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# PRACTICAL EXPONENTIAL TRACKING OF DIGITAL SYSTEMS

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ABSTRACT: In this paper we consider practical exponential tracking of nonlinear time-invariant digital systems. The definition of practical exponential tracking is introduced. Based on the definition we give and prove criterion and control algorithm that ensures practical exponential tracking. Simulations of the proposed control algorithm are performed, where a manipulator with three rotational joints (three DOF) is used as an object. The results of the simulation verify the proposed theory.
KEYWORDS: practical tracking, exponential tracking, tracking criteria and control algorithms

## INTRODUCTION

For many technical objects it is typical that their desired dynamic behavior is time varying and that the disturbances act upon them. In this case, the task of the control is to ensure a required closeness between the actual and desired behavior of the controlled plant (object). Briefly, the control has to ensure a kind of tracking, even in circumstances, when disturbances act on the object.

There exist different concepts of tracking: the absolute tracking concept (established by Grujić and Porter, [2]), the Lyapunov tracking concept (introduced by Grujić in the references [5] through [10] of the paper [3]) and practical tracking concept (defined also by Grujić, [3]). Besides these concepts, there exists $k^{th}$ order tracking [5], which incorporates the previous tracking concepts.

Practical tracking was introduced by Grujić, firstly for the continuous-time systems [3], and then for the discrete-time systems, [4]. Later, this concept was developed in papers [6], [7], given by the same author. Further contributions to the theory of practical tracking were given in [8] for continuous systems, in [10,12] for digital systems and in [11] for hybrid systems.

Tracking in the sense of Lyapunov requires the existence of a $\Delta$ neighborhood of an initial desired output $y_{d0}$, such that for each initial output $y_0$ from that neighborhood, the real object output $y(t)$ converges to the desired output $y_d(t)$, as time increases infinitely, or mathematically

$$\exists\Delta > 0 \therefore \left\|y_{d0} - y_0\right\| < \Delta \Rightarrow \lim_{t\to\infty}\left\|y_d(t) - y(t)\right\| = 0.$$

This is the classical, widely known and used tracking property called asymptotic tracking, or simply, tracking.

Different from the Lyapunov tracking concept, the practical tracking concept takes into account all technical and construction constraints of a real object, as well as the object behavior over a prespecified (possibly finite) time interval. This concept starts with the following prespecified (or to be determined) sets of output errors (which are connected neighborhoods of the zero error value): the set of initial errors $E_I$, the set of actual errors $E_A$ and the set of final errors $E_F$. Based on these sets and the set of desired outputs $S_{yd}$ we calculate the appropriate sets of the admitted real outputs $Y_I(t)$, $Y_A(t)$ and $Y_F(t)$. Now, the practical tracking is achieved if there exist control $u(t)$ that to transfer the system real output $y(t)$ from a set of initial outputs $Y_I(t)$ to a set of final outputs $Y_F(t)$, during the predefined or to be determined, time $\tau$, $(t \in [0,\tau[), \tau \in \mathrm{R}^+$, so that the system real output must not leave the set of the permitted instantaneous actual outputs $Y_A(t)$. At the same time, disturbances and controls should belong to the, in advance, permitted and realizable sets, $S_d$ and $S_u$, respectively.

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*Depending on the desired change of the instantaneous output errors $e(t)$ and their convergence from the initial to the final values, we distinguish several types of practical tracking, [10,11,12]. If that change is exponential, then it is the practical exponential tracking.*

## NOTATION

- $B \in R^{n\times m}, m \le n$; *a matrix describing the transmission of control action,*
- $e[k;e_0;y_d(\cdot),u(\cdot),z(\cdot)] = e(k) = e_k \in R^r$, $r \le m$, $e_k = y_d(k) - y(k)$; *the time evolutions of the output error vector* $e$ *related to* $e_0 = e(0) \in R^r$, $u(k)$, $z(k)$, $y_d(k)$ *at the time* $k \in Z_n$,
- $E_{(\cdot)} \in R^r$, $_{(\cdot)}=_{I,A}$; *the sets of all admitted* $e_k$ *(closed connected neighborhood of zero error* $0_e$*) on the time sets* $\{0\}$ *and* $Z_n$ *respectively,*
- $e_{m(\cdot)} = \min\{e : e \in E_{(\cdot)}\}$ *and* $e_{M(\cdot)} = \max\{e : e \in E_{(\cdot)}\}$, $_{(\cdot)}=_{I,A}$; *minimum and maximum respectively, taken for each element of the error vector* $e$, *so that, for example* $e_{(\cdot)I} = \left(e_{1(\cdot)I}\ e_{2(\cdot)I} \cdots e_{r(\cdot)I}\right)^T$, $_{(\cdot)}=_{m,M}$ *and for each ith,* $i \in [1,r]$, *component* $e_{imI} \le e_{i0} \le e_{iMI}$ *hold,*
- $k \in Z_n$; *the discrete time, the real time is* $t = kT$, $T = t_{k+1} - t_k$ *is the sample period. At the initial moment* $k = k_0 = 0$,
- $s(y) = [sign(y_1),\ldots,sign(y_r)]^T$; *the vector function, the elements of which are signs of the components of the output vector* $y(\cdot)$,
- $S_{yd}, S_u$ and $S_z$; *the sets of all accepted desired outputs* $y_d(\cdot)$, *realizable controls* $u(\cdot)$ *and permitted disturbances* $z(\cdot)$ *over the time set* $Z_n$, *respectively,*
- $u(\cdot) \in R^m$, $u_m \le u(\cdot) \le u_M$; *the control vector,* $u_m$, $u_M$ *are the minimum and the maximum of admitted control* $u(\cdot)$ *over the time set* $Z_n$, *respectively,*
- $v(\cdot)$; *the vector function from the Lurie class of functions* $v(\cdot) \in N(L)$, *so that next conditions are satisfied:* $(i)$ $v(\cdot)$ *is continuous on* $R$, $(ii)$ $v(0)=0$ *and* $(iii)$ $\frac{v(\zeta)}{\zeta} \in L$, $L \in [L_1, \Lambda L_1]$, *where are:* $L_1 = diag\{l_{11}\, l_{21} \cdots l_{r1}\}$, $\Lambda L_1 = L_2 = diag\{l_{12}\, l_{22} \cdots l_{r2}\}$ *and* $\Lambda = diag\{\alpha_1\, \alpha_2 \cdots \alpha_r\}$; $\alpha_i \ge 1$ *so that for every* $i = 1,2,\cdots,r$ *are valid* $l_{i1} \le l_{i2} \le \infty$,
- $y[k;y_0;y_d(\cdot),u(\cdot),z(\cdot)] = y(k) = y_k \in R^r$; *the real output response, which is, at the instant* $k \in Z_n$, *equal to the real output vector at the same time,*
- $Y_{(\cdot)}(k) = Y_{(\cdot)}[k;y_d(k);E_{(\cdot)}] = \{y : y(k) = y_d(k) - e(k), e(k) \in E_{(\cdot)}\}$, $_{(\cdot)}=_{I,A}$; *the set functions of all admitted vector functions* $y$ *with respect to* $y_d$ *and* $E_{(\cdot)}$, $_{(\cdot)}=_{I,A}$ *on the appropriate time sets* $\{0\}, Z_n$, *respectively,*
- $z(\cdot): Z_n \to R^p$; *the disturbance vector function defined on the time set* $Z_n$,
- $Z_n = [0, n_p[, n_p \in N$; *the discrete time set,* $n_p$ *is the discrete time up to which tracking is realized*
- $\beta, \gamma \in R^+$ *and next are valid:* $1 < \beta < \infty$ *and* $\beta \le \gamma < \infty$
- $\Gamma = diag\left\{\frac{\gamma_1 - 1}{\gamma_1} \ldots \frac{\gamma_r - 1}{\gamma_r}\right\}$;
- $\mathbf{1} = (1\ \ 1\ \ \ldots\ \ 1)^T$; *the unity vector of appropriate dimension*

## PROBLEM STATEMENT

*In this paper we consider a digital system (called plant), which consists of an object together with all sensors and actuators, and whose mathematical model is described by a vector difference equation as*[1]

$$\left.\begin{aligned} &f[x(k), x(k+1), \ldots, x(k+\alpha), z(k)] = Bb[u(k)] \\ &y(k) = g[x(k), z(k)], \end{aligned}\right\} \qquad (1)$$

*where* $x \in R^n, z \in R^p, u \in R^m, y \in R^r$ *are the state, disturbance, input and output vectors, respectively. The vector functions* $f : R^{(\alpha+1)\times n} \times R^p \to R^n$, $g : R^n \times R^p \to R^r$ *and* $b : R^m \to R^m$,

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[1]*it is easy to prove that the system could be expressed in usual form: state equation and output equation, [10]*

*describe the system internal dynamics, output and control function, respectively. These functions satisfy the usual smoothness properties.*

*Firstly, we define perfect tracking and give an assumption.*

***Definition 1** (Perfect tracking [4]). The plant (1) exhibits perfect tracking if and only if* $y(k)=y_d(k)$ *is satisfied for every* $k\in Z_n$ .

***Assumption 1**: There exist matrices* $C\in \mathrm{R}^{m\times n}$ *and* $F\in \mathrm{R}^{r\times m}$ *such that are* $\det(CB)\neq 0$ *and* $\det(FF^T)\neq 0$.

*This means that the matrices* $B$ *and* $C$ *have the full rank,* $rankB=rankC=\min(m,n)=m\leq n$ [2]. *The requirement expresses the necessary condition for a simultaneous independent control of m different output variables.*

*From the above definition, the necessary condition for perfect tracking is* $y_0=y_{d0}$ . *In this case the corresponding control function, obtained from (1), is determined by*

$$b[u_N(k)]=(CB)^{-1}C[f(x_N(k),x_N(k+1),\ldots,x_N(k+\alpha),z_N(k))],$$

*where the index* $_N$ *denotes nominal values of state, disturbance and control vectors,* $x_N,z_N,u_N$ , *respectively.*

*However, tracking is not perfect as soon as* $y_0\neq y_{d0}$, *or equivalently* $e_0\neq 0_e$, *i.e. desired and actual output in the initial moment (k=0) are not the same. Therefore it is necessary to correct the previous nominal control law. Evidently, the correction should be related to the instantaneous error* $e(k)=y_d(k)-y(k)$, *such that control at* $k$ *th instant of time becomes*

$$b[u(k)]=(CB)^{-1}C\cdot f[x(k),x(k+1),\ldots,x(k+\alpha),z(k)]+F^T(FF^T)^{-1}\cdot p[e(k)],$$

*where a vector function* $p(\cdot)$ *denotes this correction, and the matrix* $F\in \mathrm{R}^{r\times m}$ *is such that* $\det(FF^T)\neq 0$ [3]. *The vector function* $p\in \mathrm{R}^r$ , *has the same dimension as the output vector* $y(\cdot)$ *and depends on the error* $e(\cdot)$ *(and/or on its derivatives and/or on its integral). The matrix* $F$ *adjusts dimension of the above equation.*

*By selecting the appropriate function* $p[e(k)]$, *the manner of the output error change (from the initial to the final value) is determined, and, consequently, required quality of tracking also. In this paper is given an algorithm that ensures exponential change of the error from the* $e(0)\in E_I$ *to the* $e(n_p)\in E_A=E_F$ .

*In order for the plant (1) to accomplish practical exponential tracking the following assumptions must be satisfied:*

***Assumption 2:** All the components of the output vector* $y(k)$ *and of the disturbance vector* $z(k)$ *are measurable at every instant* $k\in Z_n$ .

***Assumption 3:** Each component of the state vector* $x(k)$ *is measurable*[4] *or could be calculated as* $x(k)=g^I[y(k),z(k)]$. *The components of the state vector* $x(k+i)$, $i=1,\cdots,\alpha$ *are known for all* $k\in Z_n$

***Assumption 4**: The vector functions: of the internal dynamic* $f(\cdot)$ , *of the output* $g(\cdot)$ *and of the control* $b(\cdot)$ *are well defined. There exist the inverse function* $b^I(\cdot)$ *of the vector function* $b(\cdot)$ *related to* $u(k)$ *and it is unique, i.e.* $u(k)\equiv b^I[b(u(k))]$ .

## *DEFINITION OF PRACTICAL EXPONENTIAL TRACKING*

***Definition 2** The plant (1) controlled by* $u(\cdot)\in S_u$ *exhibits practical exponential tracking with respect to* $\{n_p,\Lambda,\beta,Y_I(\cdot),Y_A(\cdot),S_{yd},S_z\}$, *if and only if for every* $[y_d(\cdot),z(\cdot)]\in S_{yd}\times S_z$ *there exists a control* $u(\cdot)\in S_u$ *such that* $y_0\in Y_I(y_{d0};E_I)$ *implies*

$$y[k;y_0;y_d(\cdot),u(\cdot),z(\cdot)]\in Y_A(k),\ \forall k\in Z_n, \tag{2}$$

[2] *for each real system condition m ≤ n is always satisfied, or other words, the number inputs is never greater than the number of states.*

[3] *this is possible due to* $r\leq m$ , *so that is rank* $(FF^T)=r$

[4] *if all components of vector* $x(k)$ *are measurable, then measurability of the output vector* $y(k)$ *in the assumption 3 may be omitted, because* $y(k)=g[x(k),z(k)]$

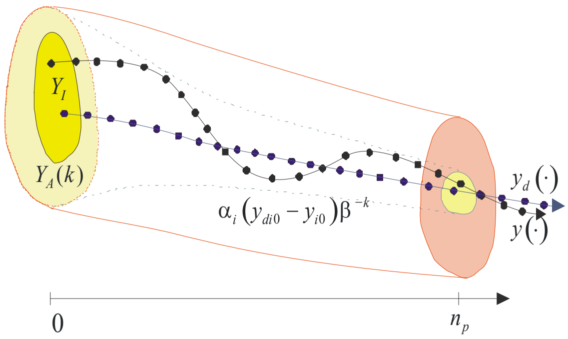

Figure 1. Practical exponential tracking

and that for every $[i,k]\in\{1,2,\cdots,r\}\times Z_n$ the following[5]:

$$y_i(k)\geq y_{di}(k)-\alpha_i(y_{di0}-y_{i0})\beta^{-k},\ y_{di0}\geq y_{i0}, \quad (3)$$

$$y_i(k)\leq y_{di}(k)-\alpha_i(y_{di0}-y_{i0})\beta^{-k},\ y_{di0}\leq y_{i0}, \quad (4)$$

hold.

The definition of practical exponential tracking for continuous-time system firstly[6] is given in [8], while the definition of practical exponential tracking for digital systems (see Figure 1) is initially given in [10]. Both definitions are given with respect to the output vector space.

## CRITERION AND ALGORITHM

In the criterion and algorithm, which follow below, we will use a vector functions $v(\cdot)$, that belong to a class of functions $V$, often called "aggregate functions" [1]. These functions can, but need not, be in general Lyapunov functions. Herein, they are not Lyapunov functions because they belong to the Lurie class of functions[7] $N(L)$, that are defined in Section 2.

## Lemma

**Lemma 1:** Let the discrete time system be given by a scalar difference equation

$$x_{k+1}=x_k-\frac{\delta-1}{\delta}x_k,\ x_k\in\mathrm{R},\ k\in Z_n,\ \delta\in[1,+\infty[. \quad (5)$$

The motion $x(\cdot;k_0;x_0)$ of the system (5) is unique and continuous in $x_0$ through every $(k_0,x_0)\in Z_n\times\mathrm{R}$ and it is determined by

$$x(k;k_0,x_0)=x_0\delta^{-(k-k_0)}. \quad (6)$$

Proof of lemma 1: Let be $x_k=c\lambda^k$; $c,\lambda\in\mathrm{R}$; $c\neq 0,\lambda>0$, than it follows that is $x_{k+1}=c\lambda^{k+1}$. Substituting both to the equation (5), we obtain

$$c\lambda^k\left(\lambda-1+\frac{\delta-1}{\delta}\right)=0\Rightarrow\lambda=\delta^{-1},$$

so that is $x_k=c\delta^{-k}$, where $c$ is unknown constant. This constant we determine using initial conditions. If $k=k_0$ then it is $x_k=x_0$, so that is $c=\delta^{k_0}$. Thus, the solution of the equation (5) is

$$x_k=x(k;k_0,x_0)=x_0\,\delta^{-(k-k_0)}.$$

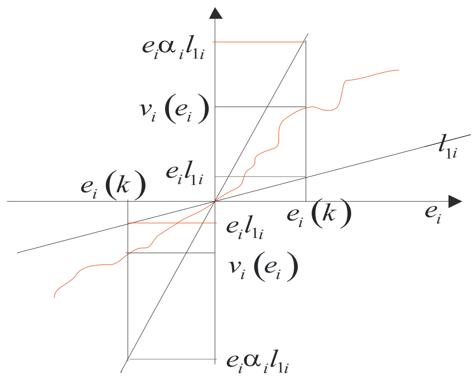

Figure 2. Functions from a class V

Therefore, the above lemma is proved.

## Criterion

**Theorem 1:** In order for the plant (1) controlled by $u(\cdot)\in S_u$ to exhibit practical exponential tracking with respect to $\{n_p,\Lambda,\beta,Y_I(\cdot),Y_A(\cdot),S_{yd},S_z\}$ it is sufficient that for the function $v(\cdot)$, $v(\cdot)\in N(L)$, $L\in[L_1,\Lambda L_1]$, the control $u(\cdot)$ ensures[8]:

$$\Delta v[e(k)]=-\Gamma v[e(k)],\ \forall[k,e_0,y_d(\cdot),z(\cdot)]\in Z_n\times E_I\times S_{yd}\times S_z, \quad (7)$$

and that for every $i\in\{1,2,\cdots,r\}$

$$\frac{e_{imA}}{e_{imI}}\leq\alpha_i\leq\frac{e_{iMA}}{e_{iMI}}. \quad (8)$$

holds.

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[5]by $y_i(k)$ is denoted $y_i[k;y_{i0};y_d(\cdot),u(\cdot),z(\cdot)]$

[6]definition of exponential tracking is the first given in [9], but for other types of tracking

[7]some of functions that belong to the Lurie class are: $\sin(\cdot)$, $(\cdot)\in[-\pi/2,\pi/2]$, $\sinh(\cdot)$, $\tanh(\cdot)$, $|(\cdot)|^{1/3}\,sign(\cdot)$, $(\cdot)^3$, $\cdots$ where is $(\cdot)\in\mathrm{R}$

[8]in the next equations by $\Delta$ is denoted first finite difference and $e(k)=e(k,e_o,y_d,u(\cdot),z(\cdot))$

***Proof of Theorem 1:*** *Let us observe the behavior of the system (7) for arbitrary values of the desired output vector* $y_d(\cdot)\in S_{yd}$ *, of the disturbance vector* $z(\cdot)\in S_z$ *, of the initial error vector* $e_0 \in E_I$ *and for an arbitrary component* $i, i\in\{1,2,\dots,r\}$ *. Now, according to the definition of the matrix* $\Gamma$ *, the vector equation (7) can be expressed in the scalar form, for the* $i$ *th component , as*

$$\Delta v_i[e_i(k)] = -\frac{\gamma_i - 1}{\gamma_i} v_i[e_i(k)]. \tag{9}$$

*The solution of this equation, according to the Lemma 1, becomes*

$$v_i[e_i(k)] = v_{i0}\gamma_i^{-k}. \tag{10}$$

*Further, we analyze the behavior of the system (7) for various values of the initial error* $e_{i0}$ *and for* $\forall k \in Z_n$ *. First we consider the case when it is* $e_{i0}\ge 0$ *:*

*Since, for the Lurie class of the functions* $v(\cdot)\in \overline{N(L)}$ *the following is valid (see Figure 2),*

$$l_{1i}e_i \le v_i(e_i) \le \alpha_i l_{1i} e_i , \tag{11}$$

*what together with* $\beta \le \gamma_i < \infty$ *(see definitions in Section 2.) and (10) gives*

$$e_i(k) \le e_{i0}\alpha_i\gamma_i^{-k} \le e_{i0}\alpha_i\beta^{-k}. \tag{12}$$

*Since* $e_{i0}\le e_{iMI}$ *is true, then the above equation becomes*

$$e_i(k) \le e_{iMI}\alpha_i\beta^{-k}. \tag{13}$$

*Based on the equations (8) and (13) we find*

$$e_i(k) \le e_{iMA}\beta^{-k};\quad \forall k\in Z_n. \tag{14}$$

*This equation expresses exponential decreasing of the error* $e_i(\cdot)$ *from the initial value* $e_{i0}$ *to the zero value, consequently the error* $e_i(k)$ *remains into the set* $E_{iA}$ *. According to the definition of the set* $Y_A$ *, that means*

$$y[k; y_0; y_d(\cdot), u(\cdot), z(\cdot)] \in Y_A(k),\ \forall k\in Z_n. \tag{15}$$

*Therefore, the first condition of the Definition 2 is satisfied.*

*Now, using the equation (12), and replacing* $e_i(k)$ *and* $e_{i0}$ *with* $y_{di}(k) - y_i(k)$ *and* $y_{di0} - y_{i0}$ *, respectively, we get*

$$y_i(k) \ge y_{di}(k) - \alpha_i(y_{di0} - y_{i0})\beta^{-k},\ y_{di0}\ge y_{i0}. \tag{16}$$

*In the similar way, for the case* $e_{i0}\le 0$ *, we obtain*

$$\overline{y[k; y_0; y_d(\cdot), u(\cdot), z(\cdot)]} \in Y_A(k),\ \forall k\in Z_n, \tag{17}$$

*and*

$$y_i(k) \le y_{di}(k) - \alpha_i(y_{di0} - y_{i0})\beta^{-k},\ y_{di0}\le y_{i0}. \tag{18}$$

*Thus, we found out that the equations (15), (16), (17) and (18) are valid for an arbitrarily chosen* $[e_{i0}, i, y_d(\cdot), z(\cdot)] \in E_{Ii}\times\{1,2,\cdots,r\}\times S_{yd}\times S_z$ *, and consequently for each mentioned value. Accordingly, we may finally conclude that the plant (1) exhibits practical exponential tracking in the sense of the Definition 2. Therefore, the theorem is proved.*

## ***Algorithm***

***Theorem 2:*** *Let the assumptions (1-4) hold, and let the set* $S_u = \{u(\cdot): u_m\mathbf{1}\le u(k)\le u_M\mathbf{1}\}$*, with the control function*

$$b[u(k)] = (CB)^{-1}C\cdot f[x(k),\cdots,x(k+\alpha),z(k)] + F^T(FF^T)^{-1}\{\Delta v[e(k-1)] + \Gamma v[e(k-1)]\},$$
$$\forall[k, e_0, y_d(\cdot), z(\cdot)] \in Z_n\times E_I\times S_{yd}\times S_z, \tag{19}$$

*where is* $v(\cdot)\in N(L),\quad L\in[L_1, \Lambda L_1]$ *.*

*The plant (1) controlled by* $u(\cdot)\in S_u$ *exhibits practical exponential tracking with respect to* $\{n_p, \Lambda, \beta, Y_I(\cdot), Y_A(\cdot), S_{yd}, S_z\}$ *if for every* $i\in\{1,2,\cdots,r\}$

$$\frac{e_{imA}}{e_{imI}} \le \alpha_i \le \frac{e_{iMA}}{e_{iMI}}. \tag{20}$$

*holds.*

***Proof of Theorem 2:*** *Multiplying the first equation of the system (1) by the matrix* $(CB)^{-1}C$ *from left-hand side, we get*

$$b[u(k)] = (CB)^{-1} C \cdot f[x(k), \cdots, x(k+\alpha), z(k)]. \tag{21}$$

*Subtracting this equation from the equation (19) we obtain*

$$F^T \left(FF^T\right)^{-1} \{\Delta v[e(k-1)] + \Gamma\, v[e(k-1)]\} = 0. \tag{22}$$

*After multiplying the above equation by matrix* $F$ *from the left-hand side and shifting it by one sampling period we have*

$$\Delta v[e(k)] = -\Gamma\, v[e(k)],\ \forall [k, e_0, y_d(\cdot), z(\cdot)] \in Z_n \times E_I \times S_{yd} \times S_z. \tag{23}$$

*This equation together with the condition (20) and the proof of the Theorem 1 proves this Theorem.*

## SIMULATION RESULTS

*For simulation of control algorithm that is proposed in Theorem 2, we use the manipulator with three rotating joints (three DOF), see Figure 3. Based on the technical features and the desired output behavior we adopt next values:*

- *the time of tracking* $\tau = 1.5\,\mathrm{sec}$; *the sample period* $T = 10^{-3}\,\mathrm{sec}$ *(it is selected by using Shenon's Theorem and linearized model of the manipulator). Based on this time we define discrete time* $n_p$ *and the time set* $Z_n$ *as:* $n_p = 1500$, $Z_n = [0, n_p[$,
- *the desired dynamical behavior of the output is determined by the set*

$$S_{yd} = \left\{ y_d \ :\ y_d(t) = \begin{pmatrix} y_{1d} = 0.9\cos(\frac{\pi t}{4})e^{-0.1t} \\ y_{2d} = 1.2\sin(\frac{\pi t}{4})e^{-0.1t} \\ y_{3d} = 0.8 + 0.2t \end{pmatrix} \right\}$$

- *the desired quality of tracking over selected time set* $Z_n$ *is determined by the initial* $E_I$ *and the actual* $E_A$ *output error sets as:*

$$E_I = E_A = \left\{ e \ :\ \begin{pmatrix} -0.10 \\ -0.08 \\ -0.05 \end{pmatrix} \le e \le \begin{pmatrix} 0.10 \\ 0.08 \\ 0.05 \end{pmatrix} \right\}$$

*where* $e_0 = (-0.08 \quad 0.06 \quad 0.04)^T$ *is selected initial output error vector*[9].

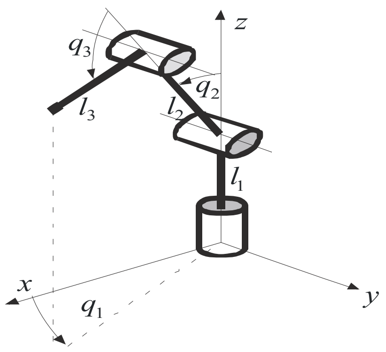

*Figure 3. Manipulator with three rotating joints*

- *the sets of admitted disturbances* $S_z$ *and realizable controls* $S_u$ *are given as:*

$$S_z = \left\{ z \ :\ z(t) = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \right\} [N]$$

*where components of the vector* $z$ *are:*

$$z_1 = \begin{cases} 0 & , \quad t \le 0.5 \\ -20 & , \quad 0.5 < t \le 1 \\ -5 & , \quad t > 1 \end{cases}$$

$$z_2 = -50 e^{-3t} \cos(3t)\, sign\,[\sin(10t)]$$

$$z_3 = -10 + 20\sin(\frac{\pi}{2} t)$$

*and the set* $S_u$ *is*

$$S_u = \{u \ : -160 \le u(t) \le 160\} [Nm]$$

- *the manner of the exponential changes of the output error vector (from the initial value toward*

[9]*all dimension of the output errors are in* $[m]$

*the zero value) is determined by the values of the next coefficients:* $\alpha_1 = \alpha_2 = \alpha_3 = 1$ *and* $\beta = 1.002$ *. Based on these values we select coefficients* $\gamma_i$ *, as:* $\gamma_1 = \beta, \quad \gamma_2 = 1.02\beta, \; \gamma_3 = 1.01\beta$ *. Further, using these data we calculate matrices* $\Lambda$ *and* $\Gamma$ *as:*

$$\Lambda = diag\{1 \quad 1 \quad 1\} \; and$$

$$\Gamma = diag\{0.0020 \quad 0.0216 \quad 0.0119\}.$$

*in this example matrix* $B = diag(40 \quad 40 \quad 20)$ *, the matrix* $F$ *we determine (see [10] and its references) as* $F = [J(q)A(q)^{-1}B]^{-1}$ *, where matrix* $J(q)$ *is Jacobian and* $A(q)$ *is matrix of inertia. The vector* $q = (q_1 \quad q_2 \quad q_3)^T$ *has components* $q_i, \, i = 1,2,3$ *, where* $q_i$ *are free rotation angles of the joints. Also, in the above proposed algorithm as aggregate function* $v \in V$ *, for each component of the output error vector, we use function* $v = \sqrt[3]{|(\cdot)|}sign(\cdot)$ *so that is* $v_i[e_i(k)] = sign(e_i(k))\sqrt[3]{|e_i(k)|}$ *.*

*The results of the simulation of the proposed algorithm in Theorem 2, by using above selected data, are given in Figure 4.*

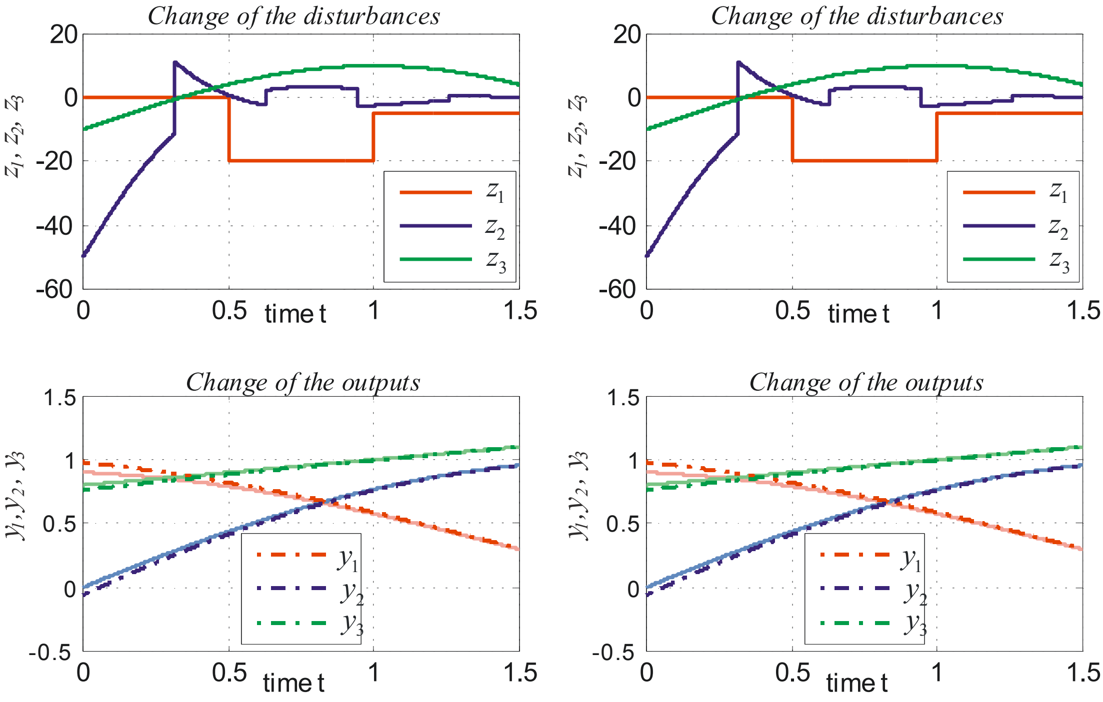


*Figure 4. Results of simulations of the proposed algoritm in Theorem 2*

## *CONCLUSIONS*

*In the paper we consider practical tracking of nonlinear time-invariant digital system. We give and prove the criterion and the control algorithm that ensure practical exponential tracking. The tracking properties are realized with respect to the prespecified sets of the times, of the permitted outputs and of the errors, of the admitted disturbances and of the realizable controls. The controls are synthesized using a digital computer which plays a role of a controller, and using negative output feedback principle, also.*

*From the results of the simulation, we see that the control which is based on the proposed algorithm forces the plant to exhibit practical exponential tracking and verifies the above proposed theory.*

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