exponent of relaxation k_D =1,28. Definition of firing angle: $\Delta\theta_d = 3{,}75\%$ and $\Delta\theta_b = 0{,}96\%$ and start of combustion angle θ_{IA} has value 349 ^{o}RAC, [10].

Equations according to the working cycle of spark ignition engine have been solved using MATLAB programming environment and calculated data was drawn from the cylinder pressure chart „p_c" depending on the angle of rotation of the crankshaft „θ".

Crank radius value is: $R_{manivelă}$ = 38,5 mm, and the value of λ_b is: $\lambda_b = 0{,}3$.

Volume displacement spark ignition engine is : V_c = 322 dm^3, and minimum chamber volume is : V_s = 0,043 dm^3. Calculate the minimum volume of the combustion chamber spark ignition engine is the relationship: $V_s = \frac{V_c}{\varepsilon - 1}$, where ε is the compression ratio to the value 8,5. The data previously determined to rewrite the law variation of the volume which is in the form:

$$v(\theta) := 0.043 + \frac{0.322}{2}\cdot\left(1 - \cos(\theta) + \frac{0.3}{2}\cdot\sin(\theta)^2\right)$$

The cylinder pressure intake valve closing time ISA until the start of combustion, is presented in fig.1.

$$p\_isa := 1.2;\ K\_c := 1.34;\ rad := \frac{\pi}{180}$$

$$V\_isa := v(242\cdot rad),\ V\_isa = 0.298;$$

$$p\_c(\theta) := p\_isa\cdot\left(\frac{V\_isa}{v(\theta)}\right)^{K\_c}$$

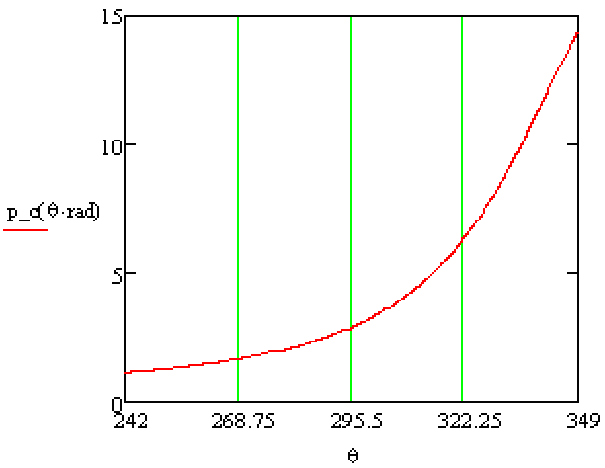

Figure 1. Graphical representation of the cylinder pressure $\theta_{ISA} < \theta < \theta_{IA}$

The cylinder pressure corresponding to the end of combustion SA until DSE, when exhaust valve opening, shown in fig.2.

$$a := 12;\ \ vibe(\theta) := 1 - e^{-a\cdot\left(\frac{\theta - \theta\_ia}{\Delta\_\theta b}\right)^{m+1}}$$

$$p\_e(\theta) := p\_3\cdot\left(\frac{v\_3}{v(\theta)}\right)^{K\_e}$$

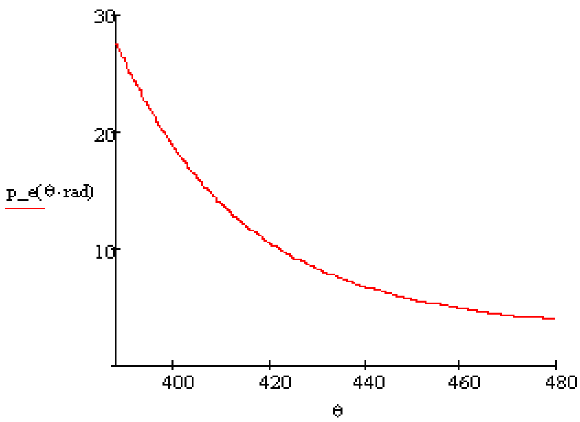

Figure 2. Graphical representation of the cylinder pressure $\theta_{SA} < \theta < \theta_{DSE}$

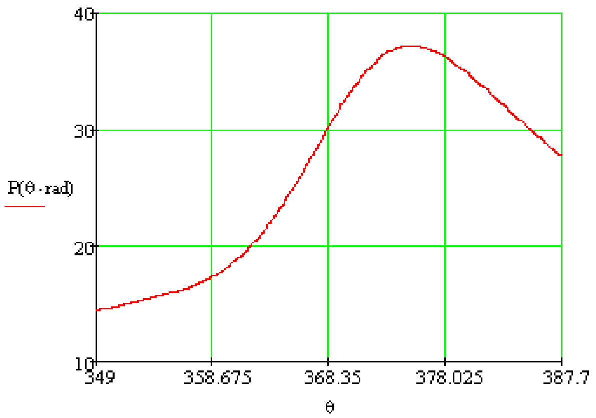

Figure 3. Graphical representation of the cylinder pressure $\theta_{IA} < \theta < \theta_{SA}$

Cylinder pressure when the firing started properly until IA corresponding to the end of combustion SA is presented in fig.3.

$$P(\theta) := [(1 - vibe(\theta))\cdot p\_c(\theta) + vibe(\theta)\cdot p\_e(\theta)]$$

$$p(\theta) := \begin{cases} p\_c(\theta) & \text{if } 242\cdot rad \le \theta < 349\cdot rad \\ [[(1 - vibe(\theta))\cdot p\_c(\theta) + vibe(\theta)\cdot p\_e(\theta)]] & \text{if } 349\cdot rad \le \theta < 387.7\cdot rad \\ p\_e(\theta) & \text{if } 387.7\cdot rad \le \theta \le 480\cdot rad \end{cases}$$

$$p\_inter(\theta) := p\_i\cdot\left(\frac{V\_i}{v(\theta)}\right)^{K\_ci}; \; p\_e\_inter(\theta) := p\_3i\cdot\left(\frac{v\_3i}{v(\theta)}\right)^{K\_ei}$$

$$p(\theta) := \begin{cases} p\_inter(\theta) & \text{if } 0 \le \theta \le 242\cdot rad \\ p\_c(\theta) & \text{if } 242\cdot rad \le \theta \le 349\cdot rad \\ [[(1 - vibe(\theta))\cdot p\_c(\theta) + vibe(\theta)\cdot p\_e(\theta)]] & \text{if } 349\cdot rad \le \theta \le 387.7\cdot rad \\ p\_e(\theta) & \text{if } 387.7\cdot rad \le \theta \le 480\cdot rad \\ p\_e\_inter(\theta) & \text{if } 480\cdot rad \le \theta \le 720\cdot rad \end{cases}$$

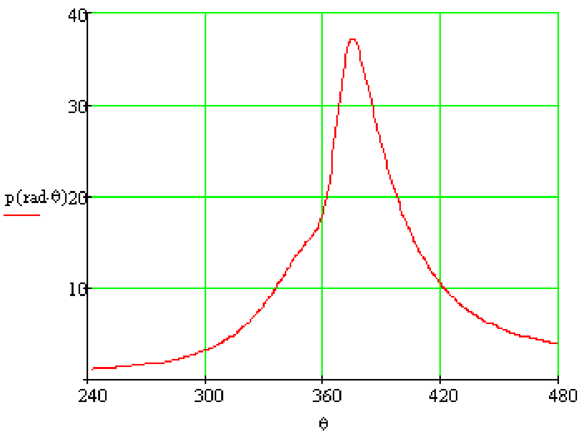

Figure 4. Graphical representation of the cylinder pressure for different values of angle „θ"

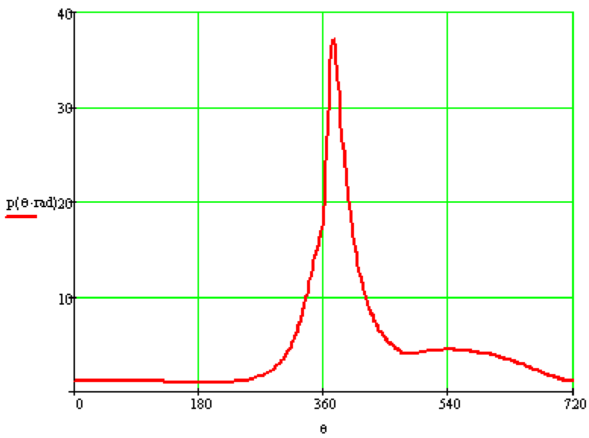

Figure 5. Graphical representation cylinder pressure for a full cycle

In fig.4 was drawn the cylinder pressure for different values of angle" θ" using diagrams in fig.1, 2 and 3. Based on the previous calculation model described in fig.5, was drawn diagram of the engine ignition using coordinates „p-θ" for a real full cycle.

CONCLUSIONS

Analytic model assess the processes occurring during a motor cycle internal combustion engine with spark ignition design which is particularly useful in providing additional information on thermal regime and engine work.

The model can be used for certain ranges of engines, so their data limit statistical variation introduced as input to be as small.

The analytical model for cylinder pressure in spark ignited engines describes the cylinder pressure developed in this. The model is expressed in closed form and the method is based on a parameterization of the ideal Otto cycles. The model is based on physical relations with components that are easy to measure and tune. The model can capture large variations in ignition timing. Cylinder pressure traces contain information about the work and emission producing process which is valuable for the engine management system.

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