ANNALS OF FACULTY ENGINEERING HUNEDOARA
– International Journal of Engineering
Tome XI (Year 2013) – FASCICULE 1 (ISSN 1584 – 2665)

[1.] Reza BABAZADEH, [2.] A. A. IMANI, [3.] Navid NEYESTANI,
[4.] D. D. GANJI, [5.] Houman B. ROKNI

AN APPROXIMATION SOLUTION OF THE 3-D HEAT LIKE EQUATION

[1.] DEPARTMENT OF CIVIL ENGINEERING, SHOMAL UNIVERSITY, AMOL, IRAN
[2.] ISLAMIC AZAD UNIVERSITY, BABOL BRANCH, BABOL, IRAN
[3.] DEPARTMENT OF ELECTRICAL ENGINEERING, SHOMAL UNIVERSITY, AMOL, IRAN
[4.] DEPARTMENT OF MECHANICAL ENGINEERING, BABOL UNIVERSITY OF TECHNOLOGY, BABOL, IRAN
[5.] UNIVERSITY OF DENVER, MECHANICAL AND MATERIALS ENGINEERING, 2390 S YORK ST., DENVER, CO, USA

ABSTRACT: There are strong and efficient techniques to find approximate solutions for the linear and nonlinear equations, that most of these equations don't have exact solution such as heat like equations. New algorithms are needed to simulate these equations. In this manuscript, the reconstruction of variational iteration method (RVIM) is applied to solve heat-like equation. This method is presented to overcome the demerit of complex calculation of other methods. Moreover the results attained in this paper compared to the exact values.
KEYWORDS: Reconstruction of Variational Iteration Method (RVIM), Heat Like Equation, Approximate solution

INTRODUCTION

As the science history in last decades indicated, the nonlinear problems are one of the most important phenomena in mathematics, physics and engineering. Efficiency of these problems show us the importance of obtaining exact or approximate solution which steel solving these problems needs better methods. Except few numbers of these problems, others do not have any exact solution and needs to be solved by approximate methods.

Researchers used new methods for this requirement such As: Adomian Decomposition Method (ADM) [1-2], Homotopy Analysis Method (HAM) [3-6], Variational Iteration Method (VIM) [7-9] Homotopy Perturbation Method (HPM) [10-13], Reconstruction of Variational Iteration Method (RVIM) [14-15], and etc. One of the well-known models of these problems is heat-like equations [16]. This model has essential role in various fields of science and engineering which is investigated widely by many researchers.

Yulita Molliq R and D.D.Ganji studied heat-like equations by means of VIM separately [17 and 18], Turgut Öziş used HPM for solving heat-like models [19], Shaher Momani researched about analytical approximate solution for fractional heat-like equations with variable coefficancients by Decomposition method [20], Abdul-Majid Wazwaz by using ADM obtained exact solutions for mentioned equations [21] and A. K. Alomari and etc. studied solutions of heat-like equations by HAM [22].

In this work we employed RVIM to solve heat-like equation. RVIM is based on Laplace-transforms in boundary condition which does not have complications of Lagrange multiplier as VIM, also does not change the equation by auxiliary parameters as Ham. At the end graphs and analytical results illuminating to us in comparison with other methods. RVIM solve the problem faster and enough accurate approximation in simple algorithm.

PROBLEM DESCRIPTION

In mathematics and engineering in order to solve the model of heat-like equations, it is considered in one-dimensional, two-dimensional and three-dimensional cases.

□ One-dimensional heat like model:

$$u_t = 1/2x^2 u_{xx} \qquad 0 < x < 1, \;\; t > 0 \tag{1}$$

□ Two-dimensional heat like model:

$$u_t = 1/2(y^2 u_{xx} + x^2 u_{yy}) \qquad x > 0, \mathrm{y} < 1, \;\; t > 0 \tag{2}$$

□ *Three-demensional heat like model; Consider the three-dimensional heat-like IBVP as,*

$$u_t = x^4 y^4 z^4 + 1/36(x^2 u_{xx} + y^2 u_{yy} + z^2 u_{zz}) \qquad x > 0,\ \ y < 1,\ \ z < 1,\ \ t > 0 \tag{3}$$

Subject to the boundary conditions and initial condition of:

$$u(0, y, z, t) = 0, \quad u(1, y, z, t) = y^4 z^4 (e^t - 1) \tag{4.1}$$

$$u(x, 0, z, t) = 0, \quad u(x, 1, z, t) = x^4 z^4 (e^t - 1) \tag{4.2}$$

$$u(x, y, 0, t) = 0, \quad u(x, y, 1, t) = x^4 y^4 (e^t - 1) \tag{4.3}$$

$$u(x, y, z, 0) = 0 \tag{5}$$

Researchers usually survey this model in one-dimensional and two- dimensional cases. But in this investigation we just considered three-dimensional equation which includes two other cases. The results obtained in solving Eq. (3) with the boundary and initial conditions shown in Eqs. (4.1 - 5) are plotted and tabulated in figures 1-3 and table 1, respectively.

DESCRIPTION OF THE METHOD

In the following section, an alternative method for finding the optimal value of the Lagrange multiplier by the use of the Laplace transform [23] will be investigated a large of problems in science and engineering involve the solution of partial differential equations. Suppose x, t are two independent variables; consider t as the principal variable and x as the secondary variable. If $u(x,t)$ *is a function of two variables x and t, when the Laplace transform is applied with t as a variable, definition of Laplace transform is*

$$L[u(x,t); s] = \int_0^\infty e^{-st} u(x,t)\,dt \tag{6}$$

We have some preliminary notations as

$$L[\frac{\partial u}{\partial t}; s] = \int_0^\infty e^{-st} \frac{\partial u}{\partial t}\,dt = sU(x,s) - u(x,0) \tag{7}$$

$$L[\frac{\partial^2 u}{\partial t^2}; s] = s^2 U(x,s) - su(x,0) - u_t(x,0) \tag{8}$$

where

$$U(x,s) = L[u(x,t); s] \tag{9}$$

We often come across functions which are not the transform of some known function, but then, they can possibly be as a product of two functions, each of which is the transform of a known function.

Thus we may be able to write the given function as $U(x,s),V(x,s)$ *where* $U(s)$ *and* $V(s)$ *are known to the transform of the functions* $U(x,t),V(x,t)$ *respectively. The convolution of* $U(x,t)$ *and* $V(x,t)$ *is written* $U(x,t)*V(x,t)$. *It is defined as the integral of the product of the two functions after one is reversed and shifted.*

Convolution Theorem: if $U(x,s),V(x,s)$ *are the Laplace transform of* $U(x,t),V(x,t)$, *when the Laplace transform is applied to t as a variable, respectively; then* $U(x,s)*V(x,s)$ *is the Laplace Transform of* $\int_0^t U(x,t-\varepsilon)V(x,\varepsilon)d\varepsilon$

$$L^{-1}[U(x,s).V(x,s)] = \int_0^t u(x,t-\varepsilon)v(x,\varepsilon)\,d\varepsilon \tag{10}$$

To facilitate our discussion of Reconstruction of Variational Iteration Method, introducing the new linear or nonlinear function $h(u(t,x)) = f(t,x) - N(u(t,x))$ *and considering the new equation, rewrite* $h(u(t,x)) = f(t,x) - N(u(t,x))$ *as*

$$L(u(t,x) = h(t,x,u)) \tag{11}$$

Now, for implementation the correctional function of VIM based on new idea of Laplace transform, applying Laplace Transform to both sides of the above equation so that we introduce artificial initial conditions to zero for main problem, then left hand side of equation after transformation is featured as

$$L[L\{u(x,t)\}] = U(x,s)P(s) \tag{12}$$

where $P(s)$ *is polynomial with the degree of the highest order derivative of linear operator.*

$$L[L[u(x,t)]] = U(x,s)P(s) = L[h\{(x,t,u)\}] \tag{13}$$

$$U(x,s) = \frac{L[h\{(x,t,u)\}]}{P(s)} \tag{14}$$

Suppose that $D(s)=\frac{1}{P(s)}$ and $L[h\{(x,t,u)\}]=H(x,s)$. Using the convolution theorem we have

$$U(x,s)=D(s).H(x,s)=L\{(d(t).h(x,t,u))\} \quad (15)$$

Taking the inverse Laplace transform on both side of Eq. (15),

$$u(x,t)=\int_0^t d(t-\varepsilon)h(x,\varepsilon,u)d\varepsilon \quad (16)$$

Thus the following reconstructed method of variational iteration formula can be obtained

$$u_{(n+1)}(x,t)=u_0(x,t)+\int_0^t d(t-\varepsilon)h(x,\varepsilon,u)d\varepsilon \quad (17)$$

And $u_0(x,t)$ is initial solution with or without unknown parameters. In absence of unknown parameters, $u_0(x,t)$ should satisfy initial boundary conditions.

APPLICATION OF THE PROPOSED METHOD

The mentioned method (RVIM) is able to solve a wide range of linear and nonlinear equations. In this paper we illustrated basic concepts of Reconstructed of Variational Iteration Method [23] as it seen in following we concentrated on solution of Eq. (3) with the boundary conditions Eqs. (4.1 – 4.3) that is placed in nonlinear equations classes.

At first rewrite Eq. (3) based on selective linear operator as

$$Lu(x,y,z,t)=u_t=\overbrace{((x^4y^4z^4+1/36(x^2u_{xx}+y^2u_{yy}+z^2u_{zz}))}^{h(u,t,x,y,z)} \quad (18)$$

Now Laplace Transform is implemented with respect to independent variable x on both sides of Eq. (18) and by using the new artificial initial conditions (which all of them are zero) we have

$$sU(t)=L\{h(u,t,x,y,z)\} \quad (19)$$

$$U(t)=\frac{L\{h(t,u)\}}{s} \quad (20)$$

and where as Laplace inverse Transform is as follows $\frac{1}{s}$

$$L^{-1}\left[\frac{1}{s}\right]=1 \quad (21)$$

By using the Laplace inverse Transform and convolution theorem, it is concluded that

$$u(t,x,y,z)=\int_0^t (t-\varepsilon)h(u,\varepsilon)d\varepsilon \quad (22)$$

Hence, we arrive at the following iterative formula for the approximate solution of (3) subject to the initial condition,

$$u_{n+1}(t,x,y,z)=u_0(x)+\int_0^x (x-E)\left[\left(x^4y^4z^4+\frac{1}{36}(x^2u_{n_{xx}}+y^2u_{n_{yy}}+z^2u_{n_{zz}}\right)\right]dE \quad (23)$$

According to above equation, for first order approximation we have:

$$u_1(t,x,y,z)=u_0(t,x,y,z)+\int_0^x (x-E)[(x^4y^4z^4+1/36(x^2u_{0_{xx}}+y^2u_{0_{yy}}+z^2u_{0_{zz}})]dE \quad (24)$$

Assuming the initial approximation as below, we have the following approximate solutions,

$$u_0(t,x,y,z)=0$$

$$u_1(t,x,y,z)=x^4y^4z^4t$$

$$u_2(t,x,y,z)=\frac{1}{2}x^4y^4z^4t^2+x^4y^4z^4t$$

$$u_3(t,x,y,z)=\frac{1}{6}x^4y^4z^4t^3+\frac{1}{2}x^4y^4z^4t^2+x^4y^4z^4t$$

By the iteration formula (23) we can calculate other values of u (such as u_4, u_5 …). We obtained results for u_8 and u_{10} to different values of x, z, y and t. it is observed when iteration goes up our accuracy increases. (Figure 1 to Figure 5).

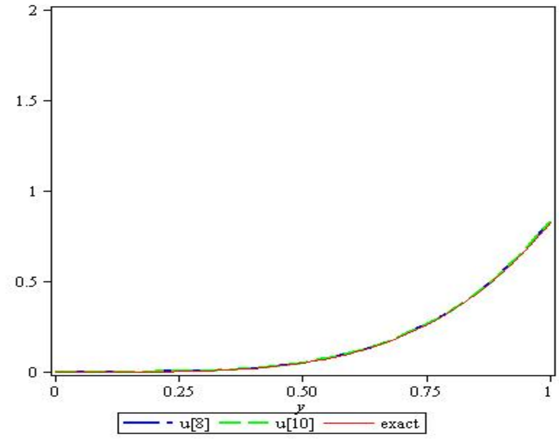

Figure 1. Comparison between the results obtained by RVIM for u_8, u_{10} *with exact solution at x=2, z=0.3 and t =2 in propagation of y*

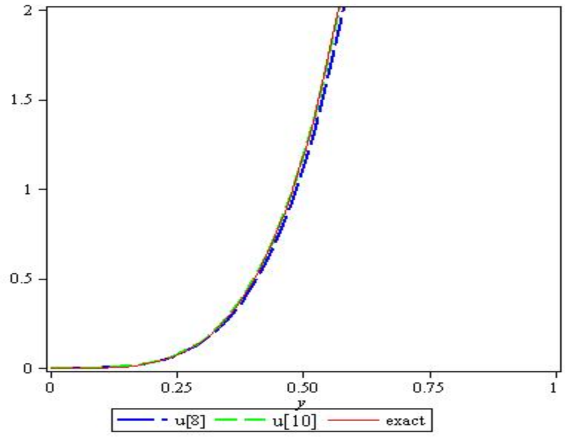

Figure 2. Comparison between the results obtained by RVIM for u_8, u_{10} *with exact solution at x=2, z=0.3 and t =5 in propagation of y*

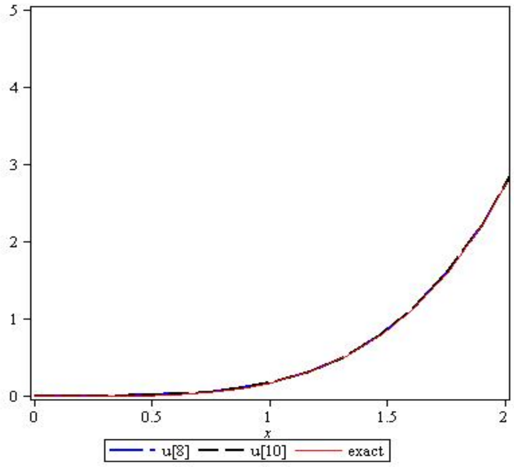

Figure 3. Comparison between the results obtained by RVIM for u_8, u_{10} *with exact solution at y=07, z=0.8 and t =1 in propagation of x*

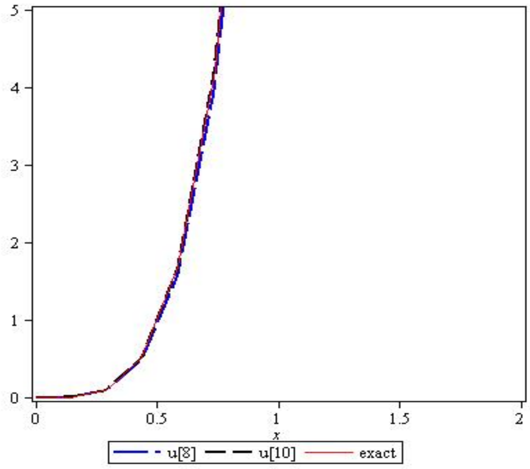

Figure 4. Comparison between the results obtained by RVIM for u_8, u_{10} *with exact solution at y=07, z=0.8 and t =5 in propagation of x*

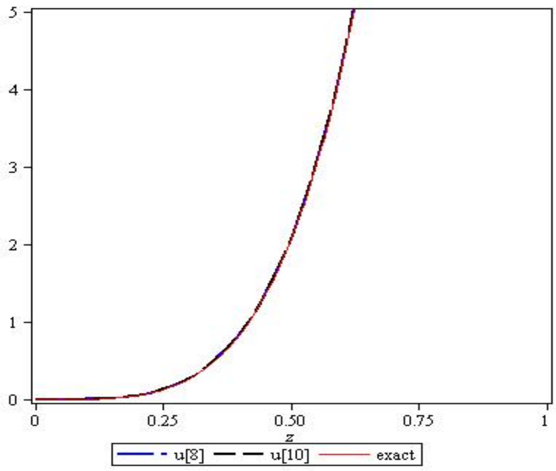

Figure 5. Comparison between the results obtained by RVIM for u_8, u_{10} *with exact solution at x=3, y=0.7 and t =1 in propagation of z*

CONCLUSIONS

In this investigation, Reconstruction of Variational Iteration Method (RVIM) has been successfully applied to solve heat-like models. Also in this work, we used maple package advantages for our calculations. High accuracy, simplicity and efficiency are main advantages of RVIM in comparison with other methods. Moreover, it reduces the size of calculations which gives this method wider applicability. All of these reasons show us RVIM is a reliable and useful method.

REFERENCES

[1.] G. Adomian, Solving Frontier Problems of Physics: The Composition Method, Kluwer, Boston, (1994).

[2.] J. I. Ramos, Piecewise-Adaptive Decomposition Methods, Chaos, Solitons and Fractals, 198(1), (2008), 92.

[3.] F. Khani, M. Ahmadzadeh Raji, and H. Hamedi Nejad, Analytical Solutions and Efficiency of the Nonlinear Fin Problem with Temperature-Dependent Thermal Conductivity and Heat Transfer Coefficient, Commun. Nonlinear Sci. Numer. Simul., 14, (2009), 3327-3338.

[4.] A. Molabahrami, F. Khani, the Homotopy Analysis Method to Solve the Burgers-Huxley Equation, Nonlinear Anal. RWA, 10(2), (2009), 589-600.

[5.] M. Sajid, I. Ahmad, T. Hayat, and M. Ayub, Unsteady Flow and Heat Transfer of a Second Grade Fluid over a Stretching Sheet, Commun. Nonlinear Sci. Numer. Simul., 14(1), (2009), 96-108.

[6.] F. Khani, M. Ahmadzadeh Raji, and S. Hamedi-Nezhad, A Series Solution of the Fin Problem with a Temperature-Dependent Thermal Conductivity, Commun. Nonlinear Sci. Numer. Simul., 14, (2009), 3007-3017.

[7.] J. H. He, Variational Iteration Method: A Kind of Nonlinear Analytical Technique: Some Example, International Journal of NonLinear Mechanics, 34(4), (1999), 699.

[8.] D. D.Ganji, Hafez Tari, and H. Babazadeh, the Application of He's Variational Iteration Method to Nonlinear Equations Arising in Heat Transfer, Physics Letters A, 363(3), (2007), 213.

[9.] M. Rafei, D. D.Ganji, H. Daniali, and H. Pashaei, The Variational Iteration Method for Nonlinear Oscillators with Discontinuities, Journal of Soundand Vibration, 305, (2007), 614.

[10.] D. D. Ganji, Houman B. Rokni, M. G. Sfahani, and S. S. Ganji, Approximate Traveling Wave Solutions for Coupled Shallow Water, Advances in Engineering Software, 41, (2010), 956-961.

[11.] M. Rafei, and D. D. Ganji, Explicit Solutions of Helmholtz Equation and Fifth-Order KdV Equation Using Homotopy Perturbation Method, International J. of Nonlinear Sci. and Numerical Simulation, 7(3), (2006), 321.

[12.] M. Dehghan, and J. Manafian, The Solution of the Variable Coefficients Fourth-Order Parabolic Partial Differential Equations by the Homotopy Perturbation Method, Zeitschriftfuer Natur for Schung, A64a, (2009), 411.

[13.] F. Shakeri, and M. Dehghan, Solution of Delay Differential Equations via a Homotopy Perturbation Method, Mathematical and Computer Modelling, 48, (2008), 486.

[14.] R. A. Usmani, "On the Numerical Integration of a Boundary Value Problem Involving a Fourth Order Linear Differential Equation," BIT, 17(2), (1977), 227-234.

[15.] R. A. Usmani, An O_h6 Finite Difference Analogue for the Solution of Some Differential Equations Occurring in Plate Deflection Theory, Journal of the Institute of Mathematics and Its Applications, 20(3), (1977), 331-333.

[16.] Cannon, John Rozier (1984), The One-Dimensional Heat Equation, Encyclopedia of Mathematics and Its Applications, 23 (1st ed.), Reading-Menlo Park-London-Don Mills–Sidney–Tokyo/ Cambridge-New York-New Rochelle-Melbourne-Sidney: Addison-Wesley Publishing Company/Cambridge University Press.

[17.] Yulita Molliq R., M.S.M. Noorani, I. Hashim "Variational iteration method for fractional heat- and Wave-like equations", Nonlinear Analysis: Real World Applications 10 (2009) 1854–1869.

[18.] A Sadighi, D D Ganji, M Gorji, N Tolou "Numerical Simulation of Heat-like Models with Variable Coefficients by the Variational Iteration Method", Journal of Physics: Conference Series 96 (2008) 012083.

[19.] Turgut Ozis, Deniz A gırseven," He's homotopy perturbation method for solving heat-like and wave-like equations With variable coefficients", Physics Letters A 372 (2008) 5944–5950.

[20.] Shaher Momani, Analytical approximate solution for Fractional heat-like and wave-like equations With variable coefficients using The decomposition method, Applied Mathematics and Computation 165 (2005) 459–472.

[21.] Abdul-Majid Wazwaz, Alice Gorguis," Exact solutions for heat-like and Wave-like equations with variable coefficients", Applied Mathematics and Computation 149 (2004) 15–29.

[22.] A. K. Alomari, M. S. M. Noorani and R. Nazar," Solutions of Heat-Like and Wave-Like Equations with Variable Coefficients by Means of the Homotopy Analysis Method", Chinese Phys. Lett. 25,(2008), 589.

[23.] A.A. Imani, D.D. Ganji, Houman B. Rokni, H. Latifizadeh, Esmail Hesameddini, M. Hadi Rafiee, Approximate traveling wave solution for shallow water wave equation, Applied Mathematical Modelling, 36 (2012) 1550–1557.